UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER 2007 EXAMINATIONS

AST221H1 (Fall 2007)

Duration: 3 hours; Aids: Calculator only; Answers in exam booklets

Marks: All five questions have equal weight, with the weight distributed equally over the subitems.

Solar mass	$1\,M_\odot$	$= 1.989 \times 10^{30}\,\mathrm{kg}$
Solar luminosity	$1\,L_\odot$	$= 3.839 \times 10^{26}\,\mathrm{W}$
Solar radius	$1\,R_\odot$	$= 6.955 \times 10^{8}\,\mathrm{m}$
Solar effective temperature	$T_\odot$	$= 5777\,\mathrm{K}$
Earth mass	$1\,M_\oplus$	$= 5.974 \times 10^{24}\,\mathrm{kg}$
Earth radius	$1\,R_\oplus$	$= 6.378 \times 10^{6}\,\mathrm{m}$
Parsec	$1\,\mathrm{pc}$	$= 3.0857 \times 10^{16}\,\mathrm{m}$
Astronomical Unit	$1\,\mathrm{AU}$	$= 1.4960 \times 10^{11}\,\mathrm{m}$
Gravitational constant	G	$= 6.673 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Speed of light	c	$= 2.9979 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck's constant	h	$= 6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar$	$= h/2\pi = 1.055 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann's constant	k	$= 1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant	σ	$= 5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Radiation constant	a	$= 4\sigma/c$
Proton mass	m_p	$= 1.67262 \times 10^{-27}\,\mathrm{kg}$
Neutron mass	m_n	$= 1.67493 \times 10^{-27}\,\mathrm{kg}$
atmoic mass unit (amu)	u	$= 1.66054 \times 10^{-27}\,\mathrm{kg}$
Electron mass	m_e	$= 9.10939 \times 10^{-31}\,\mathrm{kg}$
Hydrogen mass	m_H	$= 1.67353 \times 10^{-27}\,\mathrm{kg}$
Electron volt	$1\,\mathrm{eV}$	$= 1.6022 \times 10^{-19}\,\mathrm{J}$
Bohr radius	a_0	$= \hbar^2/m_e e^2 = 5.292 \times 10^{-11}\,\mathrm{m}$

Planet	Mass ($M_\oplus$)	Equatorial radius ($R_\oplus$)	Semi-major axis (AU)	Albedo
Mercury	0.05528	0.3825	0.3871	0.119
Venus	0.81500	0.9488	0.7233	0.750
Earth	1.00000	1.0000	1.0000	0.306
Mars	0.10745	0.5326	1.5236	0.250
Jupiter	317.83	11.209	5.2044	0.343
Saturn	95.159	9.4492	9.5826	0.342
Uranus	14.536	4.0073	19.2012	0.300
Neptune	17.147	3.8926	30.0476	0.290

1. Consider a binary at a distance of 10 pc composed of a $1\,M_\odot$ and a $0.5\,M_\odot$ star in a circular, one-year orbit.

   (a) Sketch the orbits of the two stars.

   (b) As seen from Earth, what is the maximum angular separation these stars would have? In what situation would this occur?

   (c) From Earth, what are the maximum radial velocities (Doppler shifts) of the two stars that we could measure? In what situation would these occur?

2. We derive mass-radius relations.

   (a) Use the Virial Theorem to show how radius scales with mass for balls of ideal gas that vary in mass but have the same central temperature.

   (b) Do the same for balls of degenerate gas.

3. The Hertzsprung-Russel Diagram and cluster properties.

   (a) Draw a Hertzsprung-Russel (temperature-luminosity) diagram, and sketch the zero-age-main-sequence in it. Mark the rough locations for a $10M_\odot$, a $1M_\odot$ and a $0.1M_\odot$ star on this main-sequence (be sure to label the axes, and indicate the range in temperature and luminosity by adding rough, representative values).

   (b) What information about a stellar cluster can one obtain from its Hertzsprung-Russell diagram? And how?

4. Rayleigh scattering and the colour of the sky.

   (a) Describe the concept of mean-free-path, and give the relation between scattering cross-section, the number density and the scattering mean-free-path.

   (b) In Earth's atmosphere, a violet photon ($\lambda \simeq 400\,\text{nm}$) has a mean-free-path of 30 km. How long is the mean-free-path for a green photon ($\lambda \simeq 500\,\text{nm}$) and for a red photon ($\lambda \simeq 650\,\text{nm}$)?

   (c) On a clear day, why does the horizon appear whiter than the zenith?

5. Formation of the Solar System and puzzles posed by extrasolar planets.

   (a) Describe the general idea of how our Solar System formed, and how it can reproduce its observed properties (you should include at least three properties; an example is the difference between terrestial and jovian planets).

   (b) In the context of the picture of how our Solar System formed, what properties of extra-solar planetary systems have been surprising?