UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER 2012 EXAMINATIONS

AST221H1 (Fall 2012)

Duration: 3 hours; Aids: Calculator only; Answers in exam booklets

Marks: All five questions have equal weight, with the weight distributed equally over the subitems.

Solar mass	$1\,M_\odot$	$= 1.989 \times 10^{30}\,\mathrm{kg}$
Solar luminosity	$1\,L_\odot$	$= 3.839 \times 10^{26}\,\mathrm{W}$
Solar radius	$1\,R_\odot$	$= 6.955 \times 10^{8}\,\mathrm{m}$
Solar effective temperature	$T_\odot$	$= 5777\,\mathrm{K}$
Earth mass	$1\,M_\oplus$	$= 5.974 \times 10^{24}\,\mathrm{kg}$
Earth radius	$1\,R_\oplus$	$= 6.378 \times 10^{6}\,\mathrm{m}$
Astronomical Unit	1 AU	$= 1.4960 \times 10^{11}\,\mathrm{m}$
Parsec	1 pc	$= 1\,\mathrm{AU}/\tan 1'' = 3.0857 \times 10^{16}\,\mathrm{m}$
Gravitational constant	G	$= 6.673 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Speed of light	c	$= 2.9979 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck's constant	h	$= 6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar$	$= h/2\pi = 1.055 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann's constant	k	$= 1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	$= 5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Radiation constant	a	$= 4\sigma/c = 7.5657 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K}$
Proton mass	m_p	$= 1.67262 \times 10^{-27}\,\mathrm{kg}$
Neutron mass	m_n	$= 1.67493 \times 10^{-27}\,\mathrm{kg}$
atomic mass unit (amu)	u	$= 1.66054 \times 10^{-27}\,\mathrm{kg}$
Electron mass	m_e	$= 9.10939 \times 10^{-31}\,\mathrm{kg}$
Hydrogen mass	m_H	$= 1.67353 \times 10^{-27}\,\mathrm{kg}$
Electron volt	1 eV	$= 1.6022 \times 10^{-19}\,\mathrm{J}$
Bohr radius	a_0	$= \hbar^2/m_e e^2 = 5.292 \times 10^{-11}\,\mathrm{m}$

Planet	Mass ($M_\oplus$)	Equatorial radius ($R_\oplus$)	Semi-major axis (AU)	Albedo
Mercury	0.05528	0.3825	0.3871	0.119
Venus	0.81500	0.9488	0.7233	0.750
Earth	1.00000	1.0000	1.0000	0.306
Mars	0.10745	0.5326	1.5236	0.250
Jupiter	317.83	11.209	5.2044	0.343
Saturn	95.159	9.4492	9.5826	0.342
Uranus	14.536	4.0073	19.2012	0.300
Neptune	17.147	3.8926	30.0476	0.290

1. Suppose you made an imaging survey and found a binary made up of a $20M_\odot$ and a $10M_\odot$ star, with a projected separation of 30 AU.

   (a) Estimate the orbital period (in years). Also, calculate the minimum and maximum orbital period the binary could have. Sketch the corresponding orbits.

   (b) If we measured Doppler shifts for both stars, what is the maximum possible velocity difference expected? Make a sketch of the corresponding situation.

2. We derive mass-radius relations.

   (a) Use the Virial Theorem to show how radius scales with mass for balls of ideal gas that vary in mass but have the same central temperature.

   (b) Do the same for balls of degenerate gas, both for the case that the particles are non-relativistic, and for the case that they are extremely relativistic.

3. The temperature-luminosity or Hertzsprung-Russell diagram (HRD) and the evolution of stars like the Sun.

   (a) Draw a HRD, and sketch the zero-age-main-sequence. Mark the rough locations for 10, 1, and $0.1M_\odot$ main-sequence stars. Be sure to add representative values to the axes.

   (b) Draw the evolutionary track for a $1M_\odot$ star, starting at the main sequence. Label the various stages.

   (c) For each stage, write down the source of pressure *in the core* as well as the source(s) of energy and their location(s) (e.g., on the pre-main-sequence, it would be ideal gas pressure and gravitational contraction throughout the star).

4. Rayleigh scattering and the colour of the sky.

   (a) Describe the concept of mean-free-path, and give the relation between scattering cross-section, the number density and the scattering mean-free-path.

   (b) Why is the sky blue (on Earth)? To help answer this, calculate the the mean-free-paths for green ($\lambda \simeq 500\,\mathrm{nm}$) and red ($\lambda \simeq 650\,\mathrm{nm}$) photons, using that, in Earth's atmosphere, violet photons ($\lambda \simeq 400\,\mathrm{nm}$) have a mean-free-path of $30\,\mathrm{km}$.

   (c) Why is the sky not blue on Mars? Why, instead, is it reddish?

5. Formation of the Solar System and puzzles posed by extrasolar planets.

   (a) Describe (i) the general idea of how our Solar System formed, and (ii) how our formation theories explain its observed properties (list at least three properties; an example is the difference between terrestrial and Jovian planets).

   (b) In the context of the picture of how our Solar System formed, what properties of extrasolar planetary systems have been surprising? List at least two, briefly describing why these are surprising.