IV. RADIATION GAS DYNAMICS

9. Radiatively Driven Winds from Hot Stars

References

- Castor, J. I. 1974, *MNRAS*, **169**, 279
- Castor, J. I., Abbott, D. C., & Klein, R. I. 1975, *ApJ*, **195**, 157

It has been known for a long time that massive stars on the upper main sequence lose mass at a remarkably high rate. For example, Of stars exhibit emission lines in their spectra, indicating a circumstellar envelope. Also, Wolf-Rayet stars appear to be objects that have lost part or all of their H-rich envelopes during their main sequence lifetimes of a few million years. This implies mass-loss rates of the order of $10^{-5}\,\mathrm{M_\odot\,yr^{-1}}$.

However, radiation pressure due to continuum electron scattering is not enough to overcome gravity and drive a stellar wind. That is, the luminosity of an O star is *below* the Eddington limit by a factor in the range 2-20.

Lucy and Solomon (1970, *ApJ*, **159**, 879) investigated the driving due to resonance line absorption by metals such as C III which dominates in hot atmospheres. But they found the radiation pressure arising from absorption by these lines to be inadequate by a factor of about 100. In fact, the resonance lines are less effective than the continuum in driving a wind.

In 1974, John Castor at JILA made a big break-through when he discovered that *all* the metal absorption lines – not just the strong resonance ones arising from the ground state – contribute equally to the driving. This increased the driving force by at least a factor of 100, enough to lift the radiation pressure above the acceleration of gravity. In particular, the absorption by the C III lines alone is enough to drive a wind to a terminal velocity of $1500\,\mathrm{km\,s^{-1}}$ and lead to a mass-loss rate of $6\times10^{-6}\,\mathrm{M_\odot\,yr^{-1}}$, consistent with observation.

9.1 Radiation Force Due to Metal Lines

In his 1974 paper, Castor showed that in both static and rapidly expanding atmospheres, the force due to an optically thick line is proportional to the line strength but inversely proportional (approximately) to the line optical depth and, therefore, independent of the line strength overall. This is a remarkable result because for the prevalent ion of an element like carbon (namely, C III) there are *thousands* of lines. In fact, among the first 80 terms or so of this ion, there are about 900 lines with significant equivalent width. The oscillator strengths were known to within about 25% back in 1974 which was good enough for computing a wind model. (The oscillator strengths are probably not much better known even today!)

According to Castor, the force per unit mass due to a single line is given roughly by

$$f_{\mathrm{L}} = \frac{1}{c}\,\kappa_{\mathrm{L}}\,F_c\,\Delta\nu_{\mathrm{D}}\,\min(1, 1/\tau_{\mathrm{L}})$$

where $\Delta\nu_{\mathrm{D}} \equiv \nu_o v_{th}/c$ is the Doppler width of the line. This formula is a rather crude approximation which reproduces satisfactorily Castor's non-LTE line transfer results for both static and expanding atmospheres. One just has to choose the appropriate expression for the line optical depth $\tau_{\rm L}$ in either case. The non-LTE analysis is too lengthy and involved to reproduce here. We shall rely on this approximate formula as did Castor *et al* in their 1975 paper which develops the wind model. Note that the formula indicates that the flux at the line frequency is roughly $F_{\rm L} \sim F_c/\tau_{\rm L}$ $(\tau_{\rm L} \gg 1)$. This doesn't really work in all circumstances but it is good enough to yield the approximate radiation force especially in an expanding atmosphere or wind.

For a static envelope,

$$\tau_{\rm L} = \int_r^\infty \kappa_{\rm L}\,\rho\,dr$$

whereas in an expanding one

$$\tau_{\rm L} = \kappa_{\rm L}\,\rho\,v_{th}\left|\frac{dv}{dr}\right|^{-1} = \kappa_{\rm L}\,\rho\,\Delta r_{\rm abs}$$

where the thermal velocity v_{th} enters from the Doppler width. Also, $\Delta r_{\rm abs}$ is the dimension of the absorbing region. This expression is reminiscent of the velocity gradient method that we considered recently in connection with emission lines in molecular clouds. In both cases, a large velocity gradient reduces the optical depth in the line and, therefore, increases the line flux. In the current problem, a steep velocity gradient increases the driving force responsible for the wind.

It is useful to introduce a continuum optical depth τ defined by

$$\tau \equiv \frac{\sigma_e}{\kappa_{\rm L}}\,\tau_{\rm L} = \beta\,\tau_{\rm L} \quad \text{(say)}.$$

The total radiation force due to an ensemble of absorption lines is obtained by simply summing over all the lines; thus,

$$f_{\rm rad}(\tau) = \sum_{\rm lines} f_{\rm L}(\tau) = \frac{1}{c}\sum_{\rm lines}\int_0^\infty \kappa_{\rm L}(\tau)\,F_{\rm L}(\tau),d\nu = \frac{\sigma_e F}{c}\,M(\tau)$$

where

$$F = \int_0^\infty F_c(\nu)\,d\nu \qquad \text{and} \qquad M(\tau) \equiv \sum_{\rm lines}\frac{F_c}{F}\,\Delta\nu_{\rm D}\min\left(\frac{1}{\beta},\frac{1}{\tau}\right) .$$

Note that $\sigma_e F/c$ is the continuum force due to electron scattering and $M(\tau)$ can be considered the "force multiplier" due to the lines.

Near the photosphere, $\tau \simeq 1$ and, therefore,

$$M(\tau = 1) = \sum_{\rm lines}\frac{F_c}{F}\,\Delta\nu_{\rm D} \equiv \text{fraction of the continuum blocked by lines.}$$

This fraction is small – no larger than 0.01. Thus, the lines have no effect at large (continuum) optical depths. But note that $M(\tau) \propto \tau^{-1}$ and so the radiation force can become very large at *small* optical depths like 10^{-5} or 10^{-6}. In this limit, the $1/\beta$ factor applies, making

$$M(\tau \ll 1) \simeq \sum_{\rm lines}\frac{F_c\Delta\nu_{\rm D}}{F\sigma_e}\,\kappa_{\rm L} = \frac{F_c\Delta\nu_{\rm D}}{F}\sum_{\rm lines}\frac{\kappa_{\rm L}}{\sigma_e} \sim 1000\,!$$

This value of $M_{\rm max}$ should be compared with the typical ratio of the radiation force for electron scattering alone and gravity; for an O star, one finds

$$\Gamma \equiv \frac{f_{\rm rad}}{f_{\rm grav}} = \frac{\sigma_e L}{4\pi G M c} = 0.05 \;\text{ to }\; 0.5\,.$$

Thus, the force due to absorption lines can exceed gravity by two orders of magnitude at small optical depths in an O star even if the atmosphere is static. This means that *a static atmosphere is not possible* for O stars; there is no mechanism that can prevent the ejection of the outer layers of the star.

Castor *et al* computed the force multiplier $M(\tau)$ for the C III spectrum. They assumed LTE population numbers, a black-body continuum, and a C III abundance relative to hydrogen of 10^{-3}. This is roughly equivalent to the total abundance of C, N, and O and, therefore, using this abundance for C III is effectively a quick and dirty way of including all the CNO absorption lines. They felt compelled to take this approach because the oscillator strengths of the O and N lines are not as well known as those of carbon (at least in 1974).

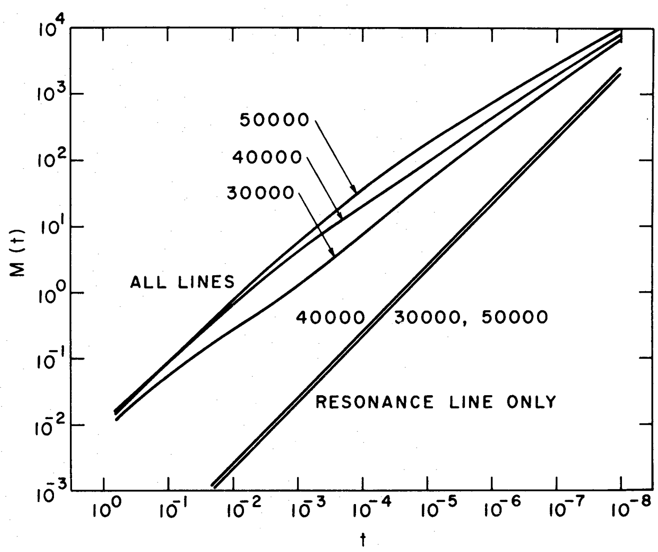

Figure 9.1 – Factor M giving the increase in the radiation force due to the lines as a function of optical depth in the continuum ($t \equiv \tau$). The curves labeled "all lines" were computed using 900 multiplets of C III; the curves "resonance line only" included just the C III resonance line $\lambda 977$. The multiplier was computed for three stellar temperatures T_e as indicated in the figure.

The values of $M(\tau)$ shown in the above figure do span the range that we estimated on the basis of evaluations at the two extremes in τ. It is interesting that the variation is almost a straight line in a log-log plot, indicating a power-law dependence on optical depth. But the slope is not -1 as you might expect because there are other quantities in the expression for $M(\tau)$ that also depend on τ. Specifically,

$$M(\tau) \simeq k\tau^{-\alpha} \quad \longrightarrow \quad \frac{1}{30}\tau^{-0.7} \quad (T_\star = 40000\,°\mathrm{K})\,.$$

The figure also shows the multiplier that would apply if only the principal resonance line of C III is allowed to contribute. Its force is about 100 times smaller than that of all the optically thick lines of C III. This explains why the Lucy & Solomon models had negligible mass loss.

Castor and company acknowledge that the factor k cannot be evaluated any more accurately than a factor 2 due to the uncertainty in the number and strength of relevant metal lines.

9.2 Castor's Stellar Wind Model

Let's suppose that the stellar envelope can be described by a steady radial flow with the temperature profile specified in advance. The forces to be included here are (i) gravity, (ii) gas pressure, (iii) continuum radiation pressure (electron scattering), and (iv) line absorption by C III ions. The justification for specifying $T(r)$ in advance is that the radiative equilibrium timescale in the flow should be shorter than the time required to move a significant distance except at large radii where the density is extremely low. In principle, the steady wind density distribution will differ from the initial hydrostatic equilibrium $\rho(r)$; this will require a modified equilibrium $T(r)$. The authors claim that only one iteration is necessary to get the correct temperature profile.

The relevant equations are

$\boxed{1}$ *mass conservation*:

$$\frac{dM}{dt} = 4\pi r^2 \rho v = \text{constant} = \dot{M}\,;$$

$\boxed{2}$ *momentum conservation*:

$$v\frac{dv}{dr} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{dP}{dr} + \frac{\sigma_e L}{4\pi r^2 c}\left[1 + M(\tau)\right];$$

$\boxed{3}$ *equation of state*:

$$P = \rho v_s^2$$

where v_s is the isothermal sound speed, a known function of radius since $T(r)$ is given. From equations 1 and 3, we can eliminate ρ and P in favor of v, v_s, r, and $\dot{M}$. Also, we describe the force multiplier $M(\tau)$ by $k\tau^{-\alpha}$ with the optical depth set equal to the expression appropriate for an expanding atmosphere:

$$\tau = \frac{\sigma_e}{\kappa_{\mathrm{L}}}\,\tau_{\mathrm{L}} = \sigma_e\,\rho\,v_{th}\left(\frac{dv}{dr}\right)^{-1}.$$

With these changes, the equation of motion for the wind becomes

$$\left(v - \frac{v_s^2}{v}\right)\frac{dv}{dr} = -\frac{GM(1-\Gamma)}{r^2} + \frac{2v_s^2}{r} - \frac{dv_s^2}{dr} + \frac{GM\Gamma k}{r^2}\left(\frac{4\pi}{\sigma_e v_{th}\dot{M}}\right)^{\alpha}\left(r^2 v\frac{dv}{dr}\right)^{\alpha}$$

where the parameter Γ was defined earlier (p154); it is the ratio of the continuum radiation pressure and gravity – typically 0.1 to 0.5 for O stars.

The foregoing wind equation doesn't look that difficult to solve but, in fact, finding the unique solution for a given L, M, and photospheric radius R requires a major effort in numerical analysis. It is actually an eigenvalue equation for $\dot{M}$ – there exists a wind solution that extends from the photosphere out to infinite radii only for a single value of the mass-loss rate (given L, M, and R). A careful analysis will reveal that there are regions in the (v, r)-plane in which dv/dr is not defined and other regions where the derivative is multivalued. A singular curve separates the two types of regions.

If one didn't know better, he might start at the photosphere with $v = 0$ and a guess for $\dot{M}$ and then try integrating out to large radii. However, his solution would quickly "crash and burn" because the integration path would inevitably run into the singular curve. It turns out that the only way to get past the singular minefield is to choose the exact value for $\dot{M}$.

To deal with this problem, Castor and company simplified the wind equation by transforming variables as follows:

$$w \equiv \frac{1}{2}v^2 \ , \qquad u \equiv -\frac{1}{r} \ , \qquad \text{and} \qquad h(u) \equiv -GM(1-\Gamma) - \frac{2v_s^2}{u} - \frac{dv_s^2}{du} \ .$$

Moreover, they grouped all the constant factors into one quantity:

$$C \equiv GM\Gamma k \left(\frac{4\pi}{\sigma_e v_{th} \dot{M}}\right)^\alpha \ .$$

With these definitions, the wind equation looks like

$$\left(1 - \frac{v_s^2}{2w}\right)\frac{dw}{du} - C\left(\frac{dw}{du}\right)^\alpha = h(u)$$

where $h(u)$ on the RHS is a "force function" which includes the effects of gravity, continuum radiation pressure, and gas pressure. The second term on the LHS is obviously the pressure due to line absorption. Note that $h(u)$ is a known function of u or radius. Also, C can be considered an unknown eigenvalue in view of its dependence on $\dot{M}$.

Examine the figures on the next page for a schematic plot of the force function $h(u)$ and maps of the (w,u)-plane which delineate five separate regions: I and III wherein dw/du is single-valued, II in which the velocity derivative is double-valued, and IV and V in which the derivative is not defined. The curve separating I and V from II is the sonic line along which $v = v_s$. Similarly, the curve dividing II and IV is the singular line along which all physical solutions must intersect *tangentially*.

Given values for L, M, and R, we find that there is only one value of $\dot{M}$ which will allow the solution to pass through a point on the singular curve (at a tangent) and pass on out into region III and large radii. This solution starts below the photosphere at some small (subsonic) velocity (point P in the figures), passes through the sonic line just above the photosphere (S), glances off the singular curve (at C), and then continues out to large r or small u.

As with many problems in physics involving regular singularities, it is not possible to integrate a solution *toward* a singular point due to inevitable (small) numerical errors. Instead, one has to determine the position of the singular or critical point (C) in advance and then start integrating from that position *inward* to the photosphere and *outward* to infinity. The correct value of the mass-loss rate $\dot{M}$ can be evaluated knowing the position of the critical point. For supersonic velocities (above S), the wind satisfies a relatively simple law:

$$v^2 = \frac{2\alpha}{1-\alpha} GM(1-\Gamma)\left(\frac{1}{r_s} - \frac{1}{r}\right)$$

where r_s is the radius of the sonic point ($\sim R$). The terminal velocity of the wind v_∞ can be easily evaluated by setting r to a large value.

The wind solution can be viewed as steady flow through a funnel but in reverse. The singular point is the upper end of the spout beyond which the funnel opens up. Below this point (actually a surface in real space), the flow is slow at high density; above, the flow is rapid at low density. It is also analogous to steam escaping through a valve or from the spout of a kettle. Nature apparently prefers the steady state (or equilibrium) solution but it doesn't necessarily have to be stable. In this particular case, it is. But in some problems, such as nova shell ejection and protostar accretion, the flow can oscillate about the steady state solution. See §10 for an illustration of the second example.

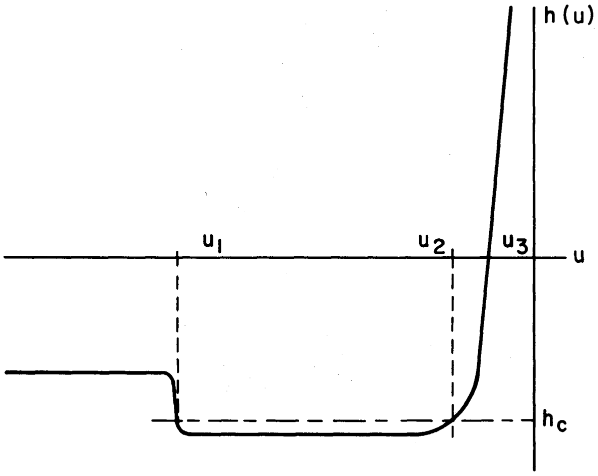

Figure 9.21 – Sketch of the force function h versus $u = -1/r$. The quantity h/r^2 is the net force per unit mass due to gravity, continuum radiation pressure, and gas pressure. The center of the star lies at infinity towards the left, and infinite radius occurs at $u = 0$ on the h-axis. The quantity h_c is the critical value of h at the boundaries of region IV (u_1 and u_2 – see figures below). At u_3, h passes through zero and goes to $+\infty$ as $u \to 0$.

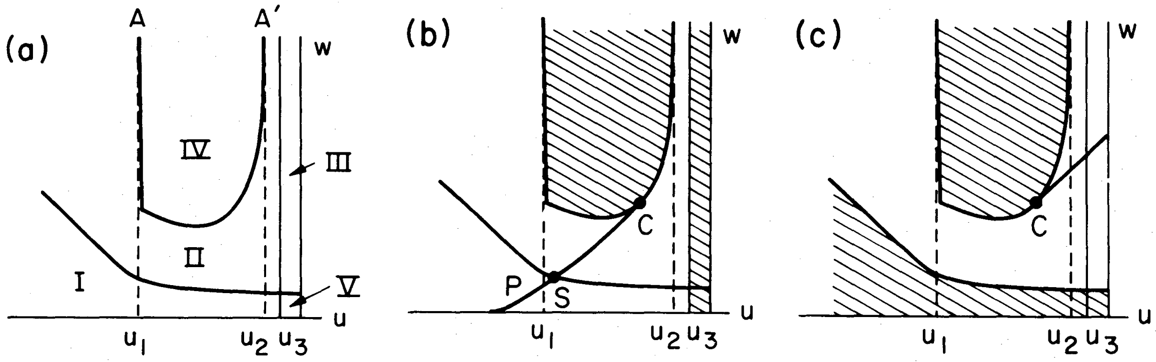

Figure 9.22 – (a) Sketch of the velocity-radius diagram in the (w, u)-plane showing five regions: in IV and V, the derivative dw/du does not exist; in I and III, the derivative is uniquely defined; and in II, it is double-valued. The curve that separates regions I and V from regions II and III is the sonic line, $v = v_s$. The curve AA' that separates regions II and IV is the locus of singular points of the wind equation. Vertical lines mark the values of u_1, u_2, and u_3 shown in Fig. 9.21. (b) The part of the (w, u)-diagram that is *not* shaded is the region in which the small-r branch of the wind solution is defined; it comprises regions I and II. This branch of the regular solution is labeled PSC where P indicates the photosphere, S is the sonic point, and C is the singular point. (c) The part of the diagram that is not shaded is the region in which the large-r branch of the solution is defined (regions II and III). The regular solution is indicated by the diagonal line extending upward from C. The curve PSC and its continuation is the unique solution that is subsonic at small radii and extends to arbitrarily large r at supersonic velocities.

9.3 Application to an O5f Star

Castor *et al* applied the foregoing theory to an O star with the following parameters:

$$M = 60\,\mathrm{M}_\odot\,, \quad L = 9.66 \times 10^5\,\mathrm{L}_\odot\,, \quad R = 13.8\,\mathrm{R}_\odot\,,$$

$$T_e = 49290\,^\circ\mathrm{K}\,, \quad \log g = 3.94\,, \quad \Gamma = 0.4\,.$$

The temperature distribution was iterated once (as mentioned earlier) to allow for the wind-induced density distribution. An LTE atmosphere code was used for this purpose even though LTE won't really apply above the sonic point. But it was determined that the wind solution is not strongly affected by $T(r)$ at large radii.

They find a mass-loss rate for this model of

$$\frac{dM}{dt} = 6.61 \times 10^{-6}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}\,.$$

This value is approximately $L/2cv_\infty$ which indicates that roughly half the original momentum carried by the radiation has been transferred to the wind and that half the total continuum flux has been blocked by the lines. It is hard to imagine a much higher driving efficiency! This result is quite remarkable; it is due, of course, to the velocity gradient which Doppler shifts the lines across most of the ultraviolet spectrum.

With this mass-loss rate, the star will lose a significant fraction of its initial mass during its own main sequence lifetime:

$$\frac{\tau_{ms}}{M}\frac{dM}{dt} \simeq \frac{1}{4} - \frac{1}{3}\,.$$

This should be enough to account for the Wolf-Rayet phenomenon in which massive stars appear to lose their entire H-rich envelopes, exposing the products of He-burning.

Study the figures on the next page; they show the variation of velocity, density, and force multiplier with radius and optical depth. Notice how the wind velocity increases by 6 orders of magnitude as it passes from below the photosphere to its terminal value of $1515\,\mathrm{km\,s^{-1}}$. This corresponds to a drop in density by 6 orders of magnitude. Note too how the force multiplier $M(\tau)$ climbs by 5 orders of magnitude, starting just below $\tau \simeq 0.1$. The steep velocity gradient in this model increases the effective M by as much as 100 compared to the static atmosphere shown in an earlier diagram.

<u>CONCLUSIONS</u>:

1. The radiation pressure force due to a large number of subordinate absorption lines can be very effective in driving a stellar wind in massive stars especially in the presence of a steep velocity gradient.

2. The mass-loss rate and the terminal wind velocity are consistent with recent UV satellite observations and are sufficiently high to explain WR stars.

3. The continuum energy distribution is almost unaffected by the wind. However, the effective photosphere of the star is lifted somewhat due to absorption in the wind. This increases the apparent radius of the star and decreases the apparent surface brightness.

4. The emission line spectra of H and He formed in the envelope are very similar to those of planetary nebulae. The line strengths of H_α and the He II lines around $\lambda 4686$ are in the range observed in Of stars.

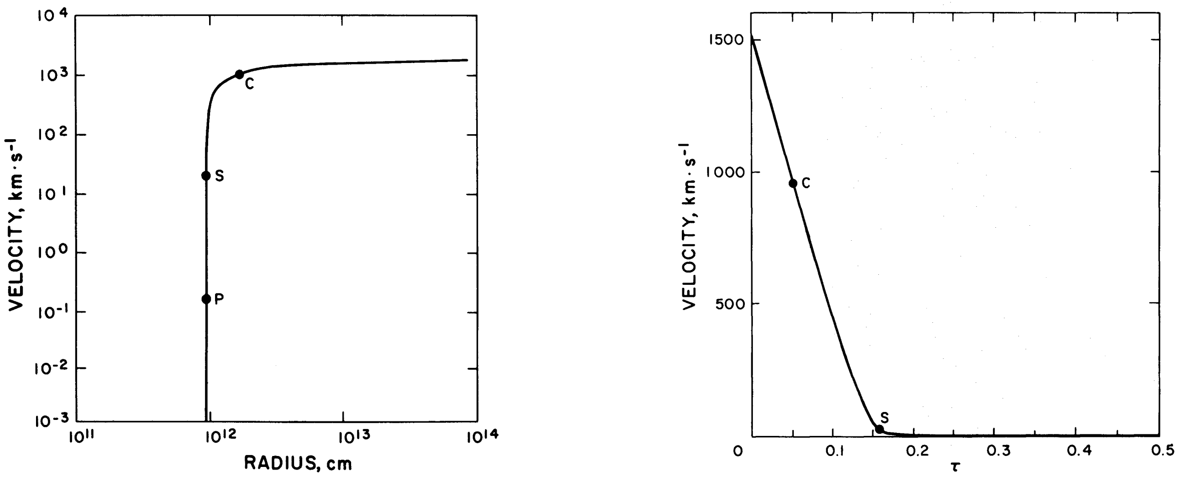

Figure 9.31 – Wind velocity versus radius (left) and continuum optical depth (right) for the O5f model described in the lecture notes. The temperature distribution is approximately self-consistent with LTE and radiative equilibrium. The point P is the photosphere, S is the sonic point, and C is the singular or critical point.

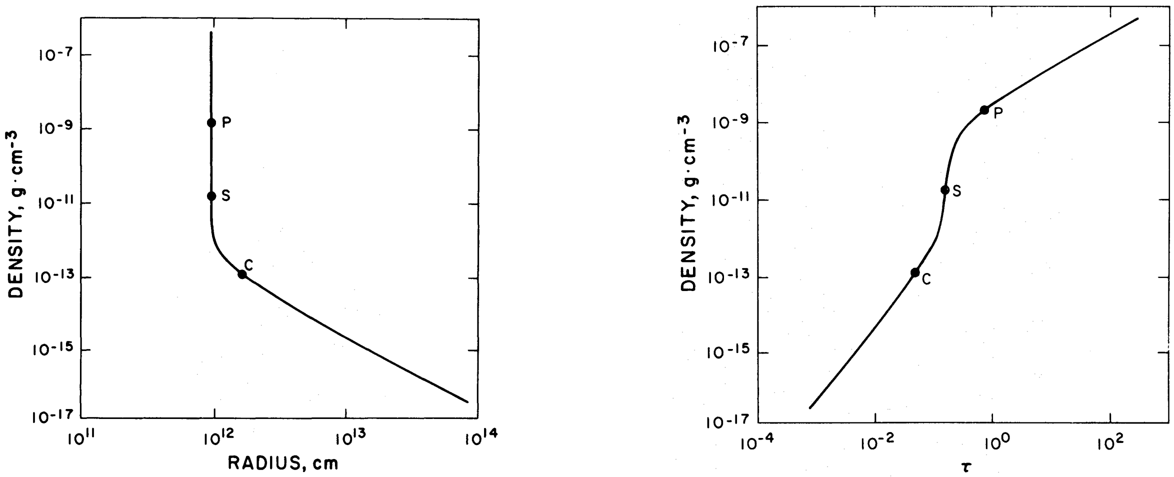

Figure 9.32 – Mass density in the wind versus radius (left) and optical depth (right). The symbols and model are the same as in Fig. 9.31.

Figure 9.33 – The force multiplier M versus the continuum optical depth. Symbols and model as in Fig. 9.31. $M + 1$ is the total radiation force in units of the force due to electron scattering alone. For this model, $1 - 0.4(M + 1)$ is the effective gravity in units of GM/r^2; it vanishes just below the sonic point S.