The Earth and Moon: from Halley to lunar ranging and shells

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Abstract. The 17 th-century English astronomer Edmond Halley studied ancient observations of lunar and solar eclipses and concluded that the month is getting shorter. We now know that this conclusion is incorrect. On the basis of a simple model for the Earth-Moon system, a possible explanation for Halley's mistake may be found. An extension of the same model predicts incorrectly that the Moon was very close to the Earth about 1.2 Gyr ago. At such a short distance, tidal forces would cause catastrophic melting of the crust of the Earth. The geologic record show no evidence for this.

To investigate this problem further, we take a look at measurements of the rotation velocity of the Earth, now and in the past. We end with a brief description of the formation of the Moon.

1. Halley and the orbit of the Moon

"And if any curious Traveller, or Merchant residing there, would please to observe, with due care, the *Phases* of the *Moon's eclipses* at *Bagdat, Aleppo* and *Alexandria*, thereby to determine their Longitudes, they could not do the Science of Astronomy a greater Service: For in and near these Places were made all the Observations whereby the Middle Motions of the *Sun* and *Moon* are limited: And I could then pronounce in what Proportion the *Moon*'s motion does Accelerate; which that it does, I think I can demonstrate, and shall (God willing) one day make it appear to the Publick."

The above sentence occurs at the end of an article that Edmond Halley[4] wrote in the November/December issue of 1695 of the Philosophical Transactions, a journal 'Giving some account of the present undertakings, studies and labours of the ingenious, in many considerable parts of the world' as its cover indicates. Apparently, Halley had concluded that the motion of the Moon along the sky is getting faster, which means that the length of the month is getting shorter. From Kepler's law, a shorter month implies a shorter distance of the Moon to the Earth. [1]

Halley's idea that the Moon was further from the Earth in the past than it is now, is remarkable, because in reality the reverse is true. It is possible to guess how Halley arrived at his conclusion from the above statement, and from the fact that Halley had written about the astronomical observations of al-Battani[3], an arabic astronomer referred to by Halley with the latinized name of Albategnius, who flourished around 890. The work by al-Battani had been translated from the arabic into latin by a certain Plato Tiburtinus who – in Halley's words – 'neither knew enough of the language, nor was instructed in the science of Astronomy'. Halley therefore emended the Latin edition, in the October 1693 issue of the Philosophical Transactions. The observations of al-Battani include several solar and lunar eclipses.

Since Halley never published the details of his reasoning, I will illustrate his probable line of argument with a hypothetical example. Because solar eclipses always occur at new Moon, the time difference between two eclipses is always an integer number of Months. Halley knew the length of the Month in his time quite accurately, and thus could compute the times on which al-Battani could have observed an eclipse. Suppose that Halley computed the time of an eclipse 800 year or about 10^4 months before his time, and found that at that time the city of Alexandria, about 15 degrees west of Bagdad, would be exactly on the line connecting the centers of the Sun and the Moon, i.e. the eclipse would be seen in this place; whereas al-Battani tells us that the eclipse on that day was seen in Bagdad. To explain this Halley would first note that due to the rotation of the Earth, Bagdad arrived on the line between the Sun and Moon one hour earlier than Alexandria. Apparently, the eclipse occurred one hour earlier than Halley computed. The conclusion is that the average month during the 800 yr interval was 10^{-4} hr longer than the month in Halley's time. For an assumed linear increase in the length of the month, the month 400 yr before Halleys time was at this average length, so that the rate of change in the length of the Month is 10^{-4} hr/400 yr, or 9 s in 10 000 yr.

[1] Alan Cook, in his 1998 biography of Halley[1], and Anton Pannekoek, in his 1951 History of Astronomy[10], agree on this interpretation. They mention that Laplace tried to explain it by computing the effect of the Sun on the orbit of the Moon, and that the computations of Laplace were shown to be in error by Adams.

Table 1. Measured parameters of the Earth and Moon system. Note that on should use the sidereal day, 86164.1 s, and the sidereal month, 27.32166 d, for the computation of the angular velocities of the Earth and of the lunar orbit. Data mainly from Dickey et al. 1994[2].

parameters of the Earth and Moon		
	Earth	Moon
mass	$M = 5.9734\,10^{27}$ g	$m = M/81.33$
radius	$R = 6.37103\,10^{8}$ cm	$r = 1.7379\,10^{8}$ cm
angular rotation velocity	$\Omega = 2\pi/(86164.1\,)$s	$\omega_m = \omega$
dimensionless ang. momentum radius	$r_{g,e} = \sqrt{0.331}$	$r_{g,m} = \sqrt{0.394}$
current orbital parameters		
semi-major axis	$a = 3.8440\,10^{10}$ cm	
derivative	$\dot{a} = 3.82$ cm/yr	
angular velocity	$\omega = 2\pi/(27.32166\,\mathrm{d})$	
eccentricity	$e = 0.05490$	

2. A simple description of the Earth-Moon system

The next step is to see in how far Halley's result can be understood in a very simple model for the Earth and the Moon. In this model, the Earth and Moon revolve around one another in a circle, and their rotation axes are assumed perpendicular to the orbital plane. We further assume that the Earth and the Moon are perfect spheres, and that their relative orbit is not perturbed by the Sun or by any of the planets.

The total angular momentum J of the Earth-Moon system then can be written

$$J = \frac{Mm}{M+m}a^2\omega + r_{g,e}^2 MR^2\Omega + r_{g,m}^2 mr^2\omega_m \tag{1}$$

The quantities occurring in this equation are explained, and their values listed, in Table 1. Since the Moon has much smaller mass and radius than the Earth, we ignore its angular momentum in what follows. In our assumption of no external perturbers, J is a constant, and its value is found by entering the values listed in Table 1 in eq. 1.

In eq. 1 a, ω and Ω are functions of time. We define

$$I \equiv r_{g,e}^2 MR^2 \tag{2}$$

and eliminate ω from eq. 1 using Kepler's law

$$\omega^2 = \frac{G(M+m)}{a^3} \tag{3}$$

to find

$$J = Mm\sqrt{\frac{Ga}{M+m}} + I\Omega \tag{4}$$

The tidal forces of the Moon slow down the rotation of the Earth, i.e. Ω is getting smaller. Eq. 4 shows that the angular momentum thus lost by the Earth is added to the angular momentum of the lunar orbit, which implies that the distance a between the Earth and the Moon increases.

We write a in terms of its current value a_n. To obtain the number of days per month, Ω/ω, we divide eq. 1 by ω, use Kepler's law again, and enter the numerical values for M, m, $r_{g,e}$ and a_n to find

$$\frac{\Omega}{\omega} \simeq 161\left(\frac{a}{a_n}\right)^{3/2} - 134\left(\frac{a}{a_n}\right)^2 \tag{5}$$

Eqs. 3 and 5 enable us to compute ω and Ω as a function of a, as illustrated in Figure 1.

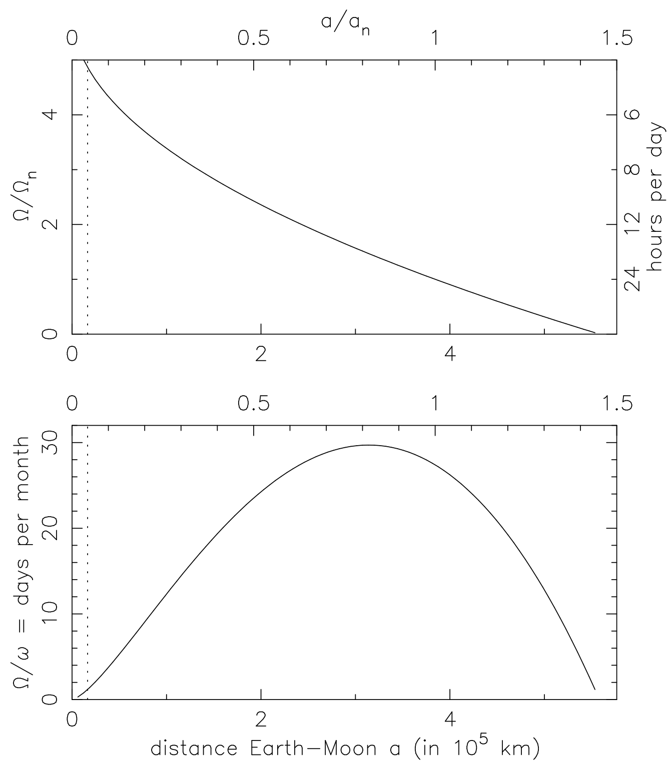

Fig. 1. The rotation velocity of the Earth Ω (in units of the current value Ω_n), and the number of days per month Ω/ω, as a function of the distance a between the Earth and Moon (in units of the current value a_n). The vertical dashed line indicates the distance at which tidal forces of the Earth would destroy the Moon. A circular orbit has been assumed.

By definition, the Earth-Moon system is now at $a = a_n$, and a is increasing. Figure 1 then tells us that the number of days per month is decreasing! The angular velocity of the rotation of the Earth is decreasing, i.e. the length of the day is increasing. Thus, even as the length of the month as measured in days is decreasing, its length as measured in constant units of time, for example in seconds, is increasing.

This I suggest is the origin of Halley's error. Halley found correctly that the number of days per month is decreasing; he assumed that the length of the day is constant, and thus concluded incorrectly that the length of the month decreases.

3. History of the orbit of the Moon: a simple description

Figure 1 shows that there would be one day per month, i.e. the length of the day would be equal to the length of the month, if the Moon and the Earth were very close to one another. Solving eq. 5 for $\Omega/\omega = 1$ we find $a = 0.038 a_n$ or about 14 600 km; and with eq. 3 it follows that the corresponding length of the month/day is 4.9 hr. Has this actually occurred in the past? To know this we must compute the distance a as a function of time, and for that we need a theory for the tidal forces. One such theory has been devised by George Darwin, the son of the famous biologist, in a series of articles starting in 1879. (A very clear recent description of this theory has been given by Piet Hut[6], and I follow his notation to a large extent.)

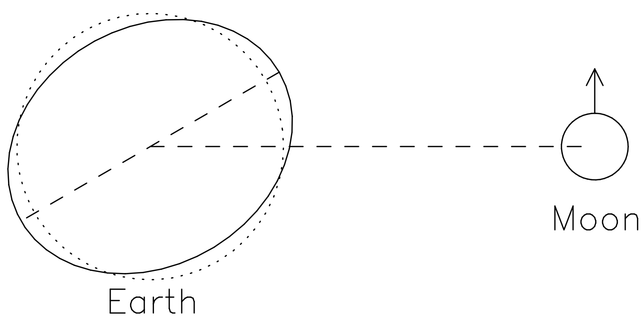

Fig. 2. The bulge caused by the Moon on the Earth, in the 'small friction' model (not to scale) leads the orbital motion by a constant angle.

The gravitational attraction of the Moon causes the Earth to bulge, both in the direction of the Moon and away from it. In Darwin's description, this bulge is not directed towards the Moon, but leads it by a *constant angle*, as shown in Figure 2. This angle is determined by the time scale of energy dissipation due to the tidal forces. Each of the bulges contains a mass μ given by

$$\mu = \frac{1}{2} k m \left(\frac{R}{a}\right)^3 \tag{6}$$

where k is a constant which depends on the mass distribution within the Earth. By integration of the forces exerted by the two bulges over the orbit, the time derivative of the semi-major axis may be computed. The result is

$$\dot{a} \equiv \frac{da}{dt} = 6 \frac{k}{T} \frac{m}{M} \left(1 + \frac{m}{M}\right) \left(\frac{R}{a}\right)^8 a \left(\frac{\Omega}{\omega} - 1\right) \tag{7}$$

where T is a time scale proportional to the time scale of the energy dissipation. In principle k/T should be derived from a detailed theory; in practice it is easier to determine k/T from the measured value of $\dot{a}$.

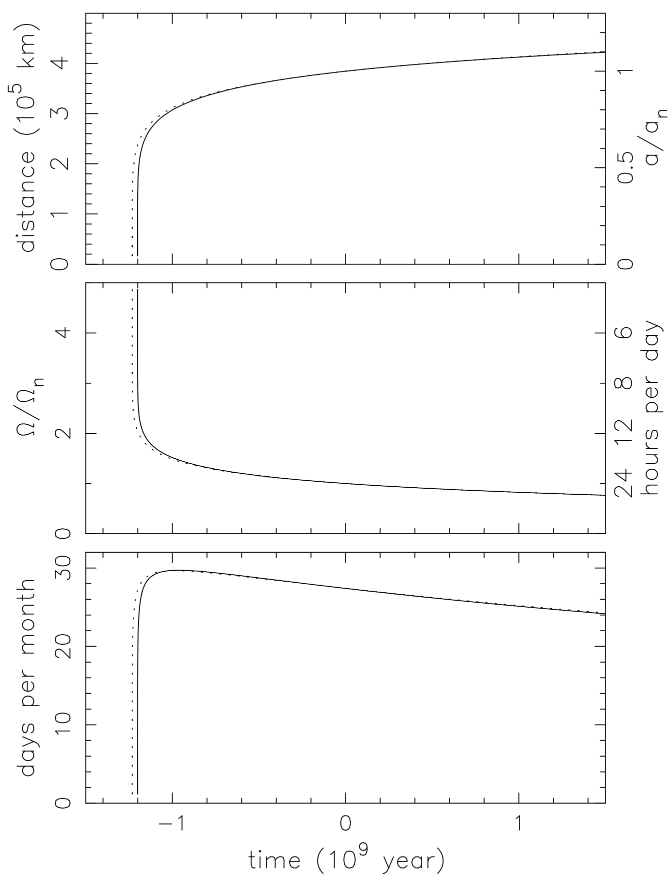

Fig. 3. The distance between the Earth and the Moon, the rotation velocity of the Earth, and the number of days per Month, in the weak friction model. The solid lines show the approximation for a circular orbit, the dashed lines include eccentricity.

In the last decades $\dot{a}$ has been measured accurately with lunar ranging. The Apollo 11, 14 and 15 astronauts have put mirrors on the Moon; one mirror was put in place by the Lunakhod 2. Pulses of laser light are sent towards the Moon, and the time intervals are measured until the detection of the returning pulses. This is not a simple measurement! Even a laser beam spreads out at the distance of the Moon, and only 1 of every 10^9 photons hits the mirrors; from these, only a similar fraction hits the telescope on Earth. With further losses in detection efficiency, in the end only about 1 of every 10^{21} photons sent to the Moon are detected after return. It is therefore a stunning achievement that the average change in distance towards the Moon has been measured[2]; its value is given in Table 1.

The differential equation 7 can be solved numerically, together with eqs. 3 and 5. The resulting evolutions of a, ω and Ω are shown in Figure 3.

For comparison, I also show the numerical solution to the equations that take into account the eccentricity of the orbit of the Moon (these equations are given by Hut[6]). An interesting aspect of the numerical solution is that the eccentricity of the lunar orbit is increasing with time, i.e. it was smaller in the past (Figure 4). The reason for this is that the transfer of angular momentum from the Earth to the lunar orbit corresponds to a transfer of energy; this energy is used partially in increasing the distance, and partially in increasing the eccentricity. (For the same angular momentum, eccentric orbits have a higher energy than circular orbits.) The small current and past eccentricity of the lunar orbit leads us to expect that the differences between the analytic solution of the simplest equation and the numerical solution are small, and this indeed is the case[15].

In particular, both numerical solutions show that the Moon was very close to the Earth about 1.2 Gyr ago. At the time of George Darwin, this was an argument for an origin of the Moon as a fragment from the Earth; once this fragment was dissociated from the Earth, the excess rotational energy of the Earth, combined with tidal forces, would expell the Moon to the larger distance where we now observe it.

4. The tidal catastrophe

To see that such a close distance has catastrophic consequences, we consider the energy equation. The total energy (again assuming a circular orbit) of the binary is

$$E = -\frac{GMm}{2a} + \frac{1}{2}I\Omega^2 \tag{8}$$

The change in energy is

$$\dot{E} = \frac{GMm}{2a^2}\dot{a} + I\Omega\dot{\Omega} \tag{9}$$

We use the time derivative of the angular momentum equation 4

$$\dot{J} = 0 = \frac{Mm}{2}\sqrt{\frac{Ga}{M+m}}\frac{\dot{a}}{a} + I\dot{\Omega} \tag{10}$$

to eliminate $\dot{a}$ from eq. 9 and find:

$$\dot{E} = I\dot{\Omega}(\Omega - \omega) \tag{11}$$

This form of the equation shows nicely that the energy change is zero in corotation, i.e. when $\Omega = \omega$; in other words, corotation is a minimum energy situation.

If we use the value of $\dot{a}$ measured by lunar ranging, we derive a required current energy dissipation of about 3.5 terawatt (i.e. $3.5\,10^{12}$ Watt). To get some idea of the importance of the energy dissipation, we compare it with the amount of solar radiation intercepted by the Earth:

$$\dot{E}_\odot = \frac{L_\odot}{4\pi \mathrm{A.U.}^2}\pi R^2 \simeq 1.75\,10^{17}\,\mathrm{Watt} \tag{12}$$

where $L_\odot$ is the solar luminosity and A.U. the distance of the Earth to the Sun. Since we know Ω and $\dot{\Omega}$ as a function of time, we can plot the energy dissipation also, and this is done in Figure 4.

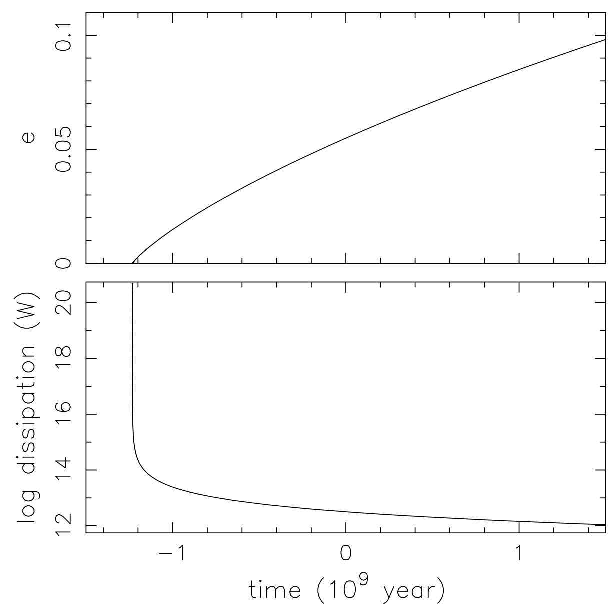

Fig. 4. Eccentricity of the lunar orbit, and the energy dissipation on the Earth associated with the tidal forces of the Moon, in the weak friction model, as a function of time.

It is seen that the tidal energy dissipation increases dramatically as the distance between the Earth and the Moon decreases; close to the smallest distance in the past the tidal energy dissipation was a thousand times higher than the energy received from the Sun. We can estimate the temperature of the crust with the black body temperature required to emit such an energy:

$$L \simeq 10^3\dot{E}_\odot = 4\pi R^2\sigma {T_c}^4 \Rightarrow T_c \simeq 1600\,\mathrm{K} \tag{13}$$

This value is high enough to completely melt the crust of the Earth. The geological record contains rock layers with ages of about 3 Gyr, however, so that we know that no such catastrophic melting can have occurred since then.

Another catastrophe may be expected at such a short distance of the Moon: the tidal force of the Earth will disrupt the Moon. A rough estimate of the distance at which this happens may be found by considering the forces on a test mass on the surface of the Moon facing the Earth. By equating the difference in the gravitational force of the

Earth on the test mass and the center of the Moon to the force exerted by the Moon on the testmass, we get:

$$\frac{GM}{(a-r)^2} - \frac{GM}{a^2} \simeq 2\frac{GM}{a^3}r = \frac{Gm}{r^2} \Rightarrow \frac{r}{a} \simeq \left(\frac{m}{2M}\right)^{1/3} \quad (14)$$

Solving this equation for a we get the distance a_t at which the Earth will disrupt the Moon: $a_t \simeq 5.5r \simeq 0.025a_n$. A more accurate estimate, due to Roche, gives $a_t \simeq 0.043a_n$, or about 16 500 km. Thus, the distance at which the tidal forces of the Moon wreak havoc with the surface of the Earth is similar to the distance at which the Earth destroys the Moon.

5. The rotation of the Earth

Since the simple model leads to a wrong conclusion, we consider another aspect of it, the rotation of the Earth. With a current value of $\dot{a}$ as measured with lunar ranging (Table 1), Eq. 10 predicts a value for the change in rotational velocity of the Earth $\dot{\Omega} = -5.60\,10^{-22}/\mathrm{s}^2$. If we write the length of the day as $P \equiv 2\pi/\Omega$, we get for the change in the length of the day

$$\Delta P = -\frac{2\pi}{\Omega^2}\Delta\Omega = -\frac{2\pi}{\Omega^2}\dot{\Omega}\Delta t \quad (15)$$

Thus, the current change of the length of the day is 0.002 s per century, according to our simple theory.

The Sun also exerts a tidal force on the Earth, and dissipation of the tides caused by the Sun also leads to a slowdown of the Earth rotation – and a corresponding increase in the orbit of the Earth around the Sun. The angular momentum of the Earth orbit is so much bigger than the rotational momentum of the Earth, that the change in the orbit is imperceptible, but the rotation of the Earth is perceptably affected. We can make a quick estimate of the change in the distance of the Earth to the Sun $\dot{\mathrm{AU}}$ (the astronomical unit) due to tides by replacing the mass of the Moon in eq.7 with the mass of the Sun $M_\odot = 1.9891 \times 10^{33}$ g. Dividing the two versions of eq.7 for the Sun and the Moon (which eliminates the factor k/T), and noting that $M_\odot \gg M \gg m$, $\Omega \gg \omega$ and that a day is much shorter than a year, we obtain

$$\begin{aligned} \dot{\mathrm{AU}} &\simeq \frac{M_\odot}{m}\frac{M_\odot}{M}\left(\frac{a}{\mathrm{AU}}\right)^7 \frac{1\mathrm{year}}{1\mathrm{month}}\,\dot{a} \simeq 0.9 \times 10^{-4}\dot{a} \\ &\simeq 3.4 \times 10^{-4}\,\mathrm{cm/yr} \end{aligned} \quad (16)$$

which is indeed negligible even on a time scale of a billion years. Entering $\dot{\mathrm{AU}}$ according to Eq.16 in Eq.10, and replacing m with $M_\odot$ in the same equation, we find the relative effects of Sun and Moon on the rotation of the Earth as

$$\frac{\dot{\Omega}_S}{\dot{\Omega}} \simeq \frac{M}{m}\sqrt{\frac{M_\odot a}{M\mathrm{AU}}}\frac{\dot{\mathrm{AU}}}{\dot{a}} \simeq 0.2 \quad (17)$$

Thus the Sun adds some 20% to the slowdown value that we derived for the Moon. (Note that we have ignored here the effect of the tides raised by the Earth on the Sun on the rotation of the Sun. This neglect can be justified by use of Eq.7, using the Earth mass for m and the mass and rotation of the Sun for M and Ω.)

A full computation, taking into account that Earth is not a perfect sphere, that its rotation axis is not perpendicular to the orbit of the Moon, and the contribution of the Sun leads to a predicted decrease of the rotation of the Earth of approximately 2.3 milliseconds per century.

Our simple theory comes remarkably close to this, and thus apparently describes the main contributions to the slowdown of the Earth rotation. We now proceed to compare this theoretical prescription with four types of measurements, on four time scales: highly accurate measurements using radio telescopes and lunar ranging during the last decades, solar eclipse measurements going back two thousand years, layer counts in shells and corals going back 400 milion year, and counting layers in mud-depositions going back 900 million year.

6. Modern measurements of the rotation of the Earth

Due to the rotation of the Earth, stars appear to move across the sky. By measuring the time interval between two moments that a star is exactly south from the observer (in the technical jargon, between two 'southern meridian passages') one can determine the length of a day. More generally, the length of the day can be derived from the speed with which a star appears to move. The larger the telescope, the more accurate the measurement of the position of a star. Not only stars, but all celestial objects may be used for such measurements, in particular also black holes in the cores of faraway galaxies, that are strong radio sources. The most accurate measurements of the rotation of the Earth are made by connecting radio telescopes across the world ('Very Long Baseline Interferometry' or VLBI) to measure the position of quasars. The advantage of using such faraway objects is that any motion that these sources themselves may have is too small to have an effect on the measurement.

The lunar ranging already discussed can be used similarly, albeit that it has to be taken into account that the Moon itself also moves. Such measurements therefore give a measurement of the sum of the rotation speed of the Earth and the revolution speed of the moon. Similar techniques may be used with artificial satellites.

Combining the three types of measurement – VLBI, lunar ranging, and satellite ranging – a very accurate record of the rotation speed of the Earth can be constructed from 1963 onwards, when these measurements were started. In discussing these, I follow a review by Hide and Dickey[5]. In Figure 5 the difference is shown between the measured length of each day since 1963 and the length of the day computed from its value at the beginning of the measurements and the assumption that the length of the day increases 1.7 millisecond per century. (Why this value is taken rather than the 2.3 milliseconds per century of the previous chapter is discussed in the next section. The offset of $\sim 2\,\mathrm{ms}$ is due to the definition that day_o-day_c was zero around 1860.)

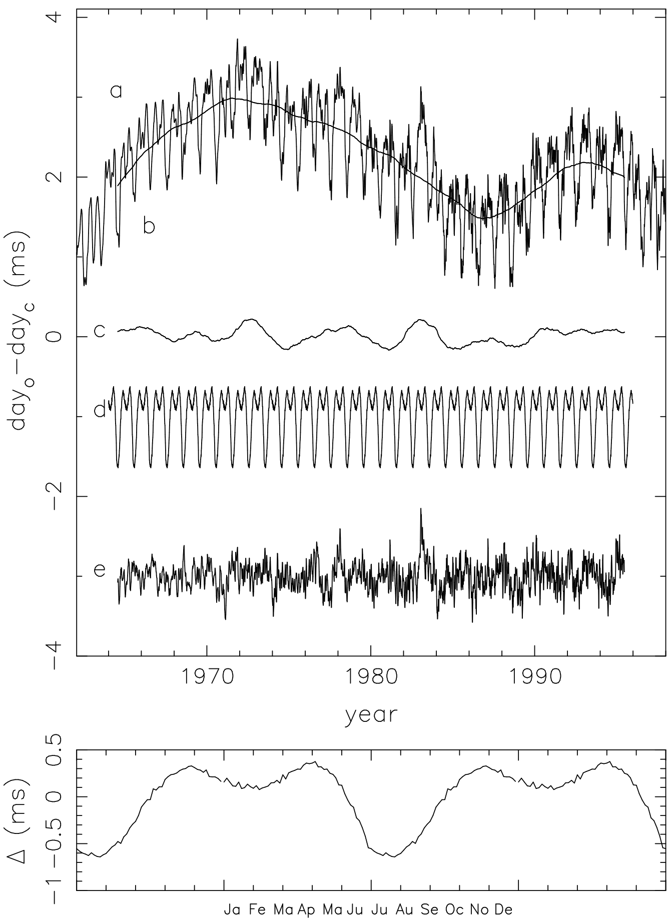

Fig. 5. Differences in milliseconds between observed length of the day, as derived from modern measurements, and the length of the day computed assuming a constant change of 1.7 millisecond per century. The top curve (a) shows the actual measurements, with a running five-year average (b) drawn through it. The curve (c) below it shows the changes from year to year, computed by taking the difference between a one-year running average and a five year average of curve (a). The variations that return every year are shown as curve (d), and enlarged in the lower frame; they are computed by subtracting curves (b) and (c) from curve (a), and by folding the remainder on a one-year period. Finally, the irregular variations (e) are found by subtracting curves (b), (c) and (d) from (a). For clarity, curves (d,e) have been shifted down by 1 and 3 ms, respectively. Data kindly provided by Dr. J.O. Dickey. Adapted from Fig. 2 of Hide & Dickey[5] (1991).

To analyse these differences, we first take a five-year running mean. This gives the mean change on a time scale of five years, and is shown as curve b) in the figure. In the second step, we subtract the five-year running mean from the one-year running mean, to determine the year-to-year variations, shown as curve c). We then subtract curves b) and c) from the measured values. The remaining values are averaged for each time in the year over all years during which measurements were made – the result gives the seasonal variation shown as curve d). Finally, the difference between this average and the values for each day is shown as curve e).

The first thing we note from the data in Figure 5 is that at a time scale of twenty years and shorter all kinds of variations are present, which prevent us from determining an accurate value for the changes on longer time scales. To put it differently, if we had subtracted 1.5 or 1.9 instead of 1.7 milliseconds per century from the measured values, Fig. 5 would still look pretty similar. We are faced with the disappointing result that *we cannot derive the long-term change in the rotation of the Earth from the extremely accurate modern measurements*! This disappointment, however, is more than compensated for by the wonderful physics that we can derive from the observed variations.

To do this, we abandon our simple model that the Earth is a single solid sphere, and acknowledge that the Earth is composed of different interacting layers. These layers and their relative contribution to the total angular momentum of the Earth are from inside out the solid core (7×10^{-4}), the fluid interior (0.1), the solid crust (0.9), the oceans (3×10^{-4}), and the atmosphere (10^{-6}).

All variations on short time scales are due to exchange of angular momentum between the solid crust and the atmosphere (and to a lesser extent the oceans). This can be verified directly because the measurements of pressure and velocity of the atmosphere are good enough to enable the meteorologists to compute the angular momentum in the atmosphere from day to day. Comparison of this momentum with that derived from the rotation velocity of the crust shows that the two vary in tandem, with opposite sign. The seasonal variations (curve d) are due to seasonal variations in the wind directions: to visualize this, think of a seasonal wind that blows on one side of a mountain during one half of the year and on the other side the other half, thus in turn braking and speeding up the rotation of the Earth crust. The other short-term variations correspond to changes in the pattern of the weather of the world, on time scales of years (curve c) and from day to day (curve e). The well-known change in weather pattern known as El Niño is evident in these curves both in 1977-1978 and in 1982-1983, the latter one giving rise to the largest changes in the angular momentum of the Earth rotation ever measured!

From the changes in angular momentum of the atmosphere, as found from meteorological data, we can compute predicted changes in the rotation of the crust; comparing these with the observed changes one finds differences of about 0.2 milliseconds. Recently it has been shown that these differences correlate well with variations in the angular momentum of the oceans, as computed from oceanographic data[8].

The variation on a time scale of 5 yr (curve b in Fig. 5) is thought to be due to exchange of angular momentum between the fluid interior of the Earth and the solid crust. At the moment the measurements of the velocities of the fluid interior are not good enough to verify this directly.

7. Historical eclipses of Sun and Moon

Let us return for a moment to the hypothetical example in Sect. 1. From the fact that the eclipse occurred in Bagdad and not in Alexandria Halley derived that the Month was longer in the past than now. However, the same observation may be explained with a different assumption, viz. that the rotation of the Earth was faster in the past. In general, the difference between a computed and observed eclipse location is due to the combined effect of changes in the rotation velocity of the Earth and the revolution velocity of the Moon.

In analyzing historical records of eclipses it is nowadays common to assume that the change in the revolution of the Moon is constant, and given from eq. 3 by

$$\begin{aligned}\dot{\omega} = -\frac{3\omega}{2a}\dot{a} &\simeq -1.26\,10^{-23}\mathrm{rad\ s}^{-2} \\ &\simeq -25.83\,\mathrm{arcsec\ cy}^{-2} \qquad (18)\end{aligned}$$

The argument underlying this assumption is that the time scale for the variation in the lunar orbit is set by the energy dissipation (see eq. 7); that this dissipation occurs in the oceans; and that there is no evidence that the oceanic currents have changed in historical times. Thus, one computes the times and locations of past eclipses taking into account the change in the length of the Month, and then interprets the remaining differences as due to changes in the length of the day.

Our information of the lunar orbit for the last three hundred years is quite accurate, thanks to the telescope. Measurements are made of the exact time that the Moon occults a star. Thus, an accurate clock is required, and such clocks were indeed available. The changes in the length of day derived from these data are shown in Figure 6. The average change between 1660 and 1980 is slower than the 2.3 ms/cy expected from theory, viz. 0.73 ms.cy. On top of this an average trend we note irregular variations. Between 1860 and 1900, in hardly forty years, the length of the day varied by 7 ms with respect to the average change, from 3 ms shorter to 4 ms longer. This variation is stronger than any seen in the modern data of Figure 5, and it would be interesting to know whether it is caused by dramatic changes in the weather.

Information of solar eclipses over the last two thousand years has been collected by Richard Stephenson[14] from records of Babylonian, Chinese, Greek and Arabic astronomers as well as European chronicles. Taking into account the change of the lunar orbit, but assuming that the day has a constant length, one can predict at what time (and in which location!) historical eclipses should be seen. The differences between the predicted times and the times at which the eclipses actually were seen are shown in Figure 7.

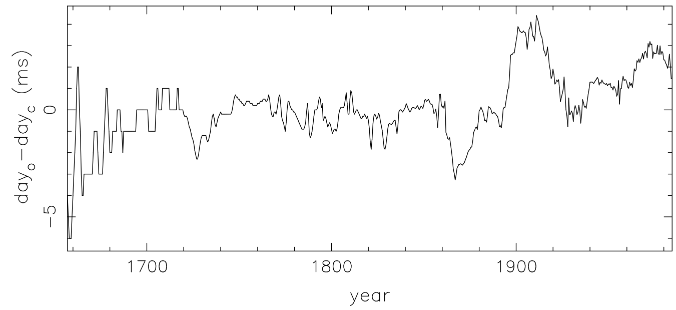

Fig. 6. Differences in milliseconds between observed length of the day, as derived from occultations of stars by the Moon observed with telescopes, and the length of the day computed assuming a constant change of 0.73 millisecond per century. Data taken from McCarthy & Babcock[9] (1986).

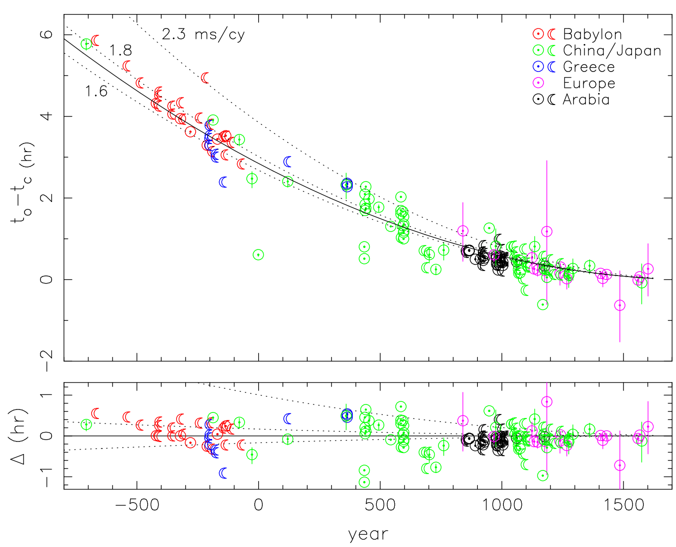

Fig. 7. Top: Differences in observed and computed time of eclipses from before the invention of the telescope, where the computed time takes into account the variation in the lunar orbit, but assumes a day of constant length. The solid line gives the differences expected if the day changes 1.7 ms/cy; dashed lines are for 1.6, 1.8 and 2.3 ms/cy. Below: Difference between observed and computed time of eclipse assuming that the day changes 1.7 ms/cy. Data taken from Stephenson (1998).

The oldest dated solar eclipse in China and lunar eclipse in Babylon, at 708 and 665 BC respectively, were observed almost six hours later than predicted for constant rotation of the Earth. From this one derives that the average change in the length of the day over the last 2700 yr is 1.7 millisecond per century. The differences between observed and computed eclipse times for this value are also shown in Figure 7, as are curves for 1.6, 1.8 and 2.3 ms/cy.

Some points are far from the curve for 1.7 ms/cy. Examples include the lunar eclipse in Babylon 214 BC, and the solar eclipse in China 1 BC. These points appear to be too far from the other observations to be explained with irregular variations in the rotation of the Earth (like those between 1860 and 1900 in Figure 6). Something must be wrong with these points: the observation itself may be wrong, its report inaccurate, or the identification with a computed date and location incorrect.

Stephenson suggests that the difference between the predicted 2.3 and the observed 1.7 ms/cy is due to the last ice age. During an ice age a thick layer of ice weighs on the poles, causing the Earth to bulge at the equator and thus increasing its angular momentum. As the ice melts, the Earth reverts to a more spherical form, and the reduction of angular momentum increases the rotation velocity. Thus, the dissipation of the tides of Moon and Sun lengthens the day by 2.3 ms/cy, and the change in form of the earth compensates for 0,6 ms/cy.

It appears to me that this cannot be the full explanation. In Figure 7 we see that the change in the length of the day varies appreciably from the average over the last 2700 years. Thus most points in the centuries surrounding 500 AD and 1000 AD lie above and below the 1.7 ms/cy curve, respectively. The latter points are shown on an expanded scale in Figure 8, together with curves for 2.3 1.7 and 0.73 ms/cy. We see that the average change over the last 1000 years lies between 0.73 and 1.7 ms/cy. Thus the changes on time scales of 1000 yr are larger in size than the 0,6 ms/cy that Stephenson seeks to explain with the end of the last ice age a hundred thousand years ago. This suggests that other, short-term effects, are at work. Perhaps changes in large oceanic streams are involved: it is known for example that the gulf-stream stopped during the so-called little ice age of 1645 to 1715.

8. Growth layers in shells and corals; mud layers

Corals and shells grow by the addition of new layers. It is thought that one layer is added each day, perhaps under the influence of the change in light. Since the amount of light at night changes with the phase of the Moon one could imagine that there is a monthly variation too, and such a variation is indeed found in fossils of corals as well as shells, going back to some 400 million year. Annual variations are also found. By carefully counting growth rings we can thus determine the number of days per month and per year.

The study of these growth rings is not without problems. For example, not all corals show yearly variation, and it is rather puzzling that no annual variation is found

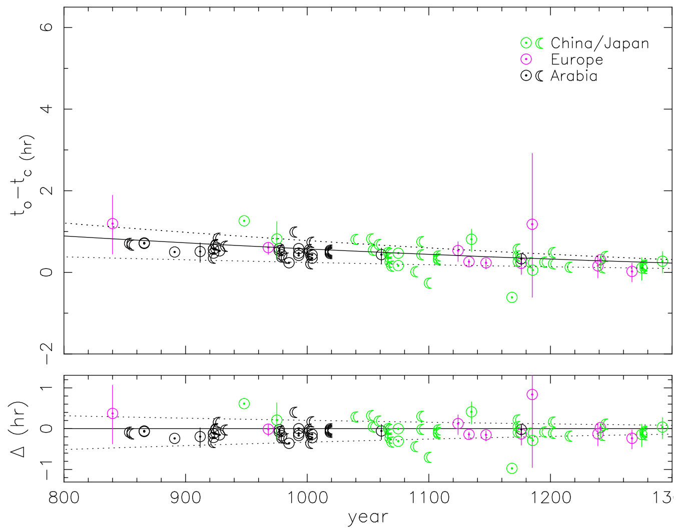

Fig. 8. As the previous figure, but enlarged for the middle ages. The solid line is for a change in length of day of 1.7 ms/cy; dashed lines for 0.73 and 2.3 ms/cy. Data taken from Stephenson (1998).

in present day relatives of those types of coral that show clear annual change in paleological times. It is not known whether the ring thickness is larger in winter or in summer. If a ring is added each day, it could be that on a day with particularly bad weather no ring is added, which would leave the month with one ring short. In the absence of good understanding of the process of growth, it is clear that the data have to be interpreted with care.

Various authors have made counts and determined the number of days per month in corals and shells from different paleological periods. In Figure 9 I show the best data available, taken from a review by C.T. Scrutton[12]. I include only those measurements for which the authors give error estimates – without which data are essentially useless.

We are now faced with a subtlety that I have so far managed to avoid: the difference between sidereal and synodic days and months (see Figure 10). Consider a place on earth which has the Sun in its Zenith. During the time that this place completes one rotation with respect to the stars around the center of the Earth, the Earth has moved a bit in it orbit around the Sun. Therefore the Earth has to rotate a bit more before our place has the Sun again in its zenith: the solar day (technically called the synodic day) is slightly longer than the stellar day (the sidereal day). The number of stellar days n in a year is exactly one higher than the number of solar days $n_s = n - 1$. A wholly analogous reasoning tells us that the number of stellar (sidereal) months in a year m is exactly one higher than the number of solar (synodic) months $m_s = m - 1$. At present the solar day D_s is 24 hours, the sidereal day D is about 4 minutes shorter; the synodic months M_s is

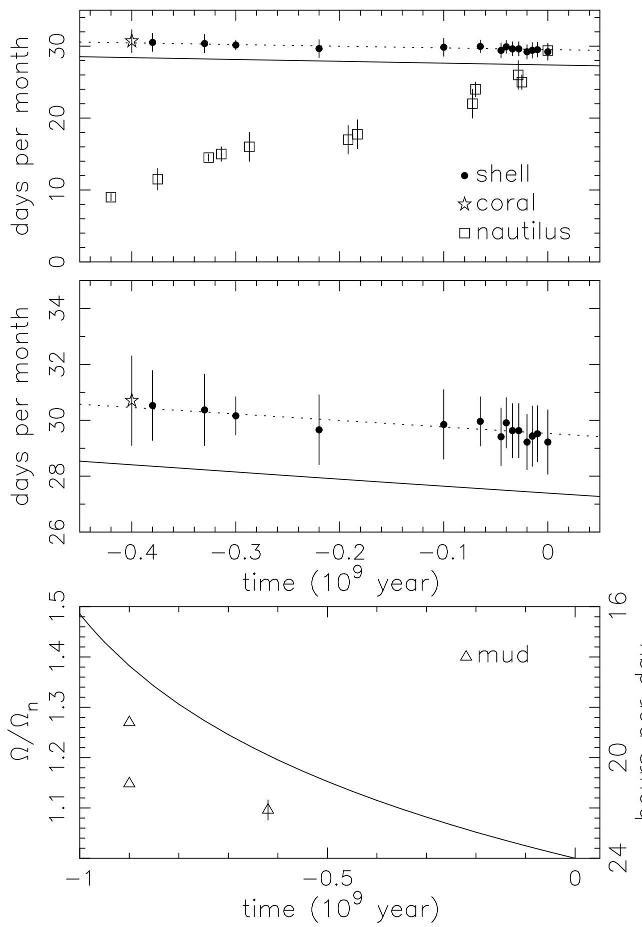

Fig. 9. Top: Number of days per month as a function of time, as derived from bivalve shells, corals, and nautilus. The solid curve gives the theoretical prediction as in Figure 3, the dashed curve gives the number of solar days per solar month. The values derived from fossil nautilus clearly are not acceptable. Data taken from Scrutton[12] (1978). Middle: enlargement of top figure. Below: rotation velocity of the Earth (in units of the current rotation velocity) according to theory (solid curve as in Figure 3) and as derived from fossil mud layers. The two points at −900 Myr indicate the difference between two published analyses.

about 29.5 (synodic) days, the sidereal month M about 27.3.

The shells and corals in the sea regulate their variations on the synodic periods. We thus must convert the number of sidereal days per sidereal month, as given by Eq. 5 to the number of synodic days per synodic month for comparison with the data from shells and corals. The conversion is given by

$$\frac{M_s}{D_s} = \frac{M}{D}\,\frac{1-\frac{1}{n}}{1-\frac{1}{m}} = \frac{\Omega}{\omega}\,\frac{\left(1-\frac{1}{n_n}\frac{\Omega_n}{\Omega}\right)}{\left(1-\frac{1}{m_n}\frac{\omega_n}{\omega}\right)} \tag{19}$$

The result is given by the dashed line in Figure 9. We see that the data are close to the line, and also that the errors are relatively large. A straight line fitted to the data of shells and corals has a slope of 2.5±2.0 Gyr^{-1} (1-σ error), i.e. the data themselves do not provide significant proof for any change in the length of the day.

Virtually all ring counts have been made by eye, and this raises the question how reliable they are. As pointed out by Scrutton[12], most publications lack details about the mode of counting and about the actual numbers of layers and days counted. It appears that the use of shells and corals for the determination of the length of the day, after a promising start in the 1970s, has come to a standstill. Little news has been published on the topic sinds 1980. It is remarkable that the one animal in which rings can be counted easily[11] gives results that contradict those for bi-valve shells and corals. The living nautilus adds a ring every day to its shell, and a section wall every month. By counting rings between walls for fossilised nautilus, one thus may determine the number of days per month in paleological times. The results are shown in Figure 9. It is seen that the nautilus had 9 days between section walls 400 Myr ago. The conclusion that the month had nine days so recently is unacceptable, however: such a fast change exacerbates the problems discussed in Sect.4.

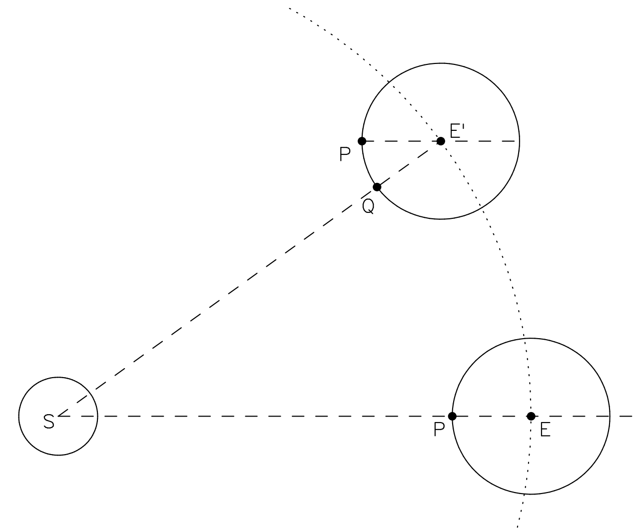

Fig. 10. Schematic drawing to explain the difference between solar and sidereal days (not to scale). When the centre of the Earth is at E, it is mid-solar-day in P; S is the center of the Sun. P rotates counter-clockwise around E, and simultaneously E moves counter-clockwise around S. After a sidereal day P has made exactly one revolution, and the centre of the Earth has arrived in E'. It is then mid-solar-day in Q; in P it is before mid-day. Only after some extra rotation will P be at mid-solar-day; the solar day thus is longer than the sidereal day.

Data for even older epochs can be obtained from variations in the thickness of mudlayers in old sediments. In some estuaria each tide is accompanied by the deposition of a layer of mud: the receding water at low-tide carries some mud and deposits it in in slightly deeper water. At spring tide, when Moon and Sun are aligned, a stronger stream may lead to extra mud deposition. This line of reasoning explains the regular variations in thickness of old sediments like those near Adelaide[16] (Australia) of 600 Myr ago, and in the Big Cottonwood Canyon[13] (Utah) of 900 Myr ago. Counting the number of layers between two extra thick layers gives the number of days per month. Yearly variations in thickness are also found, perhaps due to seasonal variations in the local sea flow.

Because the length of the year hasn't changed significantly, the number of days per year directly gives the rotation velocity of the earth. As with the shells, however, interpretation of the data is not without ambiguities, as witnessed by the fact that two different values have been derived from the same material from Big Cottonwood Canyon. Nonetheless, as shown in Figure 9, all results indicate a slower change in the rotation of the Earth in the geologic past than derived from the current rate of change. This is good news, as it implies that it is more than 1,2 Gyr ago that the Moon was very close to the Earth. More data are required for a more accurate history of the rotation of the Earth, and these may be forthcoming from suitable mud depositions.

What is the cause for the geologic variation in the rate of change of the distance to the Moon and the related change in the rotation of the Earth? Most likely it is the change in the form of the oceans, due to the drift of the continents. Maps of the location of the continents show that the form of the oceans varied dramatically over the last 500 Myr. A different form of the oceans implies a different rate of dissipation of the ocean flow. As shown by Eq. 11 this immediately affects the change in rotation of the Earth. Our simple treatment of Darwin's theory for the tides in Section 3 didn't take this into account.

9. The history of the Moon

In the last decade scientists have come to the conclusion that the Moon most probably formed after an object – possibly as big as Mars – collided with the Earth. The object was completely disrupted in the collision, together with part of the crust of the Earth. The debris moved in orbit around the Earth, the bits and pieces with higher specific weight (like iron) fell on Earth, those with lower specific weight (silicates) coagulated to form the Moon, just outside the Roche limit[7]. The model explains that the Moon has no iron core. Even though the Moon is much smaller than Mars, impact of a large body is required to explain the large angular momentum in the Earth-Moon system.

This angular momentum resided mainly in rapid rotation of the Earth, but has been transferred by tidal forces to the orbit of the Moon. Thus, the Moon which was formed at about 17 000 km from the Earth is now at 380 000 km. The rate of expansion must have been rather irregular: at first there were no oceans on the Earth. Only after the Earth cooled down, could oceans be formed and energy dissipation pick up, and each change in form and depth of the oceans has been accompanied by a change in the rate with which the Moon receded.

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