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RADIATION PROCESSES – AST1440F

Review Questions for Section 2

1. The equation for the scalar potential ϕ reduces to Poisson's equation in the limit of infinite light speed ($c \to \infty$). The term $\partial^2\phi/\partial(ct)^2$ allows for the finite propagation time of light. In principle, the same allowance should be made for the finite propagation time of gravity. Why do people doing gravitational N-body computations usually ignore this effect?

2. When one evaluates the potentials ϕ and $\boldsymbol{A}$ at time t, why must the source charges and currents be evaluated at the *retarded* time? Is there a unique retarded time for a given t?

3. For relativistic charged particles, does the power always peak in the direction of motion where $1/\kappa$ is a maximum?

4. Why does the "Coulomb" field of a particle *not* lead to radiation losses whereas the "radiation" component of $\boldsymbol{E}$ does?

5. Demonstrate that the $\boldsymbol{E}$-vector for the Coulomb field lies in the plane defined by $\boldsymbol{n}$ and $\boldsymbol{\beta}$ whereas the $\boldsymbol{E}$-vector for the radiation field lies in the plane defined by $\boldsymbol{n}$ and $\dot{\boldsymbol{\beta}}$ ($\beta \ll 1$).

6. Demonstrate that the electric field of a stationary particle is *radial* whereas the magnetic field is everywhere *zero*. (Assume that $\dot{\beta} = 0$.)

7. Consider a charged particle moving at a *constant* velocity. Sketch the E-lines in the vicinity of the particle at a given instant.

8. Demonstrate that the power radiated by an accelerated charge has a ring pattern symmetric about the $\dot{\boldsymbol{\beta}}$-axis; i.e., $P(\theta) \propto \sin^2\theta$ where θ is the angle between $\boldsymbol{n}$ and $\dot{\boldsymbol{\beta}}$. Sketch the radiation pattern for an electron scattering a plane EM-wave.

9. For the Larmor radiation spectrum, there is an extra factor of 2 in the expression for $dW/d\nu$ depending on whether or not we include the contribution of *negative*-frequencies. Do we really have a choice here?

10. In my discussion of the *relativistic* Larmor formula, the potentially difficult factor $\kappa = 1 - \boldsymbol{n} \cdot \boldsymbol{\beta}$ never appears as if by magic! What happened here?

11. Can you explain in no more than *two* sentences why the power radiated by a relativistic charged particle is invariant to a Lorentz transformation? That is, why does an observer in an inertial frame see the same power as the particle sees in its own rest frame?

12. Explain in simple terms why a *small* apparent acceleration in the direction of motion of a relativistic particle can lead to a very large radiated power. Why is there a smaller but still very substantial boost factor ($P \propto \gamma^4$) in the perpendicular direction?

13. In Figure 2.24, the power is *not* zero in the direction of $\dot{\boldsymbol{\beta}}$. Explain why.

14. Show that $P \propto E_{\rm kin}$ for non-relativistic synchrotron electrons but $E_{\rm kin}^2$ for ultra-relativistic ones ($\gamma \gg 1$).

15. The power spectrum of a large number of radiating charged particles could potentially be very difficult to calculate due to the complex phase differences arising from a distribution of retarded times. Can you see why in practice this is not generally a problem?

16. At low frequencies ($h\nu < mc^2$), we are in the classical regime with regard to electron scattering and can rely on Maxwell's equations to describe the mechanism. That is, there are no quantum effects. Explain.

17. Why is Thomson scattering coherent?

18. Can you speculate as to why the Thomson scattering cross-section is close to and slightly larger than the actual physical (classical) cross-section of the electron?

19. Strictly speaking, the Thomson cross-section *is* frequency dependent on short timescales, but averaged over one or more cycles, it is frequency *independent*. Why specifically does this happen?

20. Why is it not surprising that polarized beams (either linear, circular, or elliptical) and unpolarized beams see the same cross-section for electron scattering?

21. The scattered radiation from an unpolarized beam can be seen to be 100% polarized by an observer. Under what conditions will this happen and why physically does it occur?

22. The concept of "radiation reaction" was introduced, at least in part, to compensate for a basic flaw in Maxwell's theory. Explain.

23. In words, how is the radiation friction force defined? How does the magnitude of the force compare with the acceleration force ($m\dot{\boldsymbol{u}}$) of an electron undergoing scattering of optical radiation? Provide an order-of-magnitude estimate.

24. A charged particle undergoing simple harmonic motion will damp out due to the "radiation reaction" force or viscosity. What is the timescale of this damping for (i) optical radiation, and (ii) radio waves with a frequency of 100 MHz?

25. Thomson scattering is strictly coherent if we ignore the radiation reaction effect. But if we do allow for it and do a Fourier analysis of the damped motion, we find that the scattered radiation has a frequency distribution centered on ν_o, the incident frequency. What is the shape of the profile and what is the width (at half maximum) in angströms for (i) optical radiation, and (ii) radio emission at 100 MHz?

26. In the classical view of the atom as a bound and damped harmonic oscillator, the damping time (referred to in #24 above) is identified with the lifetime of an atomic state which radiates as it decays. How is the bandwidth of the radiation related to the atomic lifetime? Why do sharp lines correspond to metastable states?

27. The intrinsic width referred to here is much *smaller*, in general, than the other sources of line broadening in typical astrophysical situations such as stellar atmospheres. What are some of these other broadening mechanisms? Describe them physically.

28. A bound harmonic oscillator has its own natural frequency determined by the binding force. But it can be driven by radiation of a different frequency. How does the scattering cross-section depend on the two frequencies (ω_o and ω, say) and what are the limiting values for very large and very small values of ω relative to ω_o?

29. The total cross-section for scattering by a bound electron is *finite* (namely, $\pi e^2/mc$ over all frequencies). Is the same true for a free electron? (Remember that σ_{T} is a constant, independent of density.)