Assignment II: Due Friday Oct. 24th

You can hand it in by slipping it under my office door (MP1210). You can collaborate as a group to work out the problems but you should write up the answers independently. Let me know if you need help, after you have made a serious attempt at these problems.

1. Feast & Whitelock (1997, MNRAS, 291, 683) derived the galactic rotation curve using 220 Cepheids that have kinematic information from the Hipparcos catalogue. Hipparcos data allowed them to include Cepheids between 6 and 12 kpc away from the Galactic center, a big improvement in radial extent over previous works.

   (a) The theoretical basis for their work is their equation (6), where the Oort constants (A, B) are related to the measured proper motions along the galactic longitude. Derive their equation (6).

   (b) Solar peculiar motion appears in this relation because the observed proper motions of the Cepheids are caused both by the solar motion and the rotation of the Cepheids around the Galaxy.[1] The solar (U_0, V_0, W_0) values they adopted differ from that obtained in Dehnen & Binney (1998, MNRAS, 298, 387), also using Hipparcos data. In particular, they used $V_0 \approx 12$km/s as opposed to $V_0 \approx 5$ km/s. Consider the influence on the Oort constant determinations if V_0 is indeed much lower.

   (c) Using their Oort values, Feast & Whitelock derived a local rotation angular velocity of $\Omega_0 = 27.2 \pm 0.9$ km/s/kpc. This contrasts with a more modern determination (using radio observations) of $\Omega_0 = 30.3 \pm 0.9$ km/s/kpc (Reid et al, 2009, ApJ, 700, 137). However, Reid et al used the Dehnen & Binney values of $V_0 \approx 5$ km/s for their calculations. Their rotation velocity will decrease if they adopt the more modern value of $V_0 \approx 12$ km/s (Schonrich, Binney & Dehnen, 2010, MNRAS, 403, 1829, which, perhaps coincidentally, agrees with the old value adopted by Feast & Whitelock). How much would it decrease?

2. Pinning the local standard of rest is of utmost importance in determining the structure of our galaxy (an example in the last problem). Currently the situation is not yet settled, with many papers still adopting (the likely erroneous) values from Dehnen & Binney (1998, hereafter DB98). We turn now to investigate the claim by Schonrich, Binney & Dehnen (2010, hereafter SBD) that $V_0 \approx 12$ km/s. This problem is really a research project under disguise, so don't get discouraged if you don't get any obvious/clear answer.

   (a) BT equation (4.228) is the basis for determining the V_0 value. Follow through the derivation that leads to this expression. For different relaxed stellar populations, the square bracket in that equation takes essentially identical values, so that a plot of the negative mean heliocentric azimuthal velocity (excuse the obtruse definition) $v_{\rm LSR} - < v_\phi > \equiv v_a + V_0$ (where $v_{\rm LSR}$ is the LSR rotation velocity and the angle brackets

[1] The Cepheids are assumed to have small velocity dispersion themselves, or at least one that should average out.

denote averaging, v_a the rotational lag) rises linearly with σ_R^2 and yields the value of V_0 as $\sigma_R^2 \to 0$. In Fig. 4 of Dehnen & Binney (1998), data, however, deviate from a straight line at the smallest σ_R^2 bin. This was suspicious enough. SBD explains this deviation as due to the presence of two populations of stars (metal poor old stars and metal rich new stars) at the same B-V.

To convince yourself that this could be relevant, use the following toy model: let there be two populations of stars with exponential radial scale lengths 2 kpc (young) and 4 kpc (old), respectively; both satisfy velocity dispersions $\sigma_R \equiv \sqrt{< v_R^2 >} \sim 1.5\sigma_\phi$ (BT eq. 3.100) and with σ_R^2 falling off with radius exponentially like the stellar density; assume at the solar neighbourhood, the young population satisfy $\sigma_R = 10$ km/s and the old ones satisfy 40 km/s; let both have negligible $< v_R v_z >$. Take $R = 8$ kpc and $v_c = 220$ km/s. Observations are analyzed by binning stars of the same (B-V) color. If we assume that young stars make up 100% of the bluest stars (say, $B - V = -0.2$), and old stars 100% of the reddest stars (say, $B - V = 0.8$), and a smooth transition from blue to red, how do the measured σ_R^2 and $v_a + V_0$ values depend on (B-V)? how wrong can the value of V_0 be when one naively extrapolates the $v_a + V_0$ curve as a linear function of σ_R^2? You can experiment with different transition forms till you (hopefully) get something like the green squares in Fig. 3 of SBD.

(b) Both the blue crosses in Fig. 3 of SBD, as well as Fig. 4 in Reid et al (2009, using masers in star forming regions, http://adsabs.harvard.edu/abs/2009ApJ...700..137R), show that at small velocity dispersions, $v_a + V_0$ actually turns upward with decreasing σ_R^2, hitting the y-axis at 20 km/s. So the rotational lag decreases for intermediate aged stars before increasing again for older stars. Assume these metal rich young stars are indeed on nearly circular orbits. What could possibly explain the upturn, even if you have to raise V_0 to 20 km/s? e.g., can you tweak your above toy-model in any way to get this upturn?

(c) One of the main results of Reid et al is that star forming regions seem to rotate slower than the galaxy by some 15 km/s (see their Fig. 3), based on the old $V_0 = 5$ km/s value. If this is true, something is needed to support these regions against the galactic gravity. They argued: ”One explanation for this finding is that HMSFRs are born in elliptical Galactic orbits, near apocenter, with orbital eccentricity of about 0.06.” Use epicycle ideas to explain why 'apocenter' and why $e \approx 0.06$. What do you think of their claim?

(d) To explain the upturn in $v_a + V_0$, one possibility seems to be me to be a spiral arm. A spiral arm is the compression region in a self-gravitating density wave. New stars are almost exclusively born in spiral arms. The Sun is currently betwen two major spiral arms, Perseus and Scutum-Centaurus. The local velocity dispersion can be affected by a spiral arm, e.g., the non-axisymmetric potential of the spiral arm can tilt the epicycles from being symmetric about the radial/tangential axis. This tends to suppress the difference between $< v_R^2 >$ and σ_ϕ^2. How is the measured $< v_\phi >$ affected? What about $< v_R v_\phi >$? Does the measurement of $< v_R v_\phi >$, reported in DB98, support this hypothesis?

(e) If someone is willing to derive a relationship between $< v_R v_\phi >$ and $< v_\phi >$, s/he could publish the explanation and it will be a useful contribution.