Problem Set III: Luminosities

due 23 Oct 2017

For general comments about problem sets, see problem set I. For this specific one, you may find it useful to read CO Ch. 10, "The Interiors of Stars." The three parts have equal weight.

III.1. The luminosity of a star

Consider a star in hydrostatic equilibrium in which energy is transported by radiation.

1. Use the equations of radiative energy transport and hydrostatic equilibrium to derive two scaling relations for $T_{\rm c}$ in terms of other stellar properties (here, assume the gas is described by the ideal gas law). Combine the two to derive how L scales with M and R.
2. Describe in physical terms why your result is independent of the source of energy.

III.2. The Eddington Luminosity

Above a certain luminosity, called the Eddington Luminosity, radiation pressure will exceed gravity and matter will be blown away. Below, you will derive this by direct comparison of forces (for an indirect derivation, see CO, §10.6). We will consider a completely ionised, pure hydrogen gas, in which photons interact with matter (almost) exclusively through electron scattering. We will also assume the gas is optically thin, i.e., all electrons present can interact with photons.

1. Consider a single electron that scatters a single photon with energy $h\nu$ in a random direction.
   (a) On average, what momentum is imparted on the electron? What is its direction?
   (b) Use your result to show that for a given flux F and cross-section σ, the force on a single electron is $F\sigma/c$ (*Here, 'show' may be easier done in a few words rather than equations.*).
2. Consider a blob of gas with mass m at distance r from a star with mass M and luminosity L.
   (a) Write down the equations for the gravitational force and for the radiation force. Do the gravitational force and the radiation force act equally on protons and electrons? If not, what keeps the protons and electrons together?
   (b) Equate the two forces and show that your result is consistent with CO, Eq. 10.114 (*Hint: the cross section σ and opacity κ are related by $\sigma n = \kappa\rho$, where n is the number density of relevant particles and ρ the usual density.*)

III.3. Accretion by stellar corpses

'Dead' objects can be made to shine again if one dumps matter on them. We determine how bright they can be maximally, and how far away a good observatory like *XMM-Newton* can see them.

1. How much energy is released if we dump 1 kg of matter on a $M = 1.4\,M_\odot$ neutron star with a radius $R = 10\,\mathrm{km}$? Determine whether this is an efficient way to generate energy by calculating how much of the rest mass one would have to turn into energy to produce the same amount of energy. How does your result compare with nuclear fusion?
2. Calculate the luminosity (in units of the solar luminosity) one would get for a dumping rate $\dot{M} = 10^{-9}\ M_\odot\,\mathrm{yr}^{-1}$ (this *accretion* rate is typical for many binary systems). How does this compare with the Eddington luminosity? (*Use your result from the previous question, or check CO, Eq. 10.114, and use that* $\kappa = 0.04\ \mathrm{m}^2\,\mathrm{kg}^{-1}$ *for pure hydrogen.*) For what accretion rate would one reach the Eddington luminosity?
3. Now assume the neutron star accretes at the Eddington rate.
   (a) What is its effective temperature, T_{eff}? And what is the typical energy kT_{eff} (in eV) of photons? Assuming all photons have this energy, how many photons are emitted?
   (b) The *XMM-Newton* observatory has an effective area of approximately $0.3\ \mathrm{m}^2$, and can detect X-ray sources down to count rates of about $0.01\ \mathrm{s}^{-1}$. How far out (in pc) could a source accreting at the Eddington luminosity be seen? Compare with the distance to Andromeda ($\sim$800 kpc) and the Virgo cluster ($\sim 16\,\mathrm{Mpc}$).