Department of Astronomy & Astrophysics, University of Toronto

RADIATION PROCESSES – AST1440F
Review Questions for Section 4

1. Why is synchrotron radiation not usually important in strong sources of bremsstrahlung emission and vice-versa?
2. For relativistic charged particles, the power is usually the strongest in the direction of motion; that is, the radiated emission is a maximum. Under what circumstances is this not true?
3. Naively, one might expect the spectrum of a cosmic-ray electron to peak at the cyclotron or gyromagnetic frequency $\omega_{\rm B}$ which is generally very low in astrophysical situations due to small magnetic fields (typically a few μgauss). But, in fact, the peak frequency can be quite high (in the radio or even the optical in some cases). How does this happen?
4. How would you relate the E-wave signal, $E(t)$, with the radiation power pattern of a synchrotron electron? That is, can you explain why for each cycle there is *one* positive E-pulse and *two* negative ones?
5. Sketch the synchrotron spectrum of a single electron. Indicate the asymptotic behavior at high and low frequency. How does the *width* of the spectrum compare with the frequency at maximum?
6. The relativistic Larmor formula for the total emitted power has two terms, one proportional to γ^4 and the other proportional to γ^6. Yet the synchrotron power $P(E)$ goes like γ^2 or E^2. Explain.
7. Sketch the motion of a synchrotron electron with a pitch angle $\theta > \pi/2$. Can we see pulses from such a particle? When we compute the mean power $P(E)$, do we average $\sin^2\theta$ over *all* solid angles?
8. In class, I pointed out that one can express Larmor's formula in such a way as to make synchrotron emission look like the "scattering" of magnetic field energy off electrons. This makes it look as if the source of the synchrotron energy is the magnetic field itself. Does this make sense?
9. How does the half-life of a synchrotron emitting electron depend on energy? After a period much longer than both half-lives, will an (initially) high-energy electron have smaller or larger energy than an (initially) lower-energy electron?
10. In class, we saw that a relativistic electron radiates power according to $P(E) \propto E^2$. Show that the corresponding relation for a non-relativistic electron is $P(E) \propto E$. Don't use my generalized expression for $P(E)$ which involves $\gamma^2 - 1$ but instead derive it yourself. Just evaluate the acceleration of a non-relativistic electron and apply Larmor's formula.
11. Explain why the spectrum of any group of electrons (with arbitrary energy distribution) has a sharp cutoff beyond a certain energy which depends on time. Consider a supernova remnant with an age of 1000 yr and a random field of 10^{-4} gauss. What is this critical cutoff energy? What is the corresponding cutoff frequency?

12. What is $B_\perp$ in the formula for j_ν? (Remember that we have assumed *isotropic* velocities!) Can $\sin\theta$ be negative? Why isn't $\langle B_\perp \rangle$ zero? What is $\langle \sin\theta \rangle$ over a sphere?
13. Consider a point source of synchrotron radiation on the celestial sphere. Assume a particular direction for the projected field ($\boldsymbol{B}_p$, say) and indicate on a sketch the expected plane of polarization.
14. What is the expected maximum degree of linear polarization for cosmic-ray electrons with a spectral index of $\alpha = -0.75$? Why is it not 100%?
15. A nonrelativistic electron spiraling about a magnetic field line will not radiate a continuous spectrum but rather a monochromatic signal with frequency $\nu_{\rm B}$. At higher energies, it remains discrete but with many components. How does this happen? In general, however, the synchrotron radiation that we observe from astrophysical sources is *continuous*. Why?
16. Similarly, we might argue that the blackbody radiation from a solid object like an interstellar dust grain should also be discrete on some scale. But, in practice, it is observed as a continuum. Explain.
17. Consider a nonrelativistic electron radiating as it spirals about a magnetic field line. If $v/c = 0.01$, what is the power in the 5th harmonic relative to the fundamental at $\nu = \nu_{\rm B}$? For a relativistic electron with $\gamma = 100$, what is the order of the harmonic at the spectrum peak ($\nu = \nu_m$)?
18. Give three examples of synchrotron sources.
19. Why do synchrotron spectra turn over at low ν?
20. How, in principle, can we derive the absorption coefficient for synchrotron radiation given the emission coefficient derived from the classical theory of radiation?
21. How, in principle, might the synchrotron absorption coefficient be *negative*? In practice, however, this is unlikely to happen. Why not?
22. At high frequencies, F_ν increases with B at a given ν as one might expect. But at low frequencies, the flux *decreases* with larger field strength. Why does this happen?
23. Sketch the synchrotron spectrum of a source over the whole frequency range from optically thick to optically thin. Indicate the slopes at low and high ν.
24. How do the spectrum slopes at high and low frequencies differ from those for thermal radiation?
25. If the slope of the particle energy distribution were to increase by unity (from -2 to -3, say), by how much would the slope of the spectrum increase at high ν?.
26. The brightness temperatures of synchrotron sources are typically very high and obviously non-thermal. Explain.
27. List three ways of distinguishing synchrotron radiation from thermal radiation.
28. Why is it usually necessary to assume equipartition in order to estimate the magnetic field strength in a particular synchrotron source?

29. Why does the particle energy become large (for a given synchrotron luminosity L) for small magnetic field strengths?
30. Why is the spectral index of the Crab Nebula ($\alpha = -0.3$) smaller than one would expect from the cosmic-ray energy distribution?
31. What is the inferred magnetic field strength in the Crab Nebula and how does it compare with the general galactic field?
32. What is the inferred total energy of the particles and magnetic fields in the Crab and how does it compare with the original energy released in the supernova and the energy lost by the pulsar over the last 1000 yr?
33. Why does the Crab spectrum have only a "break" in the spectral index and not a sharp cutoff in energy above a particular frequency?