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CONTACT BINARIES WITH ADDITIONAL COMPONENTS. II. A SPECTROSCOPIC SEARCH FOR FAINT TERTIARIES

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ABSTRACT

It is unclear how very close binary stars form, given that during the pre-main-sequence phase the component stars would have been inside each other. One hypothesis is that they formed farther apart but were brought in closer after formation by gravitational interaction with a third member of the system. If so, all close binaries should be members of triple (or higher order) systems. As a test of this prediction, we present a search for the signature of third components in archival spectra of close binaries. In our sample of 75 objects, 23 show evidence for the presence of a third component, down to a detection limit of tertiary flux contributions of about 0.8% at 5200 Å (considering only contact and semidetached binaries, we find 20 out of 66). In a homogeneous subset of 59 contact binaries, we are fairly confident that the 15 tertiaries we have detected are all tertiaries present with mass ratios $0.28 \lesssim M_3/M_{12} \lesssim 0.75$ and implied outer periods $P \lesssim 10^6$ days. We find that if the frequency of tertiaries were the same as that of binary companions to solar-type stars, one would expect to detect about 12 tertiaries. In contrast, if all contact binaries were in triple systems, one would expect about 20. Thus, our results are not conclusive but are sufficiently suggestive to warrant further studies.

Key words: binaries: close — methods: data analysis — stars: formation — stellar dynamics

Online material: extended figure set

1. INTRODUCTION

Most stars are in binaries, yet our understanding of how these form is far from complete (for a recent review, see Tohline 2002). Particularly puzzling is the existence of very close binaries, with orbital separations of just a few stellar radii. That these cannot form independently is easily seen: during the pre-main-sequence phase, these stars would have been inside each other. Yet they exist.

One hypothesis is that they were originally single entities that spun up to breakup velocity during contraction and split in two. It is unclear, however, whether this would work: descending the Hayashi track, stars are centrally concentrated, and fission into roughly equal parts appears implausible.

If close binaries cannot form directly, could the stars perhaps form as wider binaries and be brought closer together later? One such possibility is that the binary is part of a hierarchical triple and shrinks due to interaction with the third component (Kiseleva 1998; Eggleton & Kiseleva-Eggleton 2001). This can work as follows: the tertiary induces Kozai cycles (Kozai 1962) in the inner binary, in which angular momentum is transferred between the inner and outer system, leading to cycles in eccentricity and relative inclination. For point masses this process is cyclical, but for stars, if the eccentricity becomes sufficiently high, tidal effects take over at periastron, and the orbit circularizes with a final separation of about twice the periastron distance, i.e., much smaller than the initial one.

What helps the above is that the Kozai process is weak: it only works in the absence of anything else. Thus, no cycles occur while there is tidal interaction between the stars and/or their disks (i.e., as long as the stars are young and big) nor once they have been brought in close together. The only requirements are that a sufficient number of binaries be members of triple systems and that many have sufficiently small initial inner separations and sufficiently large relative inclinations between the inner and outer binary planes. None of these constraints appear problematic: 15%–25% of all stellar systems have three or more components (Tokovinin 2004), and some well-known triples have high relative inclination (e.g., Algol; Lestrade et al. 1993).

Indeed, the mechanism has been invoked to explain the properties of a number of individual systems, such as the triple TY CrA (Beust et al. 1997) and the quadruple 41 Dra (Tokovinin et al. 2003). Furthermore, it was found by Tokovinin & Smekhov (2002) that many visual multiples had close spectroscopic subsystems, which might have formed by the above process.

The hypothesis has not, however, been taken to its logical conclusion: Do *all* close binaries form this way? The beauty of this perhaps far-fetched suggestion is that it makes a very clear prediction: all close binaries should be in hierarchical triples (or higher order systems). In this paper we investigate this possibility with a particular subset of close binaries, the W UMa contact binaries.

W UMa contact binaries—in which the two companions share an outer envelope—are the closest known binaries. While these have had a yet further phase of orbital shrinkage, likely related to magnetic braking and/or gravitational radiation, this phase could only happen if they were very close binaries to start with (Vilhu 1982; Stepien 1995; and references therein). Intriguingly, many contact binaries appear to be accompanied by tertiaries: for instance, Rucinski & Kaluzny (1982) noted the frequent presence of visual companions, while Hendry & Mochnacki (1998) found the spectral signature of a tertiary in a number of contact binaries. Furthermore, in radial-velocity studies of contact and other close binaries aimed at measuring their orbital parameters, one of us (S. M. R.) has found that about one in four binaries shows the signature of a tertiary component in its spectrum (see Rucinski 2002 and other papers in the same series).

The above led Pribulla & Rucinski (2006, hereafter Paper I) to collect the available evidence for multiplicity for contact binaries with $V < 10$. For the better observed northern sky subsample, they inferred a multiple frequency of 59% ± 8%. Since no method can detect all multiples, this is a lower limit to the true fraction,

TABLE 1
SAMPLE OF CLOSE BINARIES

Star	Reference	β[a]	Notes	Star	Reference	β[a]	Notes	Star	Reference	β[a]	Notes
AB And	IX	. . .	Y, BF	DK Cyg	II	. . .	(N)	V753 Mon	III	. . .	N
CN And	III	. . .	N, NC	V401 Cyg	VI	0.015	(Y)	V502 Oph	IX	. . .†	●
GZ And	I	0.015	*, Y, TW	V2082 Cyg	IX	0.020	*, ●	V839 Oph	II	. . .	Y
V376 And	V	. . .	N	V2150 Cyg	IV	. . .†	Y	V2357 Oph	VIII	. . .	BF
EL Aqr	V	. . .		RZ Dra	III	. . .	NC	V2377 Oph	IV	. . .	N
HV Aqr	III	0.022	*, ●	BX Dra	IX	. . .		V2388 Oph	VI	0.103	Y
V417 Aql	I	. . .		GM Dra	VI	. . .	N	V1363 Ori	IX	. . .	(N)
AH Aur	II	. . .	(Y)	FU Dra	III	. . .		BB Peg	I	0.009	*, (●)
V402 Aur	VIII	. . .	N	SV Equ	II	. . .	NC	KP Peg	IX	0.03	Y, NC
V410 Aur	VIII	0.22	Y	UX Eri	III	. . .	(Y)	V335 Peg	IX	. . .	N
44 Boo[b]	IV	0.23	Y, TB	QW Gem	VIII	. . .	(Y)	V351 Peg	V	. . .	N, BF
CK Boo	II	0.009	*, Y	V842 Her	II	. . .	N	AQ Psc	I	. . .	Y
EF Boo	V	. . .	N,	V899 Her	IV	0.725	●, TB	DV Psc	IV	. . .	NC
FI Boo	IV	0.012	*, Y	V918 Her	IX	. . .	N	OU Ser	III	. . .	N
SV Cam	VI	0.016	*, NC	V921 Her	VIII	. . .	N	EQ Tau	V	. . .	(N)
AO Cam	III	0.008	*, ●	V972 Her	VI	. . .	N	V1130 Tau	VIII	. . .	NC
DN Cam	V	. . .	Y	FG Hya	I	. . .	N	HN UMa	VIII	. . .	N
FN Cam		. . .	N	UZ Leo	II	. . .	N	HX UMa	VIII	0.023	Y
V523 Cas	VIII	. . .	(Y), BF	XZ Leo	II	. . .	(Y)	II UMa	VI	0.148	Y
V776 Cas	V	0.015	Y, TW	ET Leo	VI	0.022	*, ●	GR Vir	II	. . .	N
V445 Cep	IX	0.055	*, Y	EX Leo	IV	. . .	N	HT Vir	IV	0.282	Y
EE Cet	VI	. . .†	Y	FS Leo	VI	. . .		KZ Vir	V	. . .	NC
KR Com	VI	0.23	Y, TB	RT LMi	III	. . .		NN Vir	II	. . .	N
YY CrB	III	. . .	N	VZ Lib	IV	0.045	*, (●)	HD 93917[b]	VIII	. . .	N
SX Crv	V	. . .	N	SW Lyn	IV	0.194	NC	NSV 223[b]	VIII	. . .	

NOTES.—Asterisks indicate that the spectroscopic signature of the tertiary was not recognized in the original DDO series paper. A "Y," black dot, or "N" indicate that the system is in the sample of Paper I; parentheses indicate the extended, $V_{\rm max} > 10$ sample. (Y) Identified as a triple independently of the DDO spectra; (black dot) identified as a triple based on the DDO spectra; (N) not identified as a triple. (NC, BF, and TB) The system was not included in our statistical analysis either because it is not in contact (NC; i.e., the temperatures of the two components differ), because our procedure yields a bad fit to its spectrum and hence our sensitivity to tertiaries is poor (BF), or because the tertiary is too bright to be sure the sample is complete (TB). (TW) The system is included in our statistical analysis, but the detection of the tertiary is not counted because the tertiary is at too wide a separation, and we cannot be sure we could detect such tertiaries for all objects in our sample.

[a] The quantity β is the flux ratio ($\equiv f_3/f_{12}$) determined from our fit. (For nondetections, $\beta \lesssim 0.008$; further properties for the detected systems are listed in Table 2. A brief description can be found in the Appendix for all systems with detections, as well as some interesting triple systems we missed, marked with a dagger.)

[b] Variable names: 44 Boo = i Boo; HD 93917 = VY Sex; NSV 223 = DZ Psc.

REFERENCES.—In the series "Radial Velocity Studies of Close Binary Stars": (Paper I) Lu & Rucinski 1999; (Paper II) Rucinski & Lu 1999; (Paper III) Rucinski et al. 2000; (Paper IV) Lu et al. 2001; (Paper V) Rucinski et al. 2001; (Paper VI) Rucinski et al. 2002; (Paper VIII) Rucinski et al. 2003; (Paper IX) Pych et al. 2004.

but it is difficult if not impossible to extrapolate given the complex selection effects for various techniques and companion types.

Here we present a detailed analysis for one particular technique of searching for the spectral signature of tertiaries. We reanalyze the data sets used for the radial-velocity studies referred to above using a new technique, outlined in § 2, optimized for the detection of tertiaries. We discuss possible pitfalls and systematic effects and find that these limit us somewhat, but that we can still detect tertiaries down to flux ratios of about 1%, an improvement by a factor of 3 compared to the earlier results. In § 3 we infer properties of tertiaries and check consistency with previous work. We discuss limits and biases in our sample in § 4 and use the companion distribution of solar-type stars measured by Duquennoy & Mayor (1991, hereafter DM91) to test the hypothesis that all close binaries are in triple systems. In § 4 we also discuss what the alternative, null hypothesis should be, i.e., what one would expect if multiplicity had little or no influence on the formation of close binaries; conservatively, we assume that for this case the tertiary frequency is similar to the companion frequency of regular stars. We summarize our results and discuss future prospects in § 5.

2. DATA SET AND ANALYSIS TECHNIQUE

The data set we have available is that used in Papers I–IX of the series "Radial Velocity Studies of Close Binary Stars" (for an overview, see Rucinski 2002). Briefly, it consists of almost 4000 spectra of 75 close binaries, all taken at the David Dunlap Observatory (DDO). We list the objects in Table 1. For our purposes the important characteristics of the spectra are that they were taken with the 1800 line mm^{-1} grating centered at 5180 Å (covering about 200 Å around the Mg I triplet) and through a 1″.5 or 1″.8 slit (matched to the typical seeing of 1″.7). The resulting slit images project to 0.64 and 0.80 Å, respectively, and, including the effect of seeing, lead to effective resolutions of 35–50 km s^{-1}.

We search for the spectroscopic signature of a tertiary by fitting the spectrum of the contact binary and checking whether adding a spectrum of a fainter tertiary improves the fit significantly. For the binary spectrum we can make use of the convenient fact that the contact binary has a single spectral type (due to energy transfer from the more massive to the less massive component, by a mechanism not entirely understood; for a recent review, see Webbink 2003) and that its lines are strongly broadened by rapid rotation and orbital motion. In contrast, the lines of the third star should be narrower, since there is nothing to have prevented it, like all low-mass stars, from slowing down. (Note that there are exceptions: new DDO observations have revealed a broad-lined A-type companion for V753 Mon that entirely masks the radial velocity signatures of the binary so that only the variability is detectable. Since we remove more massive, early-type stars, this will not bias our sample.) Furthermore, any orbital

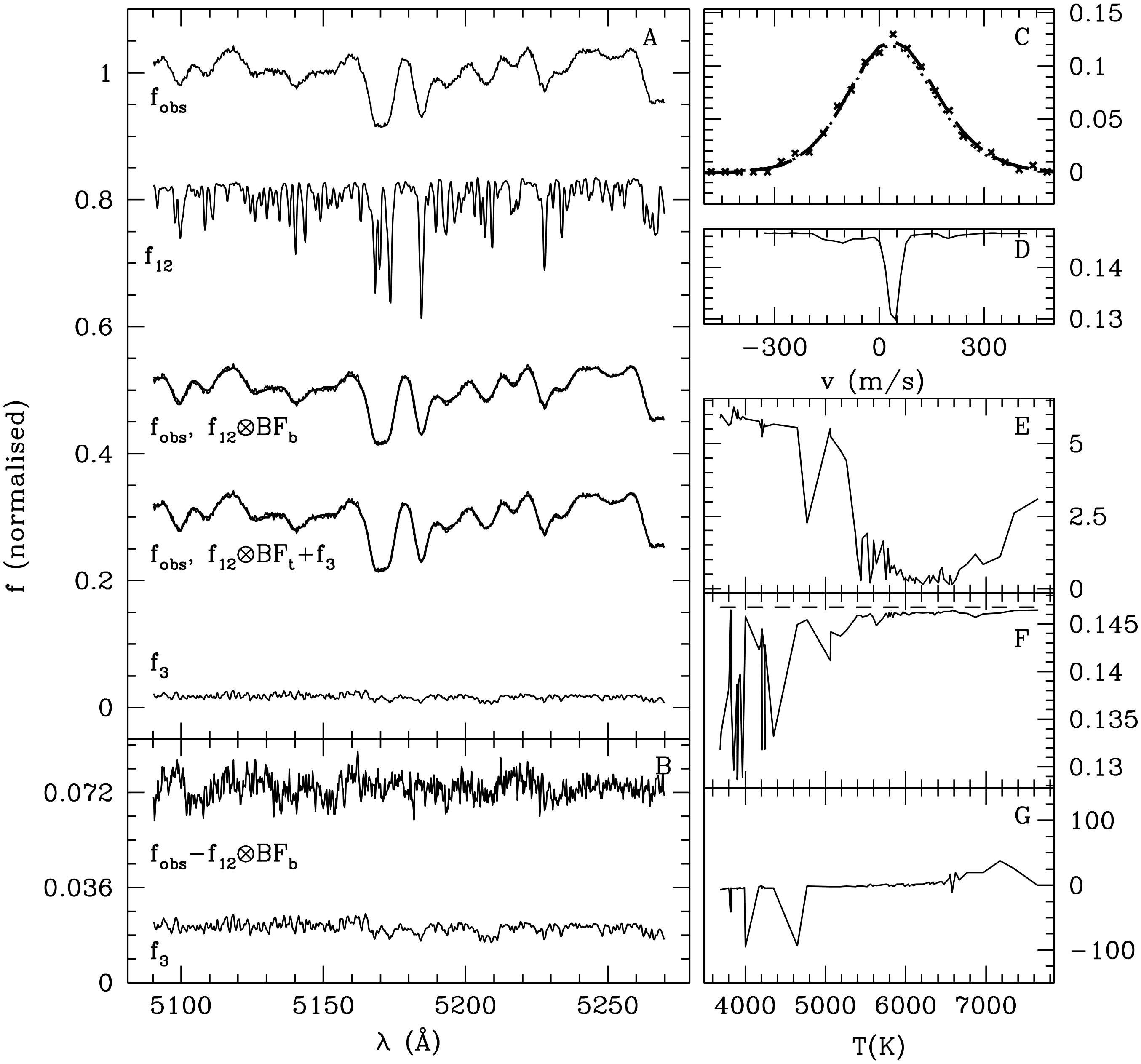

FIG. SET 1.—Search for a tertiary component in the spectra of the W UMa system CK Boo. (*a*) From top to bottom are $f_{\rm obs}$: the observed, average spectrum; f_{12}: a template of similar spectral type used to represent the contact binary; $f_{\rm obs}, f_{12} \otimes {\rm BF}_b$: the observed spectrum (*thin line*) plotted with the template convolved with a best-fit broadening function (*thick line*); $f_{\rm obs}, f_{12} \otimes {\rm BF}_t + f_3$: the observed spectrum plotted with the best-fit model composed of the template convolved with a refit broadening function, plus a tertiary spectrum; and f_3: the best-fit tertiary contribution. (*b*) Comparison between the residuals from fitting the observed spectrum without including a third star (offset by a constant value) and the best-fit tertiary spectrum. (*c*) Broadening function used to represent the line profiles of the contact binary. Crosses indicate the empirical broadening function found from least-squares decomposition (see Rucinski 2002), and the dashed line the fit to those points with our three-Gaussian model shape. This fit is used as an initial guess for the line profile; the dotted line represents the final shape, after convergence of our procedure. (*d*) Variance of the fit residuals as a function of the velocity of the tertiary. For CK Boo this shows only one minimum, which is close to the systemic velocity of the contact binary, as expected for a real tertiary. (*e, f*) Variance of the residuals as a function of temperature of the template used for the contact binary and the tertiary, respectively. For CK Boo the best-fit tertiary is substantially cooler than the binary, as expected if it is much fainter. (*g*) Best-fit relative velocity between the tertiary and contact binary as a function of tertiary temperature. For CK Boo this is close to zero, as expected for a physically associated component. [*See the electronic edition of the Journal for Figs. 1.1–1.75.*]

motion of the tertiary should be small compared to that of the contact binary, which implies that we can analyze spectra averaged over the orbital phase of the contact binary. This not only increases the signal-to-noise ratio but should also smooth out possible relatively sharp-lined features from the contact binary, such as might be caused by starspots.

Below we describe how we implemented the technique and how we modeled the line broadening in the contact binary. We then discuss the criteria we used to determine whether a detection of a tertiary is significant and real and to determine our sensitivity limits. We conclude with a discussion of the limitations of our method.

2.1. *Implementation*

To search for tertiaries we fitted the data using the procedure outlined in Figure Set 1 (figures for all the objects in our data set

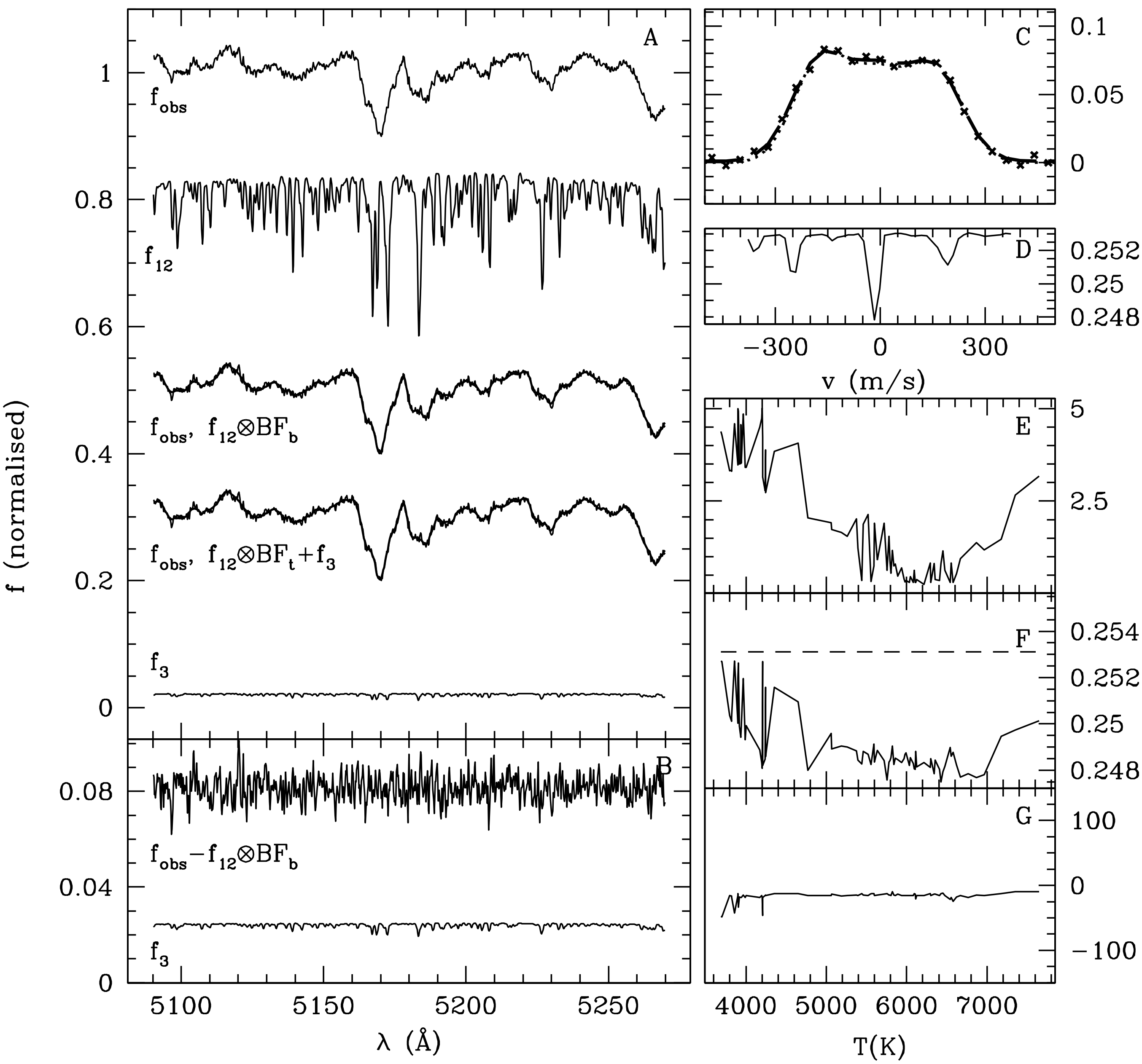

FIG. 2.—As in Fig. Set 1, but for the W UMa system EF Boo (f_{obs}), a particular object for which we do not detect a third star down to a level of 0.8%.

are available in the online version of the paper; a particularly good example of a result showing no tertiary is given in Fig. 2). Starting with the average of all normalized, barycentered spectra of a given source, we try to reproduce it with a template spectrum of a sharp-lined, slowly rotating star convolved with a model broadening function optimized to match the binary's line profile. For our templates we use the database of high-resolution, normalized stellar spectra obtained with ELODIE (Prugniel & Soubiran 2001), repeating our fit procedure for all spectra in order to find the one that matches best. The model broadening function is composed of three Gaussian curves with equal width. We choose such a model to ensure that our broadening function is wide enough that the sharp-lined signal from a putative tertiary is not removed; since this was a particularly difficult part of our analysis, we describe it in more detail in § 2.2 below.

To measure the quality of the fit, we convolve the model spectrum with a truncated Gaussian curve (to simulate the effects of seeing and transfer through the slit), regrid on the observed pixel array, multiply with a polynomial function to simulate differences in the normalization, and finally calculate the variance between the observations and model. We minimize the variance as a function of the various parameters using the downhill simplex method as described by Press et al. (1992, § 10.4).

Once the best fit to the contact binary spectrum has been determined, we add a third star to the spectrum, optimize the relative flux and velocity, and determine the resulting improvement in the fit. We again repeat this procedure for a wide range of different spectral types (from early F down to early M).

With our procedure, the final set of parameters determined is (1) the best-fit template spectrum for the contact binary or, more interestingly, its temperature; (2) the systemic velocity; (3) the width, separation, and two relative intensities for the three-Gaussian broadening profile (see § 2.2); (4) the best-fit template for the third star; (5) its fractional intensity; and (6) its velocity relative to the contact binary. In addition, there are up to 10 parameters without physical meaning, viz., those that describe the polynomial accounting for difference in continuum normalization.

2.2. *The Broadening Profile*

We found that the most difficult part of our analysis was to accurately reproduce the line-profile shape of the contact binary. In principle, with good phase coverage, one might expect that a single Gaussian curve would suffice. In practice, however, the binaries were observed preferentially near the quadratures. The result for systems with mass ratio near unity is a double-humped line profile in the average spectrum, while for systems with extreme mass ratios the profile shows a prominent central hump (from the more massive component), as well as two "sidelobes" (from the less massive one).

Based on this structure we chose to model the broadening profile with a set of three Gaussian curves, but constrained to have identical width and separation. This leaves four free parameters: the width, the separation, and the relative intensities of the two outer Gaussian curves (since the whole profile is normalized, the intensity of the central Gaussian curve does not have to be specified).

Another problem that arose in automating our procedure was that the profiles of the different systems were so varied that it was impossible to use a single set of initial guesses for the broadening profile that worked for all systems. To circumvent this we first determined empirical broadening profiles at the instrumental resolution by least-squares decomposition (using the technique described in detail by Rucinski 2002; see Fig. Set 1, panel *c*). Next we fitted these empirical broadening profiles using our three-Gaussian model and used the resulting parameters as initial guesses for our main procedure.

2.3. *Detections and Detection Limits*

With the fit results in hand, we needed to determine whether or not the possible improvement resulting from adding a tertiary was significant. For some objects, this was trivial: the residuals from the fit with the binary model showed a clear signature of a different spectral type. Those, however, would typically have been found already in the earlier studies, since the tertiary component would lead to a narrow peak in the empirical broadening function (see Rucinski 2002).

For fainter tertiaries, one could in principle use statistical tests to determine whether the improvement in variance (or equivalently χ^2) is significant. This only works, however, if the variance is dominated by measurement noise. In practice this is not the case: the quality of the fit is usually limited by a mismatch between our model and the true contact binary's spectrum. Indeed, even for our spectra with the worst signal-to-noise ratios (such as UX Eri; Fig. 3, *bottom*), we find that systematics dominate. As a result, the quality of the fits is not good in a statistical sense, and the use of χ^2 becomes meaningless.

In Figure 3 we show the types of more severe systematic mismatches that limit our sensitivity to tertiaries. The first is a poor match to the binary's spectral type, which happens mostly for cooler temperatures for which the ELODIE archive contains relatively few suitable templates. As can be seen for the case of AH Aur in Figure 3 (*top*), the mismatch leads to low-frequency residuals and thus an increased variance. Since the residuals have long-wavelength variations, one could still tease out the signature of a tertiary, but since the limits would not be as good, we decided not to include objects in which the fit is worse than AH Aur in our statistical sample.

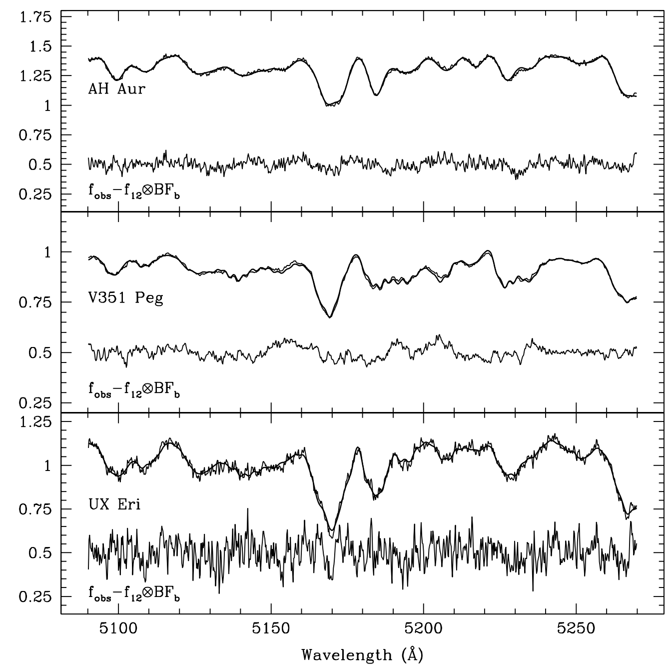

FIG. 3.—Three examples of problems encountered in fitting the contact binary, which make detecting a third star more difficult. *Top:* AH Aur and its best fit (*upper line*), with the residuals (enlarged) below. A poor match in spectral type to the contact binary leads to large-scale residuals. *Middle:* V351 Peg and the residuals from its best fit. In this case the dominant residuals come from errors in fitting the contact binary's complicated broadening profile, and so show up as high-frequency residuals. *Bottom:* UX Eri and the residuals from its best fit. In this case, the spectrum itself is noisier (in addition to the poor spectral match), and so the residuals are larger. In general, even with these effects we can detect a third star down to a level of about $\beta \simeq 0.8\%$; for the worst cases (which we exclude in our analysis) this level is closer to $\beta \simeq 2\%$–3%.

The second source for systematic error is more problematic: poor matches to the binary star's line profile. For cases such as V351 Peg (Fig. 3, *middle*), for which our three-Gaussian broadening function does not match the intrinsic profile very well, high-frequency residuals are left. In consequence, there is an obvious danger of a false detection of a "third star" that matches these residuals.

In order to avoid the above pitfalls, we decide whether or not a detection is significant using not only visual inspection of the residuals but also the following two physical arguments: First, for a faint tertiary the temperature should be substantially lower than that of the contact binary, and therefore, as a function of tertiary temperature, minimum variance should occur at low values. Second, the orbital motion of the tertiary should be relatively small, and hence, as a function of relative velocity, minimum variance should occur near zero. In the example shown in Figure 1.12, both criteria are met, and hence we consider the detection of the tertiary secure. There are also, however, a fair number of sources for which tertiary flux and the decrease in variance are similar, but for which the inferred tertiary temperature is higher than that of the contact binary and/or the radial velocity is inconsistent.

With the above procedure we find that we are able to detect tertiaries down to fluxes of 0.8% of that of the contact binary. We confirmed this by adding third stars at different flux levels to objects for which we did not detect tertiaries, and finding the level at which we could recover those: we found a limit of 2%–3% even for cases in which the match to the line profile was relatively poor, such as V523 Cas and V351 Peg.

2.4. *Limitations*

Apart from the problems addressed above in obtaining an adequate fit to the binary, our technique also has limitations inherent in the assumptions we make about the contact binary and the tertiary. For the contact binary we have assumed the same temperature for both components. While generally this is a good assumption, it does not always hold perfectly: in the recent review by Yakut & Eggleton (2005), temperature differences of up to 800 K are listed for contact binaries (including some in our sample; note, however, that different authors sometimes find rather different values). In principle, for systems in which the temperature difference is large ($\sim$400 K), it would be better to fit the binary with a composite spectrum. In practice, however, we find that the most important feature for detecting a tertiary is how well its sharp features stand out from the broadened ones of the binary. Indeed, from simulations in which we added false stars to contact binaries with and without large reported temperature differences, we found that the detection threshold was the same. Combined with the fact that we do detect a faint companion even in a noncontact system (SV Cam), we conclude that spectral type differences in the contact binary do not limit our analysis.

For the tertiary, an obvious limitation is that we cannot detect compact objects, since these would be too faint and likely not contribute any spectral features. Another is that we have few templates for late-type stars. This is not an issue for the more massive, earlier contact binaries, for which such late-type tertiaries would be undetectable. But for later type contact binaries, we might miss cool tertiaries or, more likely, overestimate their temperatures and fluxes (the latter since the strength of the band heads, etc., generally increases with decreasing temperature).

A different limitation arises from our assumption that the tertiary rotates slowly. For early-type tertiaries, this may not be correct. Those, however, would be very bright and hence noted independently (furthermore, we exclude them from our sample since we cannot be sure the sample of contact binaries with such bright tertiaries is complete; see § 4.1). For late-type tertiaries slow rotation is expected unless the star is in a close binary and is kept corotating by tidal forces. Thus, our procedure does not identify close binaries as companions (such as the quadruple system composed of two contact binaries, BV Dra and BW Dra; Rucinski & Kaluzny 1982). In order for the projected rotation velocity to be below our resolution, i.e., $v \sin i \lesssim 50$ km s^{-1}, one requires $P_{\rm rot} \gtrsim$ 1 days (for a 1 $R_\odot$ star). But at such short orbital periods, orbital velocities would be even higher, and those would smear the signal as well (at least for systems for which the spectra were obtained over an extended period of time). Orbital velocities for the tertiary were indeed found in HT Vir (Lu et al. 2001). In order not to decrease our sensitivity, a tertiary that is itself a binary should have a radial-velocity amplitude $K \lesssim 50$ km s^{-1}, which requires $P_{\rm orb} \gtrsim$ 20 days (for two 1 $M_\odot$ stars).

3. RESULTS

Out of 75 systems, we have detected tertiaries for 23, 9 of which had been missed in the original analysis of the spectra. All detections are indicated in Table 1 and described in more detail in the Appendix. Below, we compare our results to those in the literature, and then proceed to infer tertiary masses and mass ratios. A summary of observed and inferred properties of the triple systems is given in Table 2.

3.1. *Comparison to Previous Results*

In Table 1 we indicate for all stars in our sample whether it was also studied in Paper I, and if so, whether there was independent evidence for it being a member of a triple (or higher order) system. For many of our detections there is independent evidence for multiplicity, and conversely, for most systems for which we detect no tertiary, there is little evidence to the contrary. This likely reflects the fact that most methods, whether detecting companions through gravity or flux, require relatively similar minimum masses.

There are nine systems, however, for which a tertiary was found in Paper I but not in our analysis. For five cases, EE Cet, V2150 Cyg, QW Gem, AQ Psc, and AH Aur, the discrepancy is simply that the separation is too large for any light of the tertiary to have entered the slit (for EE Cet and V2150 Cyg, some light did enter the slit, which, knowing that they were visual binaries, we could detect; see the Appendix). For three others, AB And, V523 Cas, and UX Eri, the minimum mass inferred from the arrival-time variations implies a flux below our threshold (which is the case for $M_3/M_{\rm CB} \lesssim 0.28$ [§ 3.2]; V523 Cas also was fitted particularly poorly [§ 4.1]). For the remaining system, DN Cam, the identification is based on suspected multiplicity from *Hipparcos* and X-ray emission in excess of expectations. Since this does not yield a mass estimate, we cannot check whether our nondetection makes sense.

Turning now to the properties of the systems in which we detect tertiaries (Table 2), we see that the contact-binary temperatures inferred from our spectral fits (column $T_{\rm fit}$) are generally in fair agreement with those inferred from $B-V$ (T_{B-V}, found by interpolation in Table 15.7 of Cox [2000]). This gives confidence in our method. There are three exceptions, KR Com, 44 Boo and V899 Her, in all of which the tertiary contributes a significant fraction to the system's light. As a result, an incorrect temperature for the contact binary is found in the first step of our procedure. Since this temperature is not checked a posteriori, it is expected that the other results are also inaccurate (we do not use these systems in our statistical analysis; see § 4.1).

Comparing the flux ratios $\beta_{\rm fit} \equiv f_3/f_{12}$ from our fit to those from the literature ($\beta_{\rm lit}$), there are also a number of discrepancies. For resolved systems, the literature values should be reliable, and hence, we need to understand what went wrong in our procedure. We see two possible causes for errors. First, as above, for systems with bright tertiaries (44 Boo and KR Com) the temperature assigned to the contact binary is incorrect, and hence, the other parameters will be inaccurate. Second, for systems with wide separations (V776 Cas, KP Peg, and, to a lesser extent, 44 Boo) the tertiary would have been only partially in the slit, and hence, the flux would be underestimated.

Turning now to unresolved triple systems, which were all identified in the DDO program and thus have values based on the same data, we find that our flux ratios are systematically smaller, especially for fainter tertiaries. In the DDO program the tertiaries were recognized by the appearance of a sharp feature in the broadening function at the system's average radial velocity, and the flux ratio was determined from the ratio of the area under the sharp peak to the remainder of the broadening function (Rucinski et al. 2002). Since these broadening functions are derived using a least-squares decomposition based on a single template spectrum, the contribution of the tertiary was effectively measured under the assumption that it had the same spectral type as the contact binary. If its true spectrum has stronger lines, as is the case for faint tertiaries with cooler temperatures, this leads to an overestimate of its contribution. Since our procedure uses a separate spectral type to derive the contribution from the tertiary, our flux ratios should be more reliable.

The above issues allow one to understand the discrepancies between literature values and those derived here but make it difficult

TABLE 2
Observed and Inferred Properties for Close Binaries with Tertiaries

	Contact Binary							Tertiary								
Star	P (days)	M_2/M_1	$B-V$	$T_{\rm fit}$ (K)	T_{B-V} (K)	M_V	M_1 ($M_\odot$)	$\beta_{\rm lit}$	Separation (arcsec)	$\beta_{\rm fit}$	$T_{\rm fit}$ (K)	T_β[a] (K)	M_V[a]	M_3 ($M_\odot$)	M_3/M_1	M_3/M_{12}
GZ And	0.305	0.514	0.79	5600	5200	4.80	0.96	0.011[b]	2.13	0.015	3500	4000	9.69	0.42	0.43	0.29
HV Aqr	0.374	0.145	0.63	5800	5800	3.92	1.22	...	...	0.022	4000	4000	8.06	0.59	0.48	0.42
V410 Aur	0.366	0.144	0.56	5400	6000	3.75	1.29	0.38[c]	1.7	0.22	5200	5600	4.80	0.97	0.75	0.66
44 Boo	0.268	0.487	0.94	5400[d]	4800	5.50	0.85	2.08[c]	1.7	0.23[d,e]	6100[d]	5953	4.70	0.99	1.17	0.79
CK Boo	0.355	0.111	0.54	6600	6100	3.75	1.29	0.007[b]	0.12	0.009	3900	3900	9.14	0.47	0.37	0.33
FI Boo	0.390	0.372	0.64	5800	5700	3.87	1.24	...	...	0.012	3900	3800	8.67	0.52	0.42	0.31
AO Cam	0.330	0.415	0.58	5800	5900	4.01	1.18	...	...	0.008	4200	4000	9.25	0.46	0.39	0.28
V776 Cas	0.440	0.130	0.47	6500	6500	3.12	1.49	0.238[c]	5.38	0.015[e]	6100	5600	4.68	1.00	0.67	0.59
V445 Cep	0.449	0.167	0.12	7400	8400	2.03	1.95	...	...	0.055	5600	6600	5.18	0.91	0.46	0.40
KR Com	0.408	0.091	0.52	6100	6200	3.42	1.42	0.58[c]	0.119	0.23	6100	5800	4.01	1.19	0.84	0.77
V401 Cyg	0.582	0.290	0.3	6700	7300	2.07	1.93	0.03	...	0.015	4700	4700	6.63	0.73	0.38	0.29
V2082 Cyg	0.714	0.238	0.31	7000	7200	1.71	2.14	...	...	0.020	5100	5200	5.95	0.79	0.37	0.30
V899 Her	0.421	0.566	0.48	6300[d]	6400	3.24	1.44	1.5	...	0.73[d]	6400[d]	6500	3.59	1.36	0.94	0.60
ET Leo	0.347	0.342	0.61	5800	5800	4.00	1.19	...	...	0.022	3900	3900	8.15	0.58	0.49	0.36
VZ Lib	0.358	0.237	0.61	5800	5800	3.94	1.21	0.2	...	0.045	4700	4200	7.31	0.67	0.55	0.45
V2388 Oph	0.802	0.186	0.41	6100	6800	1.78	2.10	0.19[c]	0.088	0.10	5900	5900	3.59	1.36	0.65	0.55
BB Peg	0.362	0.360	0.52	5900	6200	3.65	1.33	...	...	0.009	4000	3900	8.76	0.51	0.39	0.28
HX UMa	0.379	0.291	0.44	6600	6700	3.32	1.44	0.047[c]	0.63	0.023	4400	4400	6.64	0.73	0.50	0.39
II UMa	0.825	0.172	0.4	6600	6800	1.70	2.15	0.23[c]	0.87	0.15	6100	6400	3.29	1.45	0.67	0.58
HT Vir	0.408	0.812	0.56	6100	6000	3.54	1.23	0.586[f]	0.6	0.28	6100	5900	4.12	1.15	0.93	0.52
SV Cam[g]	0.593	0.641	0.62	5800	5800	3.00	1.49	...	...	0.016	3900	3900	7.49	0.65	0.44	0.27
SW Lyn[g]	0.644	0.524	0.38	7200	6900	2.12	1.87	0.33	...	0.19	6200	6500	3.90	1.23	0.66	0.43
KP Peg[g]	0.727	0.322	0.06	7400	8900	0.92	2.68	0.52[c]	3.5	0.03[e]	7700	6600	1.63	2.19	0.82	0.62

Notes.—The binary periods (P), mass ratios (M_2/M_1), and $B-V$ are taken from the original Papers I–IX (see Table 1); $\beta_{\rm lit}$ and the separation angle are from the same sources except where indicated.

[a] For resolved triples, $\beta_{\rm lit}$ was used to infer T_β and M_V; for all others, $\beta_{\rm fit}$ was used.

[b] Paper I.

[c] Tokovinin (1997).

[d] The values inferred from our fit are inaccurate, since the tertiary is brighter than the contact binary and dominates the average spectrum.

[e] The flux ratio is inaccurate, since only a fraction of the tertiary's light fell inside the slit.

[f] Heintz (1986).

[g] SV Cam, SW Lyn, and KP Peg are not contact binaries. Therefore, the components do not have equal temperatures, and the deduced properties are less reliable. They are not used in our statistical analysis.

to estimate reliable uncertainties. From the comparison with resolved systems, uncertainties of $\lesssim 15\%$ are indicated for flux ratios between 0.05 and 0.5, but errors increase toward higher and lower values. For the brighter tertiaries, we use flux ratios from resolved observations, which should be good to $<10\%$. For fainter ones, however, one needs to keep in mind that our uncertainties increase rapidly, reaching $\sim 50\%$ for ratios below ~ 0.02.

3.2. *Inferred Properties*

Contact binaries follow a period-luminosity-color relation, which allows one to derive the absolute magnitude M_V from the period P (in days) and dereddened color $(B-V)_0$ (Rucinski 2004 and references therein):

$$M_V = -4.44 \log P + 3.02(B-V)_0 + 0.12. \quad (1)$$

With the tertiary flux ratio, this yields the absolute magnitude of the tertiary. Next we use the fact that contact binaries are on the main sequence and that, therefore, the generally fainter tertiary should be on the main sequence as well. Then, with the main-sequence mass-luminosity relation (we use Cox 2000, Tables 15.7 and 15.8), the tertiary mass M_3 follows from the absolute V-band magnitude. The results of this procedure are listed in Table 2. Here we did not correct for the typically very small reddening ($E_{B-V} = 0.00$–0.03; S. M. Rucinski 2005, unpublished). The resulting errors in the absolute magnitude inferred from equation (1) are on the order of 0.1 mag, substantially below the 0.25 mag scatter in the period-luminosity relation.[1]

The uncertainty in the derived masses has contributions from all steps. The magnitudes predicted from the period-luminosity-color relation are uncertain by ~ 0.25 mag (Rucinski 2004). For relatively bright tertiaries, the uncertainty in the tertiary flux ratio is smaller ($\lesssim 15\%$, or 0.15 mag). With a total uncertainty in M_V of ~ 0.3 mag, this leads to an uncertainty in the derived masses of $\lesssim 10\%$. Evolution along the main sequence will likely contribute less, except for the brightest tertiaries. For fainter tertiaries, the uncertainty in the flux ratio leads to an error in M_V of as much as 0.7 mag, but since the mass-luminosity relation becomes much steeper for fainter objects, we still expect the final uncertainty in the mass to be around $\sim 10\%$.

In order to verify the above, we also derived tertiary temperatures from M_V (column T_β). These should be similar to those inferred from the spectral fits ($T_{\rm fit}$); from Table 2 one sees that this is indeed the case. We note, however, that this is not a strong test, since temperature does not vary strongly with stellar mass.[2]

[1] Ignoring reddening leads to a small systematic underestimate of the brightness and thus the mass of the tertiary. Since the binary's mass will be underestimated as well, however, the effect on the mass ratio should be small.

[2] For the same reason, it is not very useful to infer masses from the fitted temperatures.

Finally, in order to derive mass ratios, we also need an estimate for the masses of the stars in the contact binary. For this purpose we use that the contact binary's luminosity will be the sum of the luminosities of its two roughly main-sequence components. Since the luminosity depends steeply on mass, this implies that for low mass ratios $q \equiv M_2/M_1$, the luminosity is simply that of the primary, while for higher ones there is a contribution from the secondary. To estimate the primary's absolute magnitude, $M_{V,1}$, we use that for main-sequence stars, the V-band luminosity scales as $M^{4.4}$ (inferred from Cox 2000, Tables 15.7 and 15.8; for a more detailed analysis see Mochnacki 1981), so that

$$M_{V,1} \simeq M_V + 2.5 \log\left(1 + q^{4.4}\right). \qquad (2)$$

Next we estimate the primary's mass M_1 using the main-sequence mass-luminosity relation (Cox 2000, Tables 15.7 and 15.8). Including the uncertainty in how far the star has evolved on the main sequence, we expect these masses to be accurate to $\lesssim$20%.

4. STATISTICAL ANALYSIS

In order to determine whether the number of tertiaries we find is consistent with the hypothesis that all close binaries form in triple systems, we need to ensure that our sample is homogeneous. Thus, we need to remove systems for which our procedure did not work properly and consider for which separations and masses (or mass ratios) we can be certain we would have detected a tertiary if one were present. We flag all systems we exclude from our sample in Table 1 and discuss our reasons in more detail below. Next we compare our results for tertiaries with those found for secondaries for solar-type stars, trying to extrapolate the tertiary frequency to masses and separations to which we are not sensitive, and testing the hypothesis that all contact binaries are in triple systems.

4.1. *Limits and Biases in our Sample*

Among the sample of close binaries observed at DDO, most are contact binaries, but nine are not: CN And (somewhat uncertain; Rucinski et al. 2000), SV Cam, RZ Dra, SV Equ, SW Lyn, KP Peg, DV Psc, V1130 Tau, and KZ Vir. For these the assumption of a single spectral type for both stars is inappropriate, and hence, our procedure does not work optimally.[3] We thus exclude all nine systems in our statistical analysis.

We exclude a further four systems because the fits to the binaries' spectra are too poor to detect a third star down to flux ratios of 0.008. For one of these, V523 Cas, the temperature is low, and we do not have a good template in our library. For the other three, AB And, V2357 Oph, and V351 Peg, the match to the line profile is poor.

We now turn to physical limits and biases. First, since our method is based on spectra taken through a 1.″8 slit, we are only able to detect tertiaries at relatively close separations. For separations in excess of ~1.″8, the contribution of tertiary light is reduced, and hence, we only detect very bright objects (such as V776 Cas). At a typical distance of ~100 pc, and including a statistical correction factor of $10^{0.13} = 1.35$ (as in DM91) for projection effects, this corresponds to a separation of ~240 AU or, assuming a total mass of the system of ~2 $M_\odot$, an orbital period of ~2600 yr $\simeq 10^6$ days.

Second, a tertiary needs to be sufficiently bright. For main-sequence stars, we cannot detect tertiaries with flux ratios below ~0.008. From Table 2 one sees that this corresponds to mass ratios $M_3/M_{12} \simeq 0.28$. The value does not appear to depend much on the properties of the contact binary. To see why, we consider three possible configurations spanning the extremes of the range of contact-binary properties seen, with masses $M_1 = \{1\ M_\odot,\ 1\ M_\odot,\ 2\ M_\odot\}$ and $M_2/M_1 = \{0.3,\ 0.8,\ 0.3\}$. For those parameters, the absolute magnitudes would be $M_{V,12} \simeq \{4.7, 4.4, 2.0\}$, and a tertiary with, e.g., $\beta = 1\%$ would be at $M_{V,3} \simeq \{9.7, 9.4, 7.0\}$. This corresponds to tertiary masses $M_3 \simeq \{0.42\ M_\odot,\ 0.45\ M_\odot,\ 0.70\ M_\odot\}$ and thus to mass ratios $M_3/M_{12} \simeq \{0.32, 0.25, 0.27\}$. We conclude that we could detect all tertiaries with mass ratios in excess of 0.28.

Third, independent of our method we must somehow be able to observe a contact binary. For very bright tertiaries, the contact binary would be completely outshone, and it likely would not be detected unless the tertiary were a star that appears interesting on its own accord and is studied in detail. But there is a bias even if the tertiary is less bright, contributing, for instance, only half the flux. In such a case the variability would still be detectable, but the system might well be misclassified, since the narrow lines of the tertiary would stand out in the spectrum while the broad ones from the contact binary would be more difficult to detect (a good example of such a system is TU UMi; Rucinski et al. 2005). Given the above, we expect the sample of known contact binaries to be biased against systems having tertiaries brighter than the contact binary (which roughly corresponds to a mass in excess of that of the primary, or a mass ratio $M_3/M_{12} \gtrsim 0.75$).

Finally, more generally, since we rely on the spectroscopic signature of a tertiary we cannot detect white dwarfs, neutron stars, or black holes.

In summary, out of a sample of 75 close binaries there are 62 contact binaries for which our method worked well, among which we detected 20 tertiaries. Among these, however, three (44 Boo, V899 Her, and KR Com) should not be included, since the tertiary is at least half as bright as the binary and we cannot be confident that our sample of contact binaries is unbiased for such systems. Furthermore, V776 Cas and GZ And should not be counted as tertiaries, since the separation between the binary and third star is too large, making us incomplete. Thus, we are left with a sample of 59 contact binaries, for which we can be reasonably confident that our 15 detected tertiaries constitute all main-sequence tertiaries with $0.28 \lesssim M_3/M_{\rm CB} \lesssim 0.75$ and $P_3 \lesssim 10^6$ days.

4.2. *Comparison with Solar-Type Binaries*

Our method misses triple systems with tertiaries that are at large separations, have low mass, and/or are compact. To estimate their number we need to extrapolate, but we do not know a priori the mass-ratio and separation distributions of the tertiary. By way of an estimate, we assume that these are the same as those found for binary companions to solar-type stars. Along the way we try to test the alternate hypothesis that not all contact binaries have tertiaries, but rather that the companion frequency is similar to that of solar-type stars.

Before proceeding we note that our choice of alternate hypothesis is somewhat arbitrary. One would like to test the hypothesis that the number of tertiaries we find is consistent with what one expects from the formation of multiples. This number, however, is not known: multiplicity among very young stars is very poorly constrained, and even among older stars there is no complete census (for a status report, see Tokovinin 2004). Our choice of comparing with solar-type stars corresponds to an implicit assumption that the companion frequency is independent of whether or not an inner system is a single star or a binary. It seems likely that this is a conservative assumption; i.e., assuming that multiplicity plays no role in the formation of close binaries,

[3] We believe the detections of tertiaries in SV Cam, SW Lyn, and KP Peg are reliable despite the fact that these systems are not in contact.

the companion frequency of contact binaries is unlikely to be higher than that for solar-type stars.

For the solar-type stars we use the DM91 sample of 164 stars of spectral type F7–G9 (masses $\sim$0.8–1.3 $M_\odot$). Using their distributions of mass ratio and period, we mimic the selection effects present in our sample. Before doing so, however, two complications need to be mentioned. One is that among the 81 orbits used in the mass-ratio and period distributions, six are second orbits from triple systems,[4] and four are second and third orbits from two quadruple systems. For our purpose of estimating probabilities of finding a companion with certain parameters, this leads to an overestimate (e.g., for the full DM91 sample, the number of single stars is 93, not $164 - 81 = 83$). To avoid biasing ourselves we treat the DM91 sample as consisting of 174 targets (some of which are stars, some binaries, and some triples), 81 of which have a companion.

A second complication is that the sample of DM91 is divided into two groups: spectroscopic and resolved (visual and common proper motion) binaries, which correspond to binaries with periods $P < 10^4$ days and $P > 10^4$ days, respectively. These groups differ in that white dwarfs are present among the (single-lined) spectroscopic binaries but not among the resolved ones. In their statistical analysis of mass ratios, DM91 inserted eight "fake" white dwarf systems into the long-period group: two each in the four mass bins with $q_{\rm max} = \{0.5,\ 0.6,\ 0.7,\ 0.8\}$. We remove these, since we cannot detect white dwarf companions. We also correct for the presence of white dwarfs among the spectroscopic binaries.

We now turn to the application of the selection effects present in our sample. First, to mimic the incompleteness among known contact binaries, we ignore all binaries with mass ratios $q \equiv M_2/M_1 > 0.75$; from lines 3 and 5 in Table 7 of DM91, after correction for one fake white dwarf in line 5, we find that this reduces the sample by 6.15 spectroscopic and 7.5 resolved binaries. (Here the numbers are noninteger, since we had to split the $q = 0.7$–0.8 bin and since for the spectroscopic binaries, DM91 corrected for the distribution of orbital inclinations.) This leaves a sample of 160.35 targets.

Second, to reproduce our detection limit we count all binaries with mass ratio $q > 0.28$ among the remaining targets. Again from lines 3 and 5, after correction for seven fake white dwarfs, we find 22.27 and 31.8 binaries, respectively.

Third, to account for our separation limit, we select systems with periods $P < 10^6$ days. For this purpose we use the fact that from the period distribution (Fig. 7 in DM91), among the resolved systems with $P > 10^4$ days, 31 out of 65 have $P < 10^6$ days. Thus, only 15.17 out of the 31.8 long-period binaries remain, and the total implied detection rate for a survey like ours would be $(22.27 + 15.17)/160.35 = 23\%$.

In the above, we still need to correct for the presence of white dwarfs among the spectroscopic binaries. DM91 mention that from statistical considerations of stellar populations, one expects "about two white dwarfs per decade of period." This would imply that about eight are present among their sample of 34 spectroscopic binaries. Seven of these would be included in the 22.27 binaries with $0.28 < q < 0.75$ selected above, implying a reduced detection rate of 19%. This may be an overestimate. On the other hand, the selected DM91 sample includes a number of companions with periods below 10 days, which could not exist around contact binaries.[5] Since these numbers are no more than guesses, we use a rounded expected detection rate of 20% for our analysis below.

In summary, we conclude that if companions to contact binaries occurred at the same frequency as those to solar-type stars, and if their properties followed the same mass-ratio and period distributions, we should have detected tertiaries for about 20% of our sample, or 12 out of 59. In reality, we found 15 tertiaries. This is three more than expected, but the difference is not highly significant: there is a 19% probability of finding 15 or more tertiaries out of 59 systems when the expected tertiary rate is 20%.

To calculate a similar probability for our hypothesis that all close binaries are in triple systems, we need to estimate the probability that a tertiary will have the correct properties to be detected with our method. For this purpose we use again the DM91 sample and first estimate the companion fraction for all systems with $q < 0.75$, again by adding up lines 3 and 5 in Table 7 (we thus include the white dwarfs). Without the highly uncertain $q < 0.1$ bin, we find 34.75 spectroscopic and 59 resolved binaries, respectively, or an implied companion fraction of $93.75/160.35 = 58\%$. If we include the estimated 5.6 spectroscopic and 14 resolved binaries with $q < 0.1$, this rises to 67%. Thus, between 30% and 34% of all companions have $q > 0.28$ and $P < 10^6$ days, and are not white dwarfs. If the tertiary rate were 100%, these would be the expected detection rates, and hence, we would expect to have found between 17 and 20 tertiaries in our sample of 59. Conservatively assuming the expected tertiary rate is 34%, we find that there is a 10% probability of finding 15 or fewer systems. Thus, our results are also consistent with the hypothesis that all close binaries are in triple systems.

Finally, for all our above estimates we assumed that the tertiaries followed the same mass-ratio and period distributions as those of solar-type companions. We do not have sufficient objects to test this rigorously but can at least verify this hypothesis. For the ranges

$$q = \{[0.28,\ 0.4\rangle,\ [0.4,\ 0.5\rangle,\ [0.5,\ 0.6\rangle,\ [0.6,\ 0.75\rangle\},$$

we found $\{8,\ 3,\ 3,\ 1\}$ tertiaries. Consulting Table 7 of DM91 and scaling to the same total number of companions (15), we infer $\{6.4,\ 3.2,\ 3.5,\ 1.9\}$ binaries (where, as above, we reduced the long-period bin by a factor of 31/65 to correct for periods $>10^6$ days and deducted one system in the three higher mass-ratio bins in order to correct for white dwarfs among the spectroscopic binaries). Clearly, within the limited statistics, the two distributions are consistent.

5. CONCLUSIONS AND PROSPECTS

We searched a sample of 75 close binaries for the spectroscopic signature of tertiaries and identified 23 triple systems, implying a ratio of almost one in three. For a homogeneous subset of 59 contact binaries, we are fairly confident our 15 tertiaries are all those that have periods $\lesssim 10^6$ days and mass ratios $0.28 \lesssim M_3/M_{\rm CB} \lesssim 0.75$.

We compared our results with expectations under two hypotheses: that the incidence of tertiaries is similar to the incidence of companions to solar-type stars, and that all close binaries are in triple systems. The latter hypothesis is expected to hold if close binaries form via the Kozai mechanism; the former appears to be a conservative upper limit for the case in which the formation of close binaries is unrelated to multiplicity. Using the DM91 sample of companions to solar-type stars to infer mass-ratio and period distributions, we find that, for the two hypotheses, the expected numbers of triple systems among our sample are 12 and

[4] DM91 mention seven triple systems but do not use the extremely wide outer orbit of the triple HD 122660.

[5] One of these, 44 Boo, is in our sample as well.

20, respectively. Finding 15 systems is consistent with either hypothesis.

While inconclusive in terms of testing the role of multiplicity in the formation of close binaries, we feel the relatively large fraction of triple systems found is encouraging, especially since in Paper I, from a variety of methods, a high tertiary fraction of 59% $\pm$ 8% was observed as well. To make progress, a larger fraction of tertiary parameter space needs to be covered. For the method presented here, this is not difficult since the archive observations used here were not optimized for the search for faint tertiaries. By using higher resolution spectra spanning a larger wavelength range, one can improve the contrast in the spectra and increase the sensitivity, and by observing at longer wavelength one will be sensitive to lower mass tertiaries for a given limiting contrast ratio. All three improvements are possible with echelle spectrographs. Furthermore, with adaptive optics in the near infrared, one can reach even lower mass tertiaries (although only on relatively long orbits). We hope to follow both routes in the future.

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APPENDIX

INDIVIDUAL SYSTEMS

Below, we briefly summarize the properties of all binaries for which we detected the spectroscopic signature of a tertiary, as well as for a number of systems without detections for which the results warrant further discussion (marked with a dagger in flux ratio column, β, of Table 1). The roman numeral directly following the system's name is the paper in the series "Radial Velocity Studies of Close Binary Stars" from which our data were drawn; see the references at the end of Table 1. For data on resolved multiples we generally rely on the Multiple Star Catalogue (MSC; 2005 June update; Tokovinin 1997). Individual figures (Fig. Set 1), as well as a brief description of each object in our data set, are available in the electronic edition of the paper.

GZ And (I) is a W UMa binary and the brightest component of a visual multiple system. Despite the noisiness of the spectrum and the relatively poor match for our best temperature of 5600 K, we clearly detect a third companion, with $\beta = 0.015$ and $T_3 = 3500$ K. Of all visual components, the only one that this could correspond to is component E of Paper I, which is at a separation of 2ʺ13 and has $\Delta H = 2.6$ and $\Delta K = 2.4$. Using Table 2 and Cox (2000, Tables 7.6, 15.7, and 15.8), we infer $M_K = 3.3$ for GZ And. Thus, component E has $M_K \simeq 5.7$ and is likely an $\sim$0.42 $M_\odot$ M2 star with $T_3 \simeq 3500$ K, $V-K \simeq 4.0$, and $M_{V,3} \simeq 9.7$. The temperature agrees very well with our measurement, and hence, we are confident we have detected component E. The implied flux ratio of $\beta \simeq 0.011$ is somewhat smaller than what we measure, the opposite of what is expected given that the tertiary would not have been completely in the slit. Likely, our measurement is biased by the relatively poor fit.

HV Aqr (III) is an A-type contact binary system and is one of the best examples of our program's ability to detect components at flux ratios of only a few percent. Although visually there appears to be little improvement between the fit with and without a third star and it is difficult to see the third star's contribution in the residuals from the binary-only fit, there are distinct minima in the variance as a function of T_3 and Δv_3. Furthermore, the fitted temperature, $T_{3,\mathrm{fit}} = 4000$ K, matches that inferred from the flux ratio, $T_{3,\beta} = 4000$ K. In Paper I no other indicators for multiplicity were found.

V410 Aur (VIII) is a W UMa–type binary in a known triple system, with a tertiary at 1ʺ7 that is fainter by $\Delta V = 1.04$ (MSC), corresponding to $\beta = 0.38$. The signal of the tertiary is obvious in the spectra, and from its contribution to the broadening function a flux ratio of 0.26 was inferred, while from our routine we infer 0.22. Likely, both numbers are lower than the true flux ratio because some of the light fell outside the slit. The temperature $T_{3,\mathrm{fit}} = 5200$ K is consistent with that inferred from the flux ratio, $T_{3,\beta} = 5600$ K.

44 Boo B (IV) is the contact binary nearest to Earth, and its spectrum is dominated by a brighter star ($\Delta V = 0.78$) at a separation of 1ʺ7. In the spectra used to analyze this system, some of the light from the third component was blocked by the slit, resulting in a flux ratio of 0.4–0.7 inferred from the broadening function. Our procedure yields a somewhat lower flux ratio of 0.23, although the program fits part of the third star as if it were the contact binary. The fit yields very sharp, well-defined minima in variance as a function of T_3 and Δv_3, but our results are nevertheless poorly defined, since the initial fit to the contact binary was biased greatly by the presence of the third star (since it dominates the spectrum). As a result, the temperature inferred for the contact binary is too high, and that for the tertiary too low. Since we use the observed flux ratio, however, our inferred tertiary mass and mass ratio should be accurate.

CK Boo (II) is an A-type W UMa system. We find a good fit to the spectrum, and while there are some systematic residuals, a clear signature of a third star is present, with a very low flux ratio $\beta = 0.009$. The fitted temperature is consistent with that inferred from the flux ratio, $T_{3,\beta} = 3900$ K. The tertiary is also detected in the adaptive-optics observations described in Paper I. The separation is only 0ʺ12, and hence, the magnitude difference $\Delta K \simeq 2.8$ is rather uncertain. We nevertheless tried to verify consistency: using Table 1 and Cox (2000, Tables 7.6, 15.7, and 15.8), we infer $M_K = 2.6$ for CK Boo and thus $M_{K,3} \simeq 5.4$. The latter implies that the tertiary would be an $\sim$0.48 $M_\odot$ M1 star with $T_3 \simeq 3800$ K, $V-K \simeq 3.8$, and $M_{V,3} \simeq 9.2$. The temperature and implied flux ratio of 0.007 agree well with our measurements. We note that from arrival-time variations in Paper I the possibility of a companion in an $\sim$5 AU orbit was mentioned. The inferred minimum mass of 1.5 $M_\odot$, however, is inconsistent with our results, unless it is a neutron star or black hole.

FI Boo (IV) is a W-type contact binary system. We clearly detect a faint third component, with $\beta = 0.012$ and $T_{3,\mathrm{fit}} = 3900$ K. The fit to the binary has relatively large systematic residuals, which dominate the variance; as a result, adding a third star does not change the variance as much as might be expected if the fit were better. Despite these limitations, there is an obvious minimum in the variance as a function of T_3 at a much later spectral type than that of the main binary. Furthermore, the variance shows a sharp drop at $\Delta v_3 \simeq 0$. The presence of a tertiary is also inferred from stochastic residuals in *Hipparcos* measurements (Paper I).

AO Cam (III) is a W-type system. The fit to the binary leaves fairly large residuals. In contrast to most other systems, these appear not to be due to inaccuracies in the broadening function but rather a relatively poor match to the template. Despite these limitations, the variance shows clear minima as a function of both T_3 and Δv_3, which leads us to conclude that there is a faint, $\beta = 0.008$ third component in the system. The temperature inferred from the fit, $T_{3,\,\rm fit} = 4200$ K, agrees well with that inferred from the flux ratio, $T_{3,\,\beta} = 4000$ K. In Paper I no other evidence for a companion was found.

SV Cam (VI) is a detached binary, for which the likely presence of a third body in a 41 or 58 yr orbit was inferred from arrival-time variations (Lehmann et al. 2002; Borkovits et al. 2004); the implied separation is a few 0ʺ1, and the minimum mass is around 0.2 $M_\odot$. The different temperatures of the binary components make it less suited to our method of analysis, and our fit to the average spectrum is relatively poor. We nevertheless clearly detect a tertiary and infer a temperature $T_{3,\,\beta} = 3900$ K that agrees nicely with the one derived from the fit, $T_{3,\,\rm fit} = 3900$ K. We note, however, that because of the poor fit, there is a marked decrease in variance for all tertiary spectral types: likely, the third star is being fitted to some of the residuals left from fitting the main binary. As a result, the fitted flux ratio of 0.016 may be somewhat higher than it would be if we were fitting only the third star. Since the temperature is close to the lower limit of our range of templates, the real temperature may well be lower. An independent indication for a lower temperature would be that the inferred mass of $M_3 = 0.65\ M_\odot$ is somewhat high compared with that inferred from the arrival-time orbit: it would require an inclination $i_3 \lesssim 20°$, which has an a priori probability of $\lesssim$5%. Furthermore, Lehmann et al. (2002) mention that "masses of $\geq$0.60 $M_\odot$ should be excluded because a third stellar spectrum would be visible in the observations, which is not the case." Since SV Cam is not a contact binary, we do not include it in our statistical analysis.

V776 Cas (V), an A-type contact binary, is the brighter member of a visual binary, with an angular separation of 5ʺ38 and $\Delta V = 1.56$ (MSC). Our fit to the contact binary is fair, and we easily detect the third star. Our flux ratio is much lower than the measured one, since most of the tertiary's light fell outside of the slit (indeed, many of the individual spectra of V776 Cas were taken intentionally excluding the third star to make it easier to calculate the radial velocity for the individual components). The fitted temperature $T_{3,\,\rm fit} = 6100$ K is somewhat higher than that inferred from the flux ratio.

EE Cet (VI) is a component of a visual binary in which the second component, at 5ʺ6, is another close binary (hence, the system is a quadruple). The contact binary is the fainter component, by $\Delta V = 0.36$ (MSC). The data used in our observations were taken excluding as much light from the other binary as possible, but our program nevertheless picks out scattered light from the companions and identifies it as a very faint third star. We do not list it in Table 1, since without knowing that the system was a multiple we would not have identified it as such. For reference, we note that our fit yields $T_{12} = 6200$ K and $T_3 = 6800$ K.

V445 Cep (IX) is an A-type contact binary with very shallow eclipses. It is a hot system; we measure $T_{12} = 7400$ K for the main binary. We also find a third star in the system, with $T_{3,\rm fit} = 6600$ K and relative flux $\beta = 0.056$. This flux is high enough that one would have expected the tertiary to have been detected in previous surveys, so it is a bit puzzling that it has not. Nonetheless, the fact that there is a clear minimum in the variance when a third star of a very different spectral type is added leads us to conclude that there is a third component in the system.

KR Com (VI) is an A-type contact binary in a visual binary, with a companion at 0ʺ119 that is fainter by $\Delta V = 0.59$, or $\beta = 0.58$ (MSC). From the broadening function, a flux ratio of 0.56 was inferred, while we find a much lower value of 0.23, likely because our fit is biased by the fact that the tertiary is so bright. Because of this, we use the observed flux ratio to infer the tertiary's parameters.

V401 Cyg (VI) is a contact binary for which the spectral signature of the tertiary, despite being only at the 3% level, was already seen in the broadening function. Our fit to the binary is poor in a somewhat surprising fashion: in some parts it reproduces the spectrum very well, while in others, particularly around 5170 Å, it fails utterly. Despite the resulting uncertainties, we clearly recover the third star with $\beta = 0.015$. The temperatures inferred from the fit and the flux ratio are both 4700 K. The presence of a close-in tertiary is also inferred from stochastic residuals in *Hipparcos* measurements (Paper I); it cannot be the object at a separation of 18ʺ0 in the adaptive-optics observations presented in Paper I.

V2082 Cyg (IX) is likely an A-type contact binary, although a detached configuration cannot be completely excluded. We measure $T_{12} = 7000$ K and obtain a fairly good fit, although there are a number of features, at 5235, 5195, and 5167 Å, that are stronger than in the template. Nevertheless, we are able to detect a faint third companion, with $\beta = 0.02$ and $T_{3,\,\rm fit} = 5100$ K (the latter consistent with $T_{3,\,\beta} = 5200$ K inferred from the flux ratio). The only other indication for the presence of a tertiary found in Paper I was that the X-ray flux was stronger than expected for the early-type binary.

V2150 Cyg (IV) is an A-type contact binary that has a much fainter, $\Delta V = 3.35$ visual companion at 3ʺ68 (Paper I). The faint companion star was outside the slit in our spectra, and unlike for brighter, well-separated visual doubles in our sample (such as EE Cet), we detect no scattered light from the third companion, nor do we find any closer companion. Our detection limit is at the 1% level, despite the fact that the binary's spectrum is not fitted all that well, with relatively large, broad-scale deviations, as well as some higher frequency residuals. This may partly be because the temperature of about 7200 K places V2150 Cyg near the upper end of our temperature range, where we have relatively few template spectra.

V899 Her B (IV) is an A-type contact binary for which the broadening profiles indicate it is the fainter component of a spectroscopic triple: $\beta = 1.5$. The brighter star is itself a radial-velocity variable as well. From our procedure, we infer a smaller value for the flux ratio, 0.725, but this is likely because the initial fit to the spectrum is biased by the dominating flux from the third star. Hence, the temperature for the binary is overestimated. Nevertheless, the overall fit is fairly good. In Paper I no other evidence for the presence of a tertiary is listed.

ET Leo (VI) is a low-amplitude contact binary presumably seen at low inclination. Our initial fit to the binary spectrum left rather strong, large-scale residuals; these were reduced but not altogether removed using a 10th-order polynomial fit to the continuum. Our best-fit temperature for the binary is 5800 K. This is substantially higher than what would be inferred from the spectral type of G8 assigned by Rucinski et al. (2002) but consistent with the temperature inferred from $B - V$. The residuals from the best fit show a clear signature of an M-type tertiary, and this is confirmed by the variance as a function of T_3 and Δv_3. The inferred flux ratio and temperature are $\beta = 0.022$ and $T_{3,\,\rm fit} = 3900$ K. The presence of the tertiary was also suspected based on residuals of *Hipparcos* measurements (Paper I).

VZ Lib (IV) is a contact binary for which the presence of a fainter tertiary component was already indicated by its signature in the broadening function. No other indicators for a tertiary were found in Paper I. We recover the third star unambiguously but find $\beta = 0.045$, which is much fainter than the value of 0.2 estimated from the broadening function. This may be because the previous measurement effectively measured the tertiary's flux assuming it had the same spectral type as the binary, while in reality it is later and hence has stronger lines (§ 3.1). Our overall fit is good, although some of the sharp-lined features (particularly at 5182 Å) are reproduced relatively poorly, indicating that the tertiary spectral type or metallicity may not be entirely correct. The tertiary's temperature estimates are also not quite in agreement: $T_{3,\beta} = 4200$ K from the flux ratio and $T_{3,\mathrm{fit}} = 4700$ K from the temperature.

SW Lyn (IV) is a close, semidetached binary in a "reversed-Algol" configuration, with the more massive component filling its Roche lobe. Despite the fact that the components are not in contact and therefore have different temperatures, we easily recover the tertiary that was identified from the broadening function. We find $\beta = 0.19$, substantially less than the value of 0.33 inferred before (likely because the latter does not take into account that the tertiary has a different spectral type; § 3.1). Overall the fit is good, although some mismatches remain. Since this system is not a contact binary, it was not used in the statistical analysis.

V502 Oph (IX) is a W-type contact binary for which there are several pieces of evidence pointing to a third companion. First, Hughes & McLean (1984) detected two radio sources near the source, separated by only 2.″6. Second, Derman & Demircan (1992) found that the arrival times of the minima showed a modulation with a period of about 35 yr (which, however, is too short for a companion separated by 2.″6). Third, Hendry & Mochnacki (1998) detected stationary Na I lines in trailed spectra. The latter detection is the most convincing and is the basis of the identification as a triple in Paper I. Unexpectedly, our procedure does not unambiguously recover the tertiary. We find a reasonable fit for a binary temperature $T_{12,\mathrm{fit}} = 5800$ K, but with some rather odd residuals throughout the spectrum, indicating that our template is not a good match. Adding a third star, we do see an improvement in the quality of the fit, but the minimum in variance does not occur close to zero relative velocity. Furthermore, taken at face value, the relative flux of $\beta = 0.007$ indicates a tertiary temperature much lower than the fitted value, $T_{3,\mathrm{fit}} = 6900$ K. For this reason we have not counted this system as a detection. We note, however, that the discrepancy might be reduced if the separation is really 2.″6, since in that case much of the tertiary's light might have fallen outside the slit.

V2388 Oph (VI) is a W UMa member of a bright visual binary, with a separation of 0.″087 and magnitude difference $\Delta V = 1.75$ (MSC). It is possibly seen in arrival-time and astrometric variations as well (Paper I). From the broadening function, a tertiary flux ratio $\beta = 0.2$ was found, while our procedure yields a value of 0.10. We find we can reproduce the spectrum very well, although the initial fit (without a third star) largely incorporates the light from the third star. Hence, the inferred binary temperature, $T_{12,\mathrm{fit}} = 6100$ K, is biased somewhat (the fact that the temperature inferred from $B - V$ is similar likely reflects the fact that the color is contaminated by the tertiary as well). Nonetheless, we find a very different spectral type for the tertiary, with $T_{3,\mathrm{fit}} = 5900$ K, consistent with what is inferred from the flux ratio.

BB Peg (I) is a W-type contact binary in which the combination of a light-curve fit and radial-velocity orbits allowed the masses of both components to be measured: $M_1 = 1.38\ M_\odot$ and $M_2 = 0.5\ M_\odot$. The primary mass is in good agreement with the mass inferred from the absolute magnitude through the period-luminosity-color relation. The average spectrum, which is among the more noisy we analyzed, is reproduced fairly well by our procedure, although some small deviations on relatively large scales remain. There is a clear drop in the variance as cooler third stars are added to the system, indicating the presence of a third component in the system with a relative flux $\beta = 0.009$ and temperature $T_{3,\mathrm{fit}} = 3900$ K. A third component is also suspected from arrival-time variations (Paper I).

KP Peg (IX) is a β Lyrae–type binary and is the brighter component of a visual binary with a separation of 3.″5 and $\Delta V = 1.6$. It has one of the earliest spectral types, A2, implying a temperature that is outside the range covered by our templates. Nevertheless, we find a fairly good fit to the binary and easily detect the third star. We find $\beta = 0.031$, which is dimmer than the known flux ratio since most of the light of the third star did not enter the slit. The fitted tertiary temperature of $T_{3,\mathrm{fit}} = 7700$ K is not consistent with what is expected for the observed magnitude difference.

V335 Peg (IX) is an A-type contact binary with the second component contributing only 5% of the total flux. We measure $T_{12} = 6400$ K and see some minor residuals. The variance profile for the third star declines for later type stars, indicating a third star with $T_3 = 3700$ K. The estimated flux it finds for the third star is only $\beta = 0.006$, however, which is right at our detection limit. Hence, we classify it as an interesting nondetection.

HX UMa (VIII) is an A-type contact binary with a previously identified, fainter ($\Delta V = 3.31$) companion at a separation of 0.″626 (MSC). The tertiary's signal was also detected in the broadening function, yielding $\beta = 0.049$. Our procedure provides an excellent fit to the main binary, with the fit improving even further with the addition of a third star. We find $\beta = 0.023$, which is somewhat lower than indicated by the magnitude difference and inconsistent with the measurement from the broadening function. It is possible that this results partly from light not entering the slit and partly from the bandpass being blueward of V. (If so, the good agreement found earlier would be due to a fortuitous cancellation of the light loss by the overestimate of the flux resulting from the assumption that the tertiary had the same spectral type as the binary; § 3.1.) The fitted tertiary temperature is $T_{3,\mathrm{fit}} = 4400$ K, which is in excellent agreement with that inferred from the observed flux ratio.

II UMa (VI) is an A-type contact binary that has a fainter, $\Delta V = 1.64$, companion at a separation of 0.″87 (MSC). The tertiary was obvious already in the broadening function, yielding $\beta = 0.17$, and our procedure easily recovers it. We find $\beta = 0.15$. Unlike other cases, this is consistent with the value found from the broadening function, since the spectral types of the contact binary and the tertiary are very similar.

HT Vir B (IV) is part of a close visual binary, with a period of 274 yr and semimajor axis of 1.″01 (Heintz 1986; current separation of ∼0.″6), and the spectrum shows strong lines from the third star. The contact binary is brighter than the companion during its maxima ($\Delta V = 0.63$) but fainter during the minima. Lu et al. (2001) found radial-velocity variations in the third component, indicating that it is in a binary itself and hence that the system is a quadruple. Our procedure yields a flux ratio $\beta = 0.28$, lower than the value of 0.52 inferred from the broadening function. Our fit is fairly good, and we infer temperatures around 6000 K for both components, consistent with the values inferred from the colors and flux ratio.

REFERENCES

Beust, H., Corporon, P., Siess, L., Forestini, M., & Lagrange, A.-M. 1997, A&A, 320, 478
Borkovits, T., Patkos, L., & Csizmadia, S. 2004, in ASP Conf. Ser. 318, Spectroscopically and Spatially Resolving the Components of the Close Binary Stars, ed. R. W. Hilditch, H. Hensberge, & K. Pavlovski (San Francisco: ASP), 245
Cox, A. N. 2000, Allen's Astrophysical Quantities (New York: AIP)
Derman, E., & Demircan, O. 1992, AJ, 103, 1658
Duquennoy, A., & Mayor, M. 1991, A&A, 248, 485 (DM91)
Eggleton, P. P., & Kiseleva-Eggleton, L. 2001, ApJ, 562, 1012
Heintz, W. D. 1986, A&AS, 64, 1
Hendry, P. D., & Mochnacki, S. W. 1998, ApJ, 504, 978
Hughes, V. A., & McLean, B. J. 1984, ApJ, 278, 716
Kiseleva, L. 1998, in IAU Colloq. 172, Impact of Modern Dynamics in Astronomy, ed. J. Henrard & S. Ferraz-Mello (Dordrecht: Kluwer), 47
Kozai, Y. 1962, AJ, 67, 591
Lehmann, H., Hempelmann, A., & Wolter, U. 2002, A&A, 392, 963
Lestrade, J., Phillips, R. B., Hodges, M. W., & Preston, R. A. 1993, ApJ, 410, 808
Lu, W., & Rucinski, S. M. 1999, AJ, 118, 515
Lu, W., Rucinski, S. M., & Ogłoza, W. 2001, AJ, 122, 402
Mochnacki, S. W. 1981, ApJ, 245, 650
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes in FORTRAN: The Art of Scientific Computing (2nd ed.; Cambridge: Cambridge Univ. Press)
Pribulla, T., & Rucinski, S. M. 2006, AJ, 131, 2986 (Paper I)
Prugniel, P., & Soubiran, C. 2001, A&A, 369, 1048
Pych, W., et al. 2004, AJ, 127, 1712
Rucinski, S. M. 2002, AJ, 124, 1746
———. 2004, NewA Rev., 48, 703
Rucinski, S. M., & Kaluzny, J. 1982, Ap&SS, 88, 433
Rucinski, S. M., & Lu, W. 1999, AJ, 118, 2451
Rucinski, S. M., Lu, W., Capobianco, C. C., Mochnacki, S. W., Blake, R. M., Thomson, J. R., Ogłoza, W., & Stachowski, G. 2002, AJ, 124, 1738
Rucinski, S. M., Lu, W., & Mochnacki, S. W. 2000, AJ, 120, 1133
Rucinski, S. M., Lu, W., Mochnacki, S. W., Ogłoza, W., & Stachowski, G. 2001, AJ, 122, 1974
Rucinski, S. M., et al. 2003, AJ, 125, 3258
———. 2005, AJ, 130, 767
Stepien, K. 1995, MNRAS, 274, 1019
Tohline, J. E. 2002, ARA&A, 40, 349
Tokovinin, A. A. 1997, A&AS, 124, 75
Tokovinin, A. 2004, Rev. Mex. AA Conf. Ser., 21, 7
Tokovinin, A., Balega, Y. Y., Pluzhnik, E. A., Shatsky, N. I., Gorynya, N. A., & Weigelt, G. 2003, A&A, 409, 245
Tokovinin, A. A., & Smekhov, M. G. 2002, A&A, 382, 118
Vilhu, O. 1982, A&A, 109, 17
Webbink, R. F. 2003, in ASP Conf. Ser. 293, 3D Stellar Evolution, ed. S. Turcotte, S. C. Keller, & R. M. Cavallo (San Francisco: ASP), 76
Yakut, K., & Eggleton, P. P. 2005, ApJ, 629, 1055