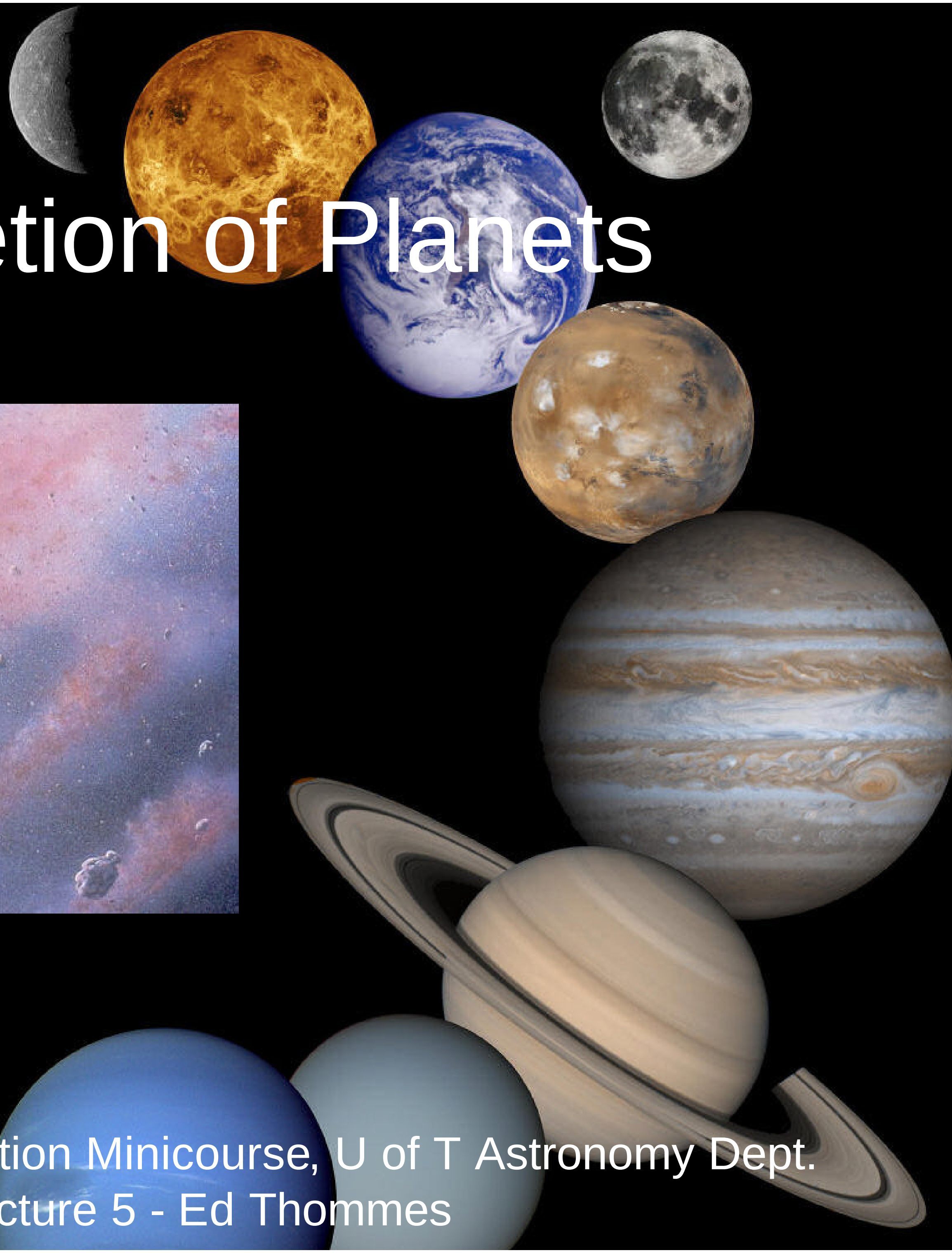
# Accretion of Planets

Star & Planet Formation Minicourse, U of T Astronomy Dept.
Lecture 5 - Ed Thommes

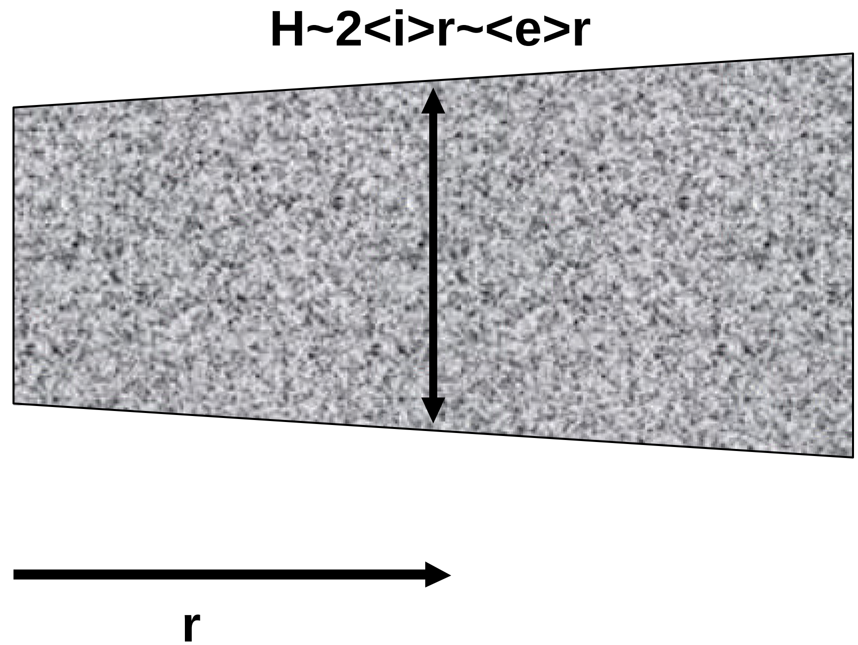
# Overview

- Start with **planetesimals**: km-size bodies, interactions are gravitational
  - (formation of planetesimals: Weidenschilling & Cuzzi, Protostars & Planets III; Experiments: Wurm, Blum & Colwell, Phys Rev E 2001)
- 3 stages of planet accretion:
  - **runaway**: ~1? $10^2$ km ($10^{-12} – 10^{-6}$ $M_\oplus$)
  - orderly or "**oligarchic**": $10^2$ km? **isolation mass** ($10^{-1} – 10^{1}$ $M_\oplus$)
    - Cores of **gas giants**, ~10 $M_\oplus$, have to be done by now!
  - Terrestrial planets: **Giant impact** phase among the isolation-mass bodies: $10^{-1}$ ? 1 $M_\oplus$
- **Extrasolar planets**: tell us wide range of outcomes possible

# The planetesimal disk

- Body on Keplerian orbit with semimajor axis ***a***, eccentricity ***e***, inclination ***i*** **?** radial excursion ***ea***, vertical excursion ~***ia***, velocity relative to Keplerian $\sim(e^2+i^2)^{1/2}\,v_{Kep}$
- Planetesimal disk typically has <i> ~ <e>/2
- **?** Disk has thickness H~2<i>r

  ~<e>r



# Estimating accretion rate

Details: e.g. Kokubo & Ida, Icarus 1996

"Particle in a box" approach: $\frac{dM}{dt} \sim r\, s\, v_{rel}$

$$r \sim \frac{\Sigma}{H} \sim \frac{\Sigma}{< e > r}, \;\; s = p\, r_M^2 \left(1 + \frac{v_{esc}^2}{v_{rel}^2}\right)$$

$$v_{rel} \sim \sqrt{2} < e > v_{Kep}$$

When the accreting body is bigger than neighbours,

**dynamical friction** makes $e_M << e_m$

$$\rightarrow v_{rel} \sim < e > v_{Kep}$$

Also, assume $v_{esc} >> v_{rel}$ $\rightarrow$ grav. focusing is effective:

$$\rightarrow \frac{dM}{dt} \sim \left(\frac{\Sigma}{< e > r}\right)\left(p\, r_M^2\right)\left(\frac{v_{esc}^2}{< e > v_{Kep}}\right)$$

# Runaway accretion

$$r_M \propto M^{1/3},\ v_{esc} \propto M^{1/3}$$

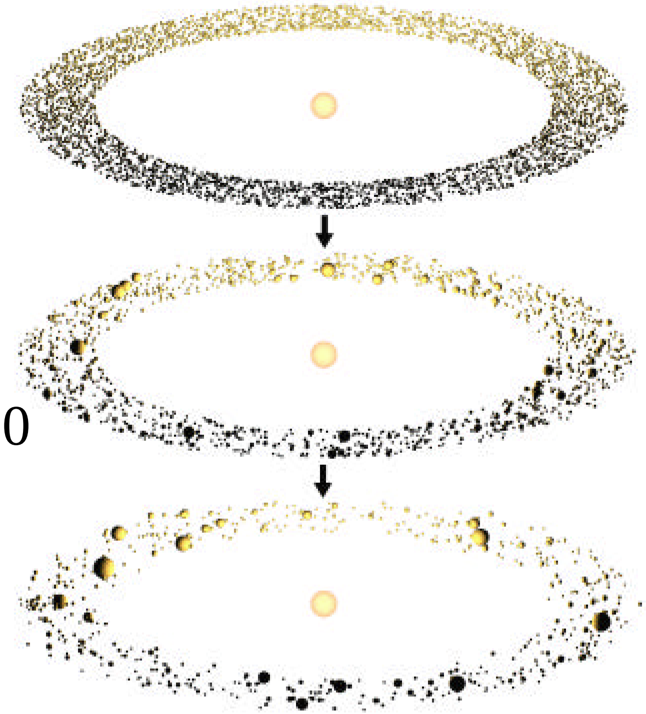
$$\rightarrow \frac{dM}{dt} \propto \frac{\Sigma M^{4/3}}{<e>^2\ r^{1/2}}$$

Now, if we have two masses, $M_1 > M_2$, then

$$\frac{d}{dt}\left(\frac{M_1}{M_2}\right) = \frac{M_1}{M_2}\left(\frac{\dot{M}_1}{M_1} - \frac{\dot{M}_2}{M_2}\right) = \frac{M_1}{M_2}\left(M_1^{1/3} - M_2^{1/3}\right) > 0$$

$\rightarrow$ Mass ratio diverges from unity, hence we have **runaway growth**

$\rightarrow$ large **protoplanets** emerge, rapidly detach from upper end of planetesimal size distribution



Kokubo & Narumi

# The end of runaway

Eventually, gravitational stirring by protoplanets dominates planetesimal random velocities (Ida & Makino, Icarus 1993)

$\rightarrow < e > \propto M^{1/3}$

$$\rightarrow \frac{dM}{dt} \propto \frac{\Sigma M^{2/3}}{r^{1/2}}$$

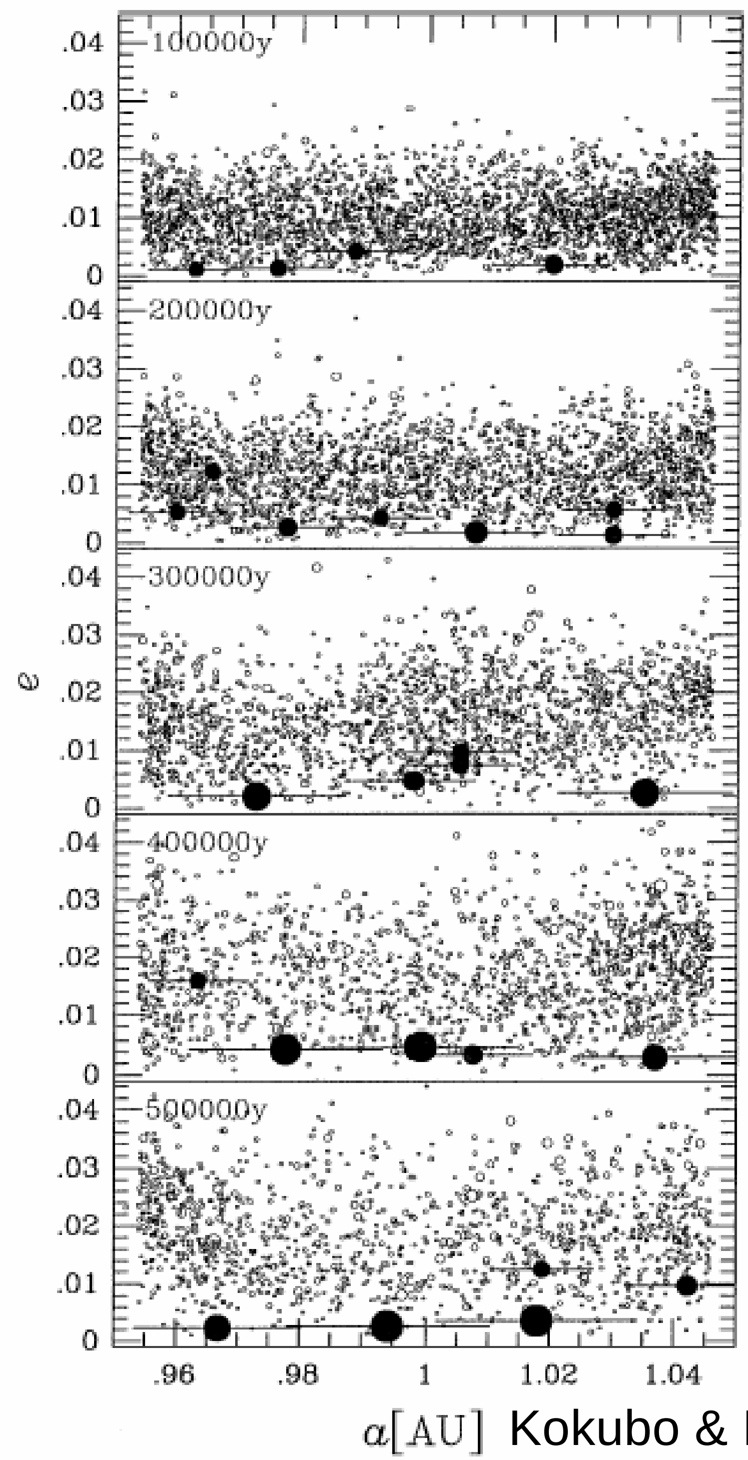
Now, if we have two masses, $M_1 > M_2$, then

$$\frac{d}{dt}\left(\frac{M_1}{M_2}\right) = \frac{M_1}{M_2}\left(M_1^{-1/3} - M_2^{-1/3}\right) < 0$$

$\rightarrow$ Mass ratio approaches unity, hence we have orderly growth. Also called **oligarchic growth** (Kokubo & Ida, Icarus 1998, 2000)

# Oligarchic growth

- Adjacent protoplanets grow at similar rates
- Hill radius: $r_H = \left(\frac{M}{3M_*}\right)^{1/3} r$
- Balance between perturbation and dynamical friction keeps ?r~5-10 $r_H$




Kokubo & Ida 2000

# Oligarchy+gas drag

Planetesimals subject to gas drag; estimate $< e >_{eq}$

by equating gravitational "stirring" timescale to

gas drag timescale (details: Ida & Makino 1993).

The result:

$$\frac{dM}{dt} \sim 4 \frac{b^{2/5} G^{1/2} M_*^{1/6} r_{gas}^{2/5} \Sigma}{r_m^{4/15} r_M^{1/3} r^{1/10} m^{2/15}} M^{2/3}$$

where spacing $= br_H$; $r_m, r_M$ = planetesimal, protoplanet bulk density;

$M_* =$ central body mass, and we've assumed for

simplicity a uniform planetesimal mass *m*.

# Isolation mass

- Oligarchic growth ends when all planetesimals used up
- Assuming spacing is maintained, can estimate the final mass:

$$M_{iso} = 2p\, r \Delta r \Sigma,\ \Delta r = b r_H \rightarrow M_{iso} = \frac{(2bp\Sigma)^{3/2} r^3}{(3M_*)^{1/2}}$$

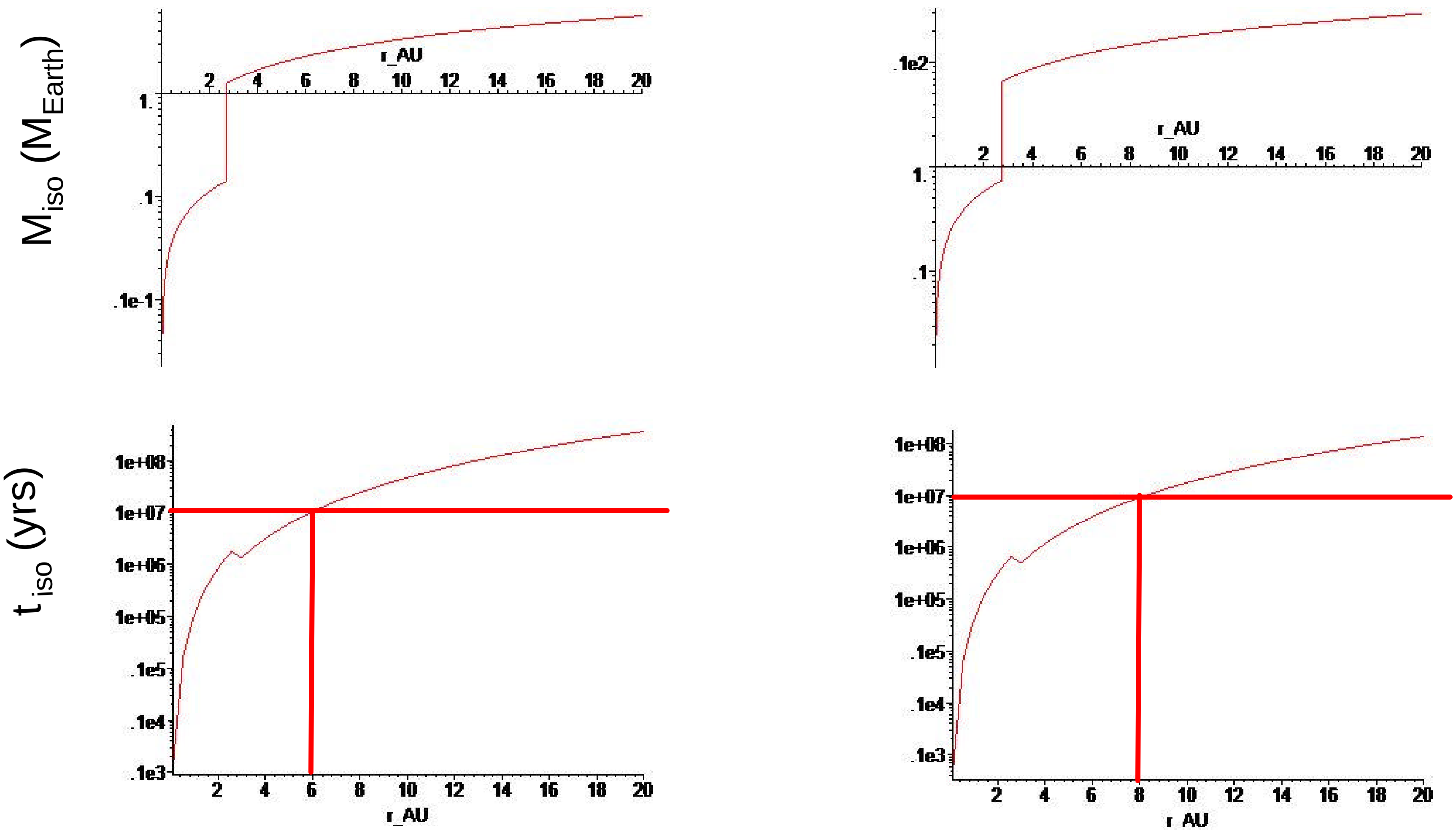
- increases with r for surface density shallower than S a r$^{-2}$

# Estimating masses and timescales

- Now we can get some actual numbers!
- Useful quantities
  - 1 AU = 1.5e13 cm
  - $M_{Sun}$=2e33 g, $M_{\oplus}$=6e27 g
  - 1 yr = 3.15e7 s
- The minimum-mass Solar nebula (MMSN) model (Hayashi 1981):
  - smear out the masses of the planets, enhance to Solar abundance with gas
  - $?_{gas}$=1.4e9(r/1 AU)$^{-3/2}$ g/cm$^3$, h/r=0.05(r/1 AU)$^{1/4}$
  - $S_{solids}$=7f(r/1 AU)$^{-3/2}$ g/cm$^2$ where f=1 inside of 2.7 AU, f=4.2 outside of 2.7 AU (**snow line**)
- Estimate $t_{iso}$ from $M_{iso}$/(dM/dt) (full time-dependent solution: Thommes, Duncan & Levison, Icarus 2003)

# Isolation mass & time: Examples

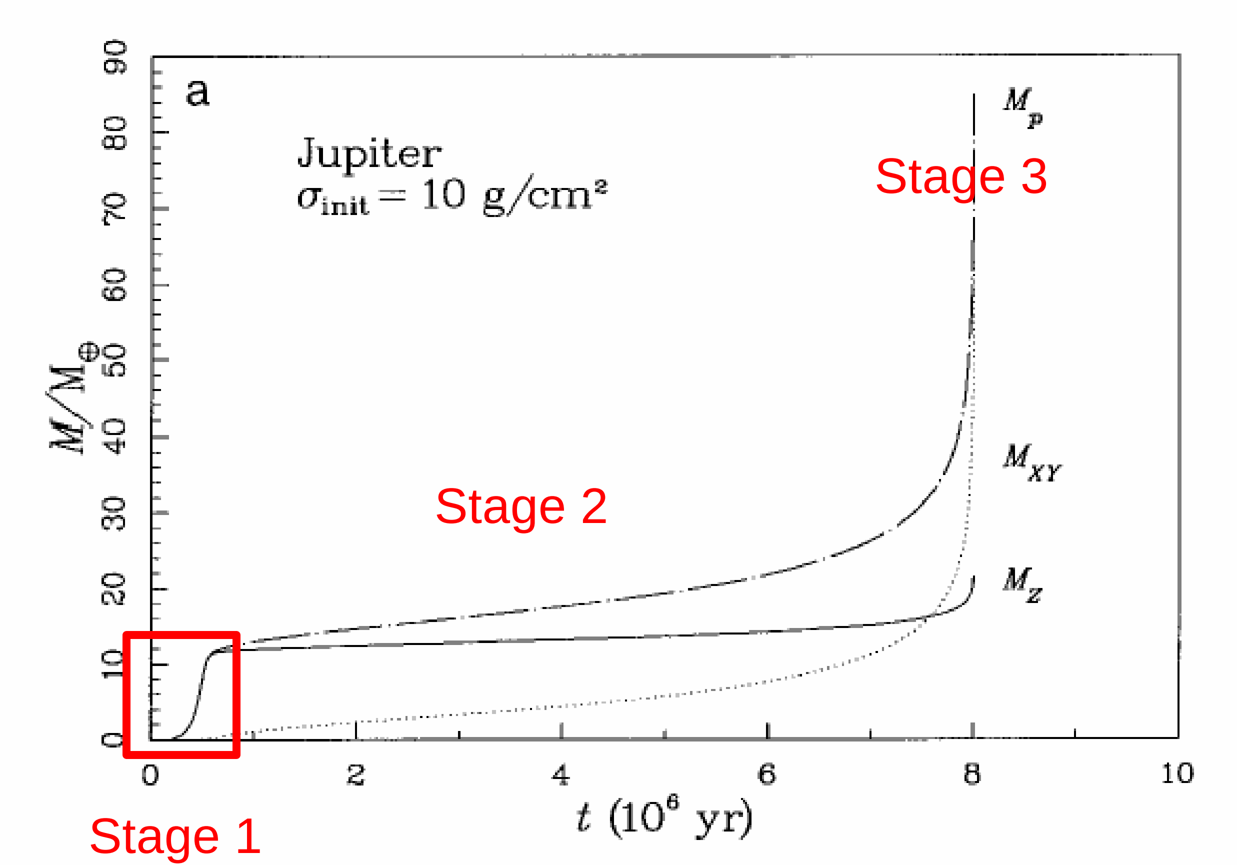
MMSN | 3 X MMSN




Other parameters: b=10, $m=10^{-9}\ M_{\oplus}$

# Gas giant formation by nucleated instability

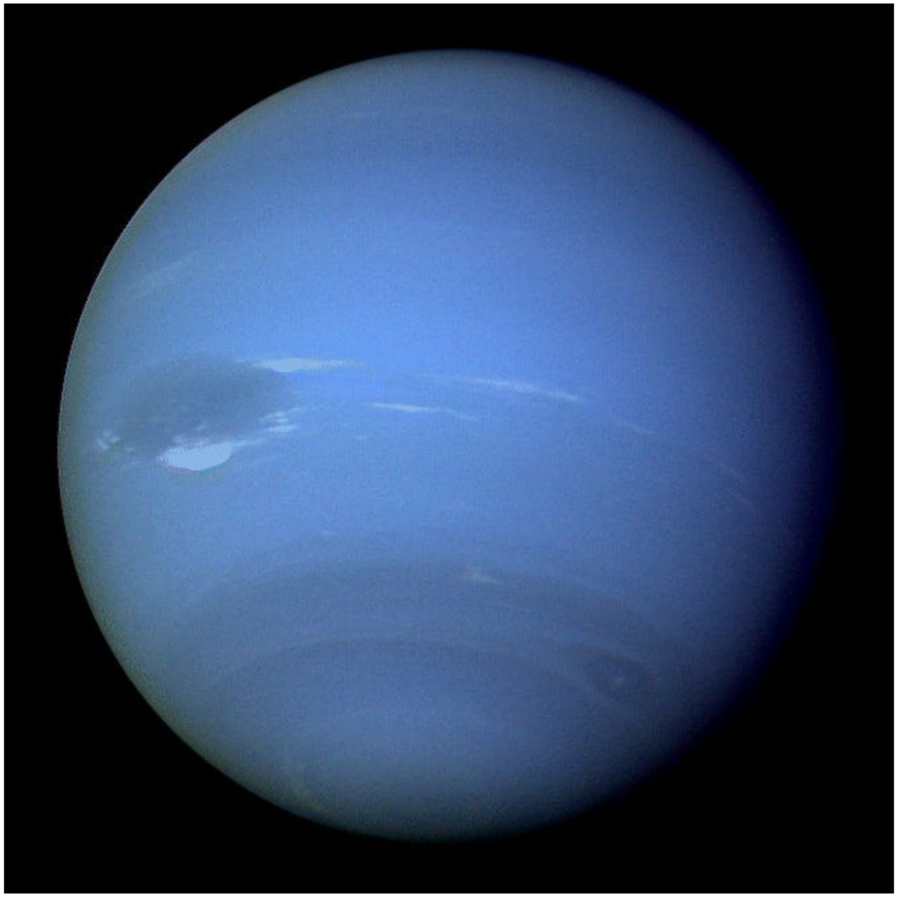
- Pollack et al, Icarus 1996: 3 gas giant formation stages
  1. core accretion (what we've been looking at)
  2. accretion of gas atmosphere until $M_{gas} \sim M_{core}$
  3. runaway accretion of gas, resulting in $M_{gas} >> M_{core}$
- Long plateau (2) can be shortened by lowering dust opacity




Pollack et al 1996

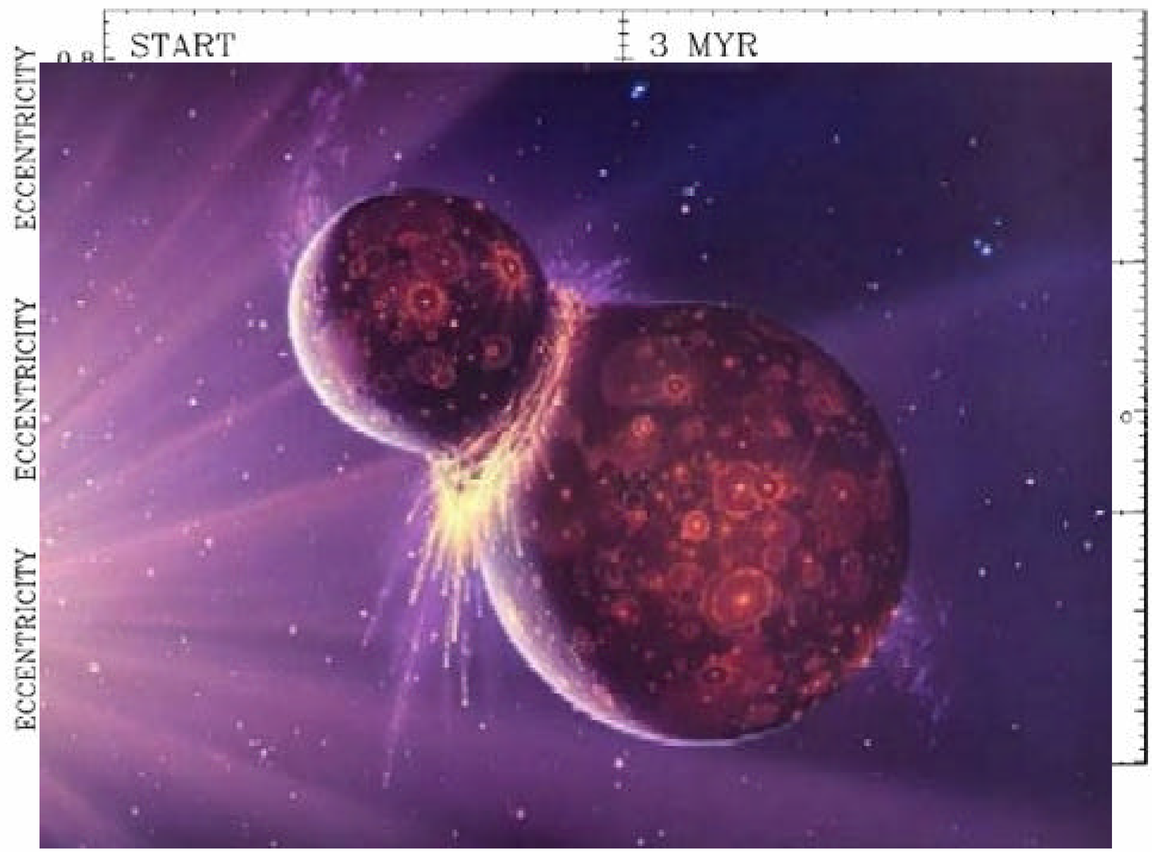
# “Ice giants” out in the cold?

- Uranus=15 $M_\oplus$; Neptune=17 $M_\oplus$
- Our estimate gives us $t_{iso}$ <~ $10^8$ yrs at 20 AU. But gas lasts at most $10^7$ yrs
- Models:
  - Jupiter/Saturn region produces excess cores, winners get gas (Jupiter & Saturn), losers get scattered (Uranus & Neptune) (Thommes, Duncan & Levison, Nature 1999, AJ 2002)
  - Planetesimals ground down to small size, collisional damping takes on role of gas damping (Goldreich, Lithwick & Sari, ARA&A 2004)



# Endgame for terrestrial planets

- Finished oligarchs in terrestrial region have mass ~$10^{-1}$ $M_{\oplus}$; need to grow by factor ~10 to get Earth, Venus
- Orbits of oligarchs have to cross
- Earth-Moon system thought to have formed from such an impact (Hartmann & Davis, Icarus 1975, Cameron & Ward 1976, Canup & Asphaug, Nature 2001)
- Standard picture: this happens after gas is gone and takes >~$10^8$ yrs (Chambers & Wetherill, Icarus 1998, Chambers, Icarus 2001) (faster scenario: Lin, Nagasawa & Thommes, in prep.)




of the semi-major axes and eccentricities of embryos from simulation 32. The symbol radius is proportional to the radius

Chambers 2001

# The extrasolar planets

- 130+ detected from radial velocity surveys
- Tell us that planet formation has **wide variety** of possible outcomes
- "**Hot Jupiters**" and pairs of planets in **mean-motion resonances**:
  - Migration (Lecture 7) probably plays major role
- **High eccentricities**:
  - Planet-planet scattering? (Rasio & Ford, Science 1996)...analogue to final terrestrial planet stage?
  - Planet-disk interactions? (Goldreich & Sari, ApJ 2003)
  - Both together? (Murray, Paskowitz & Holman, ApJ 2002; Lee & Peale, ApJ 2002
- **Us vs. them**:
  - Is our system one in which there simply wasn't much migration? If so, why?
  - Are we (low eccentricities, no hot Jupiters) the exception or the rule? Radial velocity observations still too biased to tell us (period of Jupiter = 11 yrs)

# The planetesimal disk

Body on Keplerian orbit with semimajor axis $a$ eccentricity $e$ radial