10. Protostar Accretion Via Radiative Shocks

Reference: F. H. Shu, *Gas Dynamics*, Chapters 18 and 19.

This is one of those very controversial areas of astrophysics with a long history going back to the early 1960's. The problem of turning a gas cloud into a star (or stars) is a very difficult one even if rotation and magnetic fields are ignored. It involves a range of 25 orders of magnitude in density (between cloud and star) and non-equilibrium radiation physics in the form of strong radiative accretion shocks. This makes for very difficult simulations. Historically, the two major areas of debate and argument have been (i) realistic initial conditions for protostar collapse, and (ii) appropriate boundary conditions for the accreting core. Let's consider each of them in turn. The second problem involves examining radiative transfer through an optically thin shock.

10.1 Initial Conditions for Protostar Collapse

Hayashi and his colleagues were the first to consider this problem in a serious way. In the absence of computing power, they adopted an analytical approach. Starting with an isothermal uniform density cloud, they assumed that the gas collapsed *homologously* in (supersonic) free-fall, every element hitting the center at the same instant after a time

$$\tau_f \simeq (\pi G\rho)^{-1/2} \simeq 10^6\,\mathrm{yr} \qquad (N \sim 100\,\mathrm{cm}^{-3}).$$

In this scenario, very little of the gravitational potential energy is converted into radiation and lost from the system – it all goes into the thermal energy of the accreted object. The result is a protostar in hydrostatic equilibrium with a large thermal energy content, large radius, and large luminosity. It will be convective throughout and lie well above the main sequence, high up on its "Hayashi track" in the HR diagram.

It is relatively easy to calculate the initial radius when such an object first comes into hydrostatic (virial) equilibrium. We have

$$2U + W = 0\,, \quad \text{and} \quad W + U + I + \Delta = 0$$

where U and W are the internal and gravitational energies, I is the total energy required to first dissociate the molecules and then to ionize the atoms, and Δ is the total energy radiated into space. The right-hand-side of the second equation has been set to zero on the assumption that the initial total energy of the cloud is negligible compared to that of the collapsed protostar. If the object is in convective equilibrium, it is a polytrope of index $n = 3/2$. Then

$$W = -\frac{6}{7}\frac{GM^2}{R}$$

from which we can derive

$$R = \frac{3}{7}\frac{GM^2}{(I+\Delta)} \simeq \frac{3}{7}\frac{GM}{I/M}$$

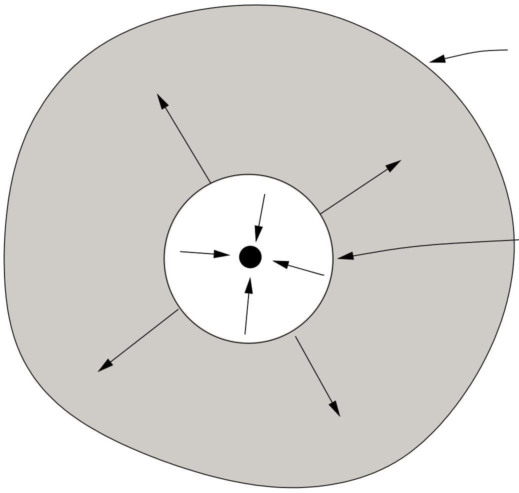
after eliminating U with the virial equilibrium relation. I have assumed here that $\Delta \sim 0$ which is what Hayashi did. To dissociate a gram of molecular hydrogen and then ionize both atoms requires

$$\frac{I}{M} = \frac{4.48\,\mathrm{ev}}{2m_{\mathrm{H}}} + \frac{13.6\,\mathrm{ev}}{m_{\mathrm{H}}} = 1.58 \times 10^{13}\,\mathrm{ergs\,gm}^{-1}\,.$$

Substituting this figure into the foregoing expression for R, we find that a $1\,\mathrm{M}_\odot$ protostar ends up on the Hayashi track with a radius of about $50\,\mathrm{R}_\odot$. Its effective temperature would be a typical red giant value ($\sim 3500\,°\mathrm{K}$), leading to a luminosity of

$$L_{\mathrm{H}} \simeq 4\pi R^2\,\sigma T_{\mathrm{RG}}^4 \simeq 325\,\mathrm{L}_\odot\,.$$

We now know that this picture is not correct. Protostars do not collapse homologously because they start off almost in hydrostatic equilibrium and, in addition, they don't have the uniform density distribution required for a gravitational force which increases linearly with radius. Instead, they collapse "inside-out". That is, they free-fall only inside an expansion front which move outwards at the velocity of sound, v_s, into a more or less equilibrium isothermal cloud. This is illustrated in the following sketch:



This is basically the Bondi accretion problem in an isothermal gas as discussed by Shu in his gas dynamics text:

$$\dot{M} \simeq 4\pi r_f^2 \rho_f v_s \simeq \frac{v_s^3}{G}$$

where ρ_f is the ambient density in a singular isothermal sphere at the expansion front radius $r_f = v_s \tau_f$:

$$\rho_f \simeq \frac{v_s^2}{4\pi G r_f^2}\,.$$

Note that the dynamical or free-fall time τ_f grows linearly with radius in the cloud. Thus, the accretion rate in an isothermal gas depends only on the sound speed or the prevailing temperature. For $T = 10\,°\mathrm{K}$, $v_s \sim 0.19\,\mathrm{km\,s^{-1}}$ and

$$\dot{M} \simeq 2 \times 10^{-6}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}\,.$$

Ironically, this is of the same order of magnitude as the accretion rate expected in the collapse of a $1\,\mathrm{M}_\odot$ cloud based solely on the free-fall timescale (see the first equation on the preceding page).

It is interesting to compare the foregoing accretion process with the problem of fluid passing through a funnel. There is an upper limit to how fast the material can move down the spout. If you pour too fast, a reservoir builds up which has to wait its turn to go down the tube. Thus, even if one were to start with a uniform density isothermal cloud, it would quickly approach an equilibrium isothermal sphere outside the expansion front which is effectively a funnel spout. This is how black holes accrete matter – at least initially when there is lots of material to be captured. In the other limit of very low densities, the material can flow down the funnel at supersonic velocities, not limited by the velocity of sound. This is called "advection dominated accretion" and is observed in some black holes at the centers of old galaxies like our own. We conclude that they are apparently running out of fuel because the black-hole luminosities are very low – much lower than Bondi accretion would imply.

The bottom line here is that the protostar builds up gradually as in-falling matter passes through an accretion shock at the outer boundary of the core in hydrostatic equilibrium. Most of the kinetic energy is thermalized in a relaxation layer behind or down-stream from the shock. From there, it gets radiated back into space. This means that the final protostar ends up with very little thermal energy. It will have a small radius (a few $R_\odot$ for a $1\,M_\odot$ object rather than the $50\,R_\odot$ estimated by Hayashi) and a small luminosity. In fact, it will bypass most of the Hayashi track which is the completely convective phase of pre-main sequence evolution.

The only problem now remaining for us to consider is the question of how to model the shock properly so as to get the correct radiation loss at the surface of the accreting protostar. This will govern the actual radius and luminosity of the final complete object that exists in virial equilibrium after accretion terminates. Note that the physics of this termination process has been completely ignored in the foregoing discussion. We don't really understand what limits the stellar mass scales but obviously stellar winds, radiation pressure, magnetic fields, and bi-polar outflows must be involved.

10.2 Boundary Temperature of an Accreting Protostar

Richard Larson (1969, *MNRAS*, **145**, 271) was the first to do a full hydrodynamical simulation of an accreting protostar. His initial conditions describe a gas at rest but not in equilibrium (unlike Hayashi who adopted supersonic collapse at $t = 0$). Larson arbitrarily chose an initial density distribution out of virial equilibrium so as to get immediate action. This resulted in a short phase of artificially rapid accretion (10^5 yr for $1\,M_\odot$), followed by a much longer and more realistic slow accretion period (10^6 yr). However, the overall evolution that he found more or less agrees with later calculations – even those which treat the shock front with radiation hydro techniques. In particular, a rarefaction front appears and moves outward at the speed of sound into a very slowly contracting medium. This is non-homologous collapse onto an accreting core in hydrostatic equilibrium.

In his text, Frank Shu argues that better initial conditions (based on observations of molecular clouds) would involve an isothermal cloud *in equilibrium* except for a small inner core that is free-falling at a constant rate through an expansion front. This spreads the accretion out uniformly over the entire 10^6 yr accretion period.

Unfortunately, the accretion shock entails non-equilibrium radiation physics and really requires a full radiation hydro calculation. Larson wasn't able to do this back in 1969 and so he adopted a simple prescription for the boundary temperature of the accreting star, namely an "effective temperature" defined by

$$4\pi R_\star^2\,\sigma T_\star^4 = L_\star + \frac{GM\dot{M}}{R_\star}\,.$$

This approach assumes that the gravitational binding energy of the accreting matter is thermalized below some radiative equilibrium layer (the "photosphere") and thrown back out into space through an optically thin shock. It turns out that this is really not such a bad assumption but it ignores the real possibility that some of the soft X-ray emission from the shock can be absorbed in the gas upstream from the shock, transformed into optical photons, and radiated into space from there. This would lower the radiative equilibrium temperature behind the shock.

The temperature profile at the surface of the protostar including the shock geometry looks like this:

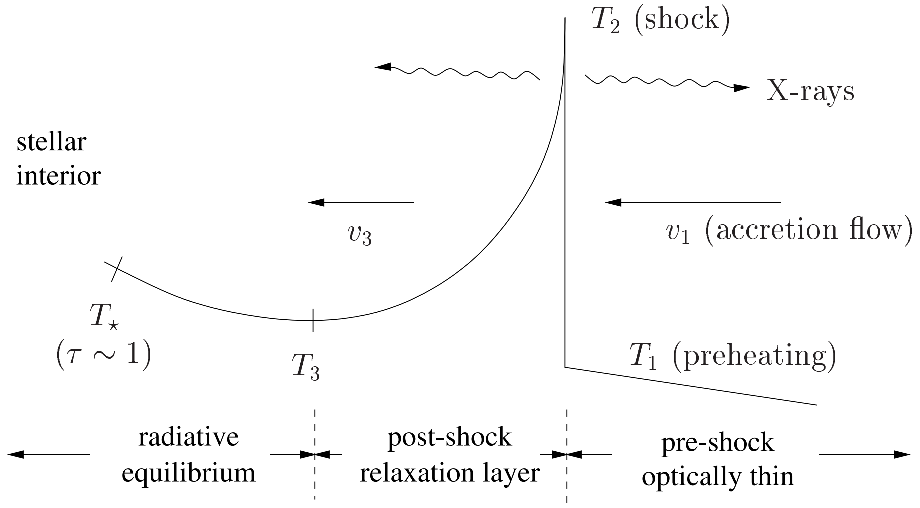


Figure 10 – Radiative processes near a strong accretion shock at the surface of a protostar start with the raising of the gas to a high viscous-shock temperature T_2. The hot gas then copiously radiates soft X-rays that travel both into and out of the star because these layers are optically thin. Strong radiative cooling brings the gas to a radiative equilibrium value T_3 at the bottom of a radiative-relaxation zone. Heating by the X-rays helps to define the preshock and post-relaxation temperatures, T_1 and T_3. The continued press of new material added on top causes the post-relaxation gas to sink gradually into a settling zone to become part of the body of the growing star. (After F. H. Shu, *Gas Dynamics*, p259.)

The point labelled T_3 indicates the temperature at the top of the radiative equilibrium layers. It's effectively the boundary temperature of an LTE continuum model like the Eddington grey atmosphere. It or $T_\star$ can serve as a boundary condition on the interior model. It will govern the rate of loss of thermal energy from the interior and determine the radius of the protostar when it first comes into hydrostatic equilibrium with its final mass.

The shock temperature is a very high, non-equilibrium value corresponding to the dissipation of the incoming kinetic energy. An absolute upper limit would be

$$\frac{3}{2}kT_2 \simeq \frac{1}{2}\mu H v_1^2 = \frac{\mu HGM}{R} .$$

Thus,

$$T_2 \simeq \frac{2\mu HGM}{3kR} \simeq 10^9\,{}^\circ\mathrm{K} \qquad (\mu \simeq 0.6) .$$

Radiative cooling will, of course, limit the temperature to a much lower value: $10^6 - 10^7\,{}^\circ$K, say. The foregoing temperature profile is reminiscent of the solar photosphere and corona except that the corona, unlike a shock, radiates very little energy compared to the solar luminosity.

The basic point here is that the shock throws the incoming gas badly out of equilibrium and it must radiate like mad in order to relax back into equilibrium as it falls down to point T_3. This temperature will be smaller than but of the same order as $T_\star$:

$$T_3 \simeq \left(\frac{GM\dot{M}}{4\pi R^3 \sigma}\right)^{1/4} \simeq 35000\,^\circ\mathrm{K} \qquad (\dot{M} \sim 10^{-6}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1})\,.$$

As the shocked gas cools down to T_3, it will emit soft X-rays which will be absorbed by the surrounding gas and re-processed into the optical where it can escape. Thus, the incoming gas can be preheated before reaching the shock. This will lower the equilibrium T_3 as we shall see.

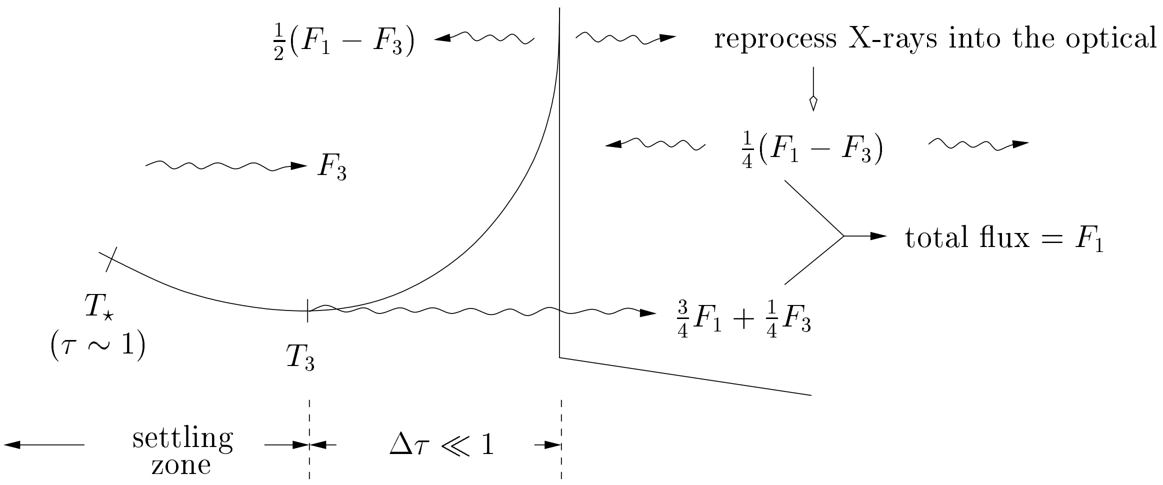
10.3 Radiative Transfer Across a Relaxation Layer

We want to estimate more carefully the radiative equilibrium temperature T_3 but avoid a full non-LTE radiative transfer calculation. The following simple transfer model is described in Shu's text (Chapter 19). The bottom line is that a better boundary condition for the effective temperature of the accreting star is

$$4\pi R_\star^2\,\sigma T_\star^4 = L_\star + \frac{3GM\dot{M}}{4R_\star}\,.$$

Note the factor '3/4' on the RHS; this is the only change from Larson's boundary condition (cf. bottom of page 163).

The relevant energy flows are shown in this energy budget diagram:



The outgoing flux seen by an external observer is F_1. The flux coming up from the interior is F_3. Thus,

$$F_1 = F_3 + \frac{GM\dot{M}}{4\pi R^3}\,.$$

Here, we assume that the accretion energy is released in the shocked layers *above* the photosphere. This is more accurate than assuming that it is released *below* the photosphere. Nevertheless, 75% of it ends up below the photosphere anyways. To see this, consider the following argument.

The X-ray flux from the shocked layers is $F_1 - F_3$. Half of this goes inward to be thermalized, and the other half goes outward to be re-processed into optical photons. Upon re-emission in an optically thin medium, half of these optical photons escape outward and half fly inward. Therefore, the total inward flux is $\frac{3}{4}(F_1 - F_3)$. This is re-thermalized, added to the outgoing flux from the interior, F_3, to give a total of $\frac{3}{4}F_1 + \frac{1}{4}F_3$ emitted by the outer equilibrium layer at temperature T_3.

To evaluate T_3, we have to do the radiative transfer problem which is not difficult if we assume LTE physics. In fact, this is a nice academic exercise for students. To start, we adopt the radiative diffusion approximation for the outward flux:

$$F_{\rm rad} = \frac{c}{3}\frac{d}{d\tau}(aT^4)$$

where $d\tau$ is an optical depth element defined with respect to a frequency-average opacity such as the Rosseland mean. Strictly speaking, the outer layers are not in radiative equilibrium because there are some gas motions as the accreting matter comes to rest in what might be called a "settling zone". We ignore such complications.

Integrate the foregoing equation to obtain

$$aT^4 = \frac{3F_3}{c}(\tau - \tau_3) + aT_3^4 \,.$$

This is just the usual LTE temperature profile. We can calculate T_3 by requiring that the flux have the right value, namely F_3. Remember that the source function in this situation is

$$S = B = J = \frac{cU}{4\pi} = \frac{acT^4}{4\pi} = \frac{\sigma T^4}{\pi} \qquad \text{(LTE grey atmosphere)}\,.$$

Therefore, the outgoing specific intensity is

$$I(\tau_3, \mu \geq 0) = \int_{\tau_3}^{\infty} \left(\frac{\sigma T^4}{\pi}\right) \mathrm{e}^{-(\tau-\tau_3)/\mu}\,\frac{d\tau}{\mu} = \frac{3F_3}{4\pi}\,\mu + \frac{\sigma T_3^4}{\pi}$$

from which we can calculate the net flux

$$\begin{aligned} F_3 &= 2\pi \int_0^1 I(\tau_3,\mu)\,\mu\,d\mu + 2\pi \int_{-1}^0 I(\tau_3,\mu)\,\mu\,d\mu \\ &= \frac{1}{2}\,F_3 + \sigma T_3^4 - \frac{3}{4}\,(F_1 - F_3) \end{aligned}$$

where I have replaced the second integral on the RHS by the incoming flux from the shocked layers; this includes the reprocessed radiation from above the shock. This integral would normally be zero in a non-accreting star. Solving for T_3, we find

$$T_3 = \left[\frac{1}{\sigma}\left(\frac{3}{4}F_1 - \frac{1}{4}F_3\right)\right]^{1/4} \,.$$

In the limit of little or no accretion,

$$T_3 = \left(\frac{F_3}{2\sigma}\right)^{1/4} = \frac{1}{2^{1/4}}\,T_\star(F_3) = 0.8409\,T_\star(F_3) \qquad (F_1 = F_3)\,,$$

where $T_\star(F_3)$ is the effective temperature of the non-accreting star. Note that T_3 is the same as the boundary temperature of an Eddington grey atmosphere.

On the other hand, if the accretion rate is very high, then

$$T_3 = \left(\frac{3F_1}{4\sigma}\right)^{1/4} = \left(\frac{3}{4}\right)^{1/4} T_\star(F_1) = 0.9306\, T_\star(F_1) \qquad (F_1 \gg F_3)\,,$$

where $T_\star(F_1)$ is the effective temperature of the accreting star: $T_\star(F_1) \gg T_\star(F_3)$.

Finally, we can confirm that the flux of photons seen by an external observer coming up from the radiative equilibrium boundary is

$$2\pi \int_0^1 I(\tau_3, \mu)\, \mu\, d\mu = \frac{1}{2}F_3 + \sigma T_3^4 = \frac{3}{4}F_1 + \frac{1}{4}F_3\,.$$

Add this to the reprocessed X-ray flux $\frac{1}{4}(F_1 - F_3)$ that the observer sees directly to get a total of F_1. Energy is conserved!

Note that our boundary temperature is 0.93 that of Larson because we allowed for the reprocessing of X-rays in the pre-shock region of the accreting flow. That is, we allowed for 25% of the accretion energy to be radiated away from the *shocked layers* rather than from the photosphere. This is really a very small effect that doesn't make a crucial difference to the protostar evolution that Larson described 30 years ago. His main error was in choosing non-equilibrium initial conditions that led to a variable accretion rate. Of course, Shu's Bondi-type accretion in a singular isothermal sphere ($\rho_c \to \infty$) is also an approximation to some degree but probably a better one than Larson used. And remember that we have ignored rotation and magnetic fields! Nevertheless, Larson's higher boundary temperature results in more energy loss and, consequently, smaller radii. Protostars will appear lower down on the Hayashi track compared to those found by Shu and his colleagues (e.g., Stahler 1983, *ApJ*, **274**, 822) who employed the approach described here.

It is also interesting to note that adopting the Bondi-style constant accretion rate makes hydrodynamical simulations unnecessary. All one needs to do is add spherical shells at a constant rate until the desired mass is reached. At the same time, of course, one follows the quasistatic growth of the core which is in hydrostatic equilibrium. This is really analogous to the problem of mass-loss from hot stars but in reverse. This approach requires only a simple radiative model for the shocked layers.

To keep everyone honest, a number of people have applied radiative hydro methods to this problem to make sure that simple radiative models are actually appropriate and valid; early examples are

- Winkler, K.-H. & Newman, M. J. 1980, *ApJ*, **236**, 201
- Langer, W., Chanmugan, G., & Shaviv, G. 1981, *ApJ Letters*, **245**, L23 (white dwarf accretion)
- Chevalier, R. & Imamura, J. 1982, *ApJ*, **261**, 543
- Bulluch, M. 1991, *A&A*, **243**, 205

As you might imagine, they encountered all sorts of numerical problems and numerical instabilities – shocks are always difficult to deal with. But over all, their results agree qualitatively with the Shu picture. In particular, everyone agrees that effectively all the kinetic energy of the accreting matter is radiated away with very little, if any, going into the thermal energy of the protostar. This was essentially an assumption made by both Larson and Shu.

I should point out that several groups have found a physical (real) overstability in connection with the shock position. This is not that surprising when you think about it. A positive perturbation in the accretion rate (say) will lead to an increase in the shock temperature T_2. This will result in an increase in the thickness of the relaxation layers since the gas needs a greater distance to cool down to equilibrium. But this expansion moves the shock upstream against the flow, leading to an increase in the relative velocity of the gas hitting the shock. This makes the shock temperature *even higher*. Thus, there is an instability which resolves itself when the relaxation layers expand enough to allow the gas to cool back to the steady-state temperature – or below in which case there could be (and apparently are) oscillations about the equilibrium shock position.