Midterm, 30 October 2006

Duration: 50 minutes

Examination aids: Anything you like, except communication devices.

Note: Answers can be brief and in point-form, but be sure that derivations can be followed.

1. At the of the Sun's life, what will be left is a $0.6\,M_\odot$ core composed of Carbon and Oxygen, in roughly equal amounts.
   (a) [10 points] Estimate how much energy will have been produced by fusion over the Sun's whole life. *Note: the binding energy per nucleon of Carbon is 7.7 MeV, and for Oxygen it is 8.0 MeV.*
   (b) [10 points] Estimate the total amount of energy that the Sun will radiate during its *main-sequence* phase. What fraction is this of the total energy you calculated above?
   (c) [10 points] Following the main sequence, the Sun will go through a number of further phases before becoming a white dwarf. In total, will these further phases last longer or shorter than the main sequence? By what order-of-magnitude ratio? (*order-of-magnitude is* 10^n*, so factor 0.01, factor 10, factor* 10^6*?*) Will the Sun be more or less luminous? By what order-of-magnitude ratio? Does this square with your results above?

2. We consider main-sequence stars with a range of mass.

   (a) [15 points] Derive the scaling relation for central temperature T_c as a function of mass M and radius R for a self-gravitating ball of ideal gas. For main sequence stars, observations of binaries show that $R \propto M^{3/4}$. Show that this implies that $T_\mathrm{c} \propto M^{1/4}$ on the main sequence.

   (b) [15 points] Observations of binaries also show that $L \propto M^4$. Combined with the weak dependence of T_c on mass, what does this imply for nuclear reactions, are they (very) sensitive to temperature or (very) insensitive? Explain your answer based on the physics of these reactions (i.e., what is required for particles to succeed in fusing, and why is that (in)sensitive to temperature?)

Bonus [10 extra points] Assume a nuclear energy generation rate per unit matter [units W/kg] that scales as $\epsilon \propto \rho T_\mathrm{c}^\nu$. Use scaling relations to infer ν.

3. Each of the panels in the figure below shows the velocity orbits of two stars. Answer the following questions [6 points per panel]:

   For each panel, indicate whether or not these two orbits could be from two stars that orbit each other (i.e., are in a binary).
   For those panels for which you think the orbits do not make sense for a real binary, explain why in a few words.
   For those panels for which you think the orbits could be from a binary, (i) write whether the orbit is (close to) circular or eccentric; (ii) indicate where periastron occurs for eccentric orbits; (iii) tell which star is more massive, the one with the solid line or the one with the dashed one.