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SPURIOUS PERIODS IN SPECTROSCOPIC BINARIES, II*

By R. W. Tanner

WHEN radial-velocity measurements are always made at nearly the same time of day, the possibility arises of representing the observations equally well by alternative periods. To determine which of these related periods is the true one, other observations made at times differing as widely as possible from the usual time are needed.

If such observations are not at hand, a study of the phase errors of the observations correlated with even slight variations in the time of observation may be informative.

A detailed account of the method, with examples of its application, will be found in the Journal of the Royal Astronomical Society of Canada, vol. 42, p. 177, 1948 (Paper I).

For each observation recorded a relative hour angle, α, measured in sidereal days, is calculated, together with the departure in phase, $\Delta\phi$, of the observation from the mean curve drawn through all the observations assembled using the published period. A correlation diagram of $\Delta\phi$ against α is made. It is readily shown that if the published period is the true one, $\Delta\phi$ is independent of α, but if the period is spurious, $\Delta\phi$ depends linearly on α.

In view of the importance of the correct determination of period a systematic survey of a large number of published spectroscopic binary orbits has been made.

The published orbits of 149 spectroscopic binaries have been surveyed for alternative periods. Four stars were found to have spurious periods, and one other was found to be better represented by an unrelated period. This paper supplements the Paper I by providing

(1) a list of the stars examined, roughly classified according to reliability of period

(2) notes on some individual stars of interest

(3) revised orbits for the four stars whose periods were spurious.

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The periods of the stars investigated are not all equally well determined, and the following classification, while unfortunately rather vague and subjective, aims at furnishing some sort of index to the degree of confidence to be placed in the periods assigned.

Class A.—If observations for radial velocity are numerous and well distributed in phase and hour angle, and if the errors of measurement are small compared with the amplitude of variation, then the correlation diagram based on the true period will show a strong concentration of points along the α-axis. No other period can so well represent the observations, and one may repose complete confidence in the published period. Such cases, 57 in number, are listed in *A* below.

Class B.—Under *B* are listed 70 stars for which the clustering along the α-axis, while not so pronounced, is yet sufficient to leave very little possibility of an alternative period. It will be realized that the classes shade off insensibly one into another; the correctness of the *A* periods is more evident prima facie than that of the *B*.

Class C.—In unfavourable cases the correlation figure may fail to give definite indication of the truth or otherwise of the published period. This may be due to a paucity of observations, large errors of measurement, little variation in the hour angle at which the star was observed, or to non-orbital variations in the lines. All such orbits were very carefully scrutinized, and although no better period than the published one could be found for 17 of them, it is believed that further observation would be desirable to put the periods beyond doubt. Perhaps some of these stars are not true binaries, and might be relegated to the appendix of Moore's Catalogue after further investigation. In concluding this explanation of the grouping below, it should be mentioned that two stars at first placed in *C* have now been included in *A* in the light of evidence subsequently available. The stars for which new periods are found are also listed under *C*.

The numbers are those of Moore's Fourth Catalogue up to 372; the others are H.D. numbers.

A	1	5	6	12	13	18	30	31	34	38
	46	48	50	61	62	65	67	69	71	73
	80	81	92	93	106	107	108	110	111	113
	116	123	126	128	129	140	148	162	171	174
	180	181	182	184	193	243	270	282	301	303
	331	336	367	22124	34762	93075		179094		

B	2	3	9	16	17	21	24	29	33	40
	41	47	55	57	58	60	63	66	70	76
	83	85	86	87	88	89	91	94	96	103
	109	120	125	133	134	139	141	142	144	145
	149	150	159	166	167	172	183	186	203	207
	227	235	246	259	260	288	311	317	324	328
	361	363	364	372	9312	96528	99967	181144		
	209813	213389								
C	27	28	32	35	49	52	56	95	99	105
	161	165	221	254	325	65626	218066			
Spurious	68	230	278	1826						
Erroneous	340									

The following notes on some of the stars in the last class will perhaps make the basis of classification a little clearer.

Moore 27. Although more than 100 observations with a variation in α of 0.4 day are available, the period is still doubtful. Luyten has suggested a 2.34 day period, and the Lick and Ottawa velocities indeed show a trace of the correlation appropriate to this period, but the Victoria plates do not confirm this. The residuals are worse with this period than with the published period of 1.74 days. Possibly the star is not binary?

Moore 35. A little positive correlation was seen, but both P_2 and ${}_2P_2$ fail to assemble the observations as well as the published period. There seems no possibility of a long period other than the published one.

Moore 49. This is a southerly star for which the range in α is necessarily small for northern observers, and the errors of measurement are large. No correlation was evident, and the alternative periods do not improve the fit. Trials were made for unrelated periods without success.

Moore 52. This is a double-line binary whose lines are resolved for only about 0.1 period; consequently a test for alternative period is inconclusive. The star might be examined profitably with higher dispersion.

Moore 56. There is some evidence of correlation between large hour angles and large residuals, but of conflicting sign, so that this does not appear to be due to a short period; flexure in spectrograph, or other systematic observational error, perhaps?

Moore 221. The scatter diagram vaguely suggests ${}_2P_1$, and this period represented the published observations about as well as the longer period. When inquiry was made to Victoria, Dr. Pearce made available another 19 plates for examination and measurement here. It was concluded from a study of these that the two components could be differentiated pretty consistently, thus ruling out a ${}_2P_1$ period, but the representation is still not quite satisfactory, and the star should be further investigated.

Revised Orbit, H.D. 1826

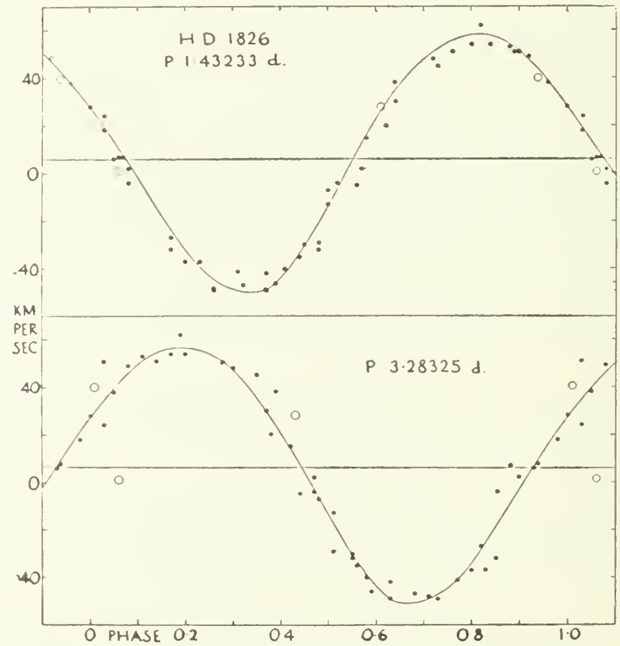
This star appeared in *Pub. D.D.O.*, vol. I, no. 6, with period 3.28 days. A strong positive correlation has indicated P_2, 1.43 days. The fit with the new period was at first only slightly better. One plate omitted from the original orbit was found:

J.D. 2429556.767	Velocity −24.3 km./sec.

Three confirmatory plates were obtained in 1947:

J.D. 2432516.590	Velocity +28.0 km./sec.
2432518.492	+40.4
2432518.660	−01.2

These enabled the period to be estimated at 1.43233 ± 0.00002 days from observations over 3100 revolutions.



A few preliminary trials showed a good fit with a circular orbit. Using the preliminary elements

T_0 2429189.573 J.D., $K = 53.3$ km./sec., $\gamma \doteq 4.5$ km./sec., $e = 0$

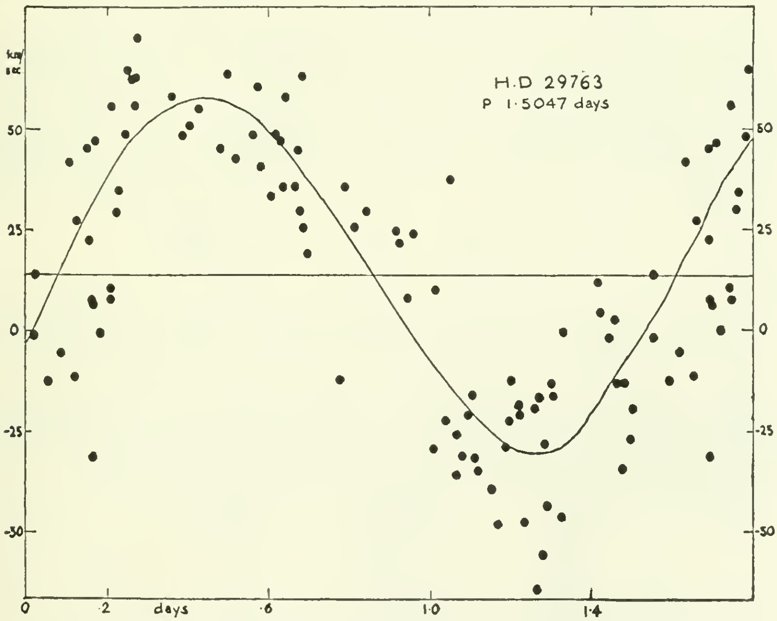
a least-squares solution by Sterne's method was made on the 48 available plates. Equal weights were employed, and the observations were not grouped.

The new elements with their mean errors follow; the former values are given on the right for comparison.

	Revised Elements	Previous Elements
P	1.43233 ± 0.00002 days	3.28325
T_0	2429189.577 ± 0.004 J.D.	
e	0.024 ± 0.017	0.056
ω	$202° \pm 39°$	152°
γ	$+4.43 \pm 0.62$ km./sec.	5.90
K	53.40 ± 0.92 km./sec.	54.5
$a \sin i$	1.05×10^6 km.	2.46×10^6
mass function	$0.023 \odot$	$0.055 \odot$
mean error single plate	± 4.22 km./sec.	± 6.1

Revised Orbit, H.D. 29763, τ Tauri

The original orbit is by Parker, *Report Chief Astronomer*, *Canada*, vol. I, p. 166, 1910. The correlation figure showed a strong positive correlation, indicating the period 2.9572 days. The observations were reassembled with this period and the sum of the squares of the

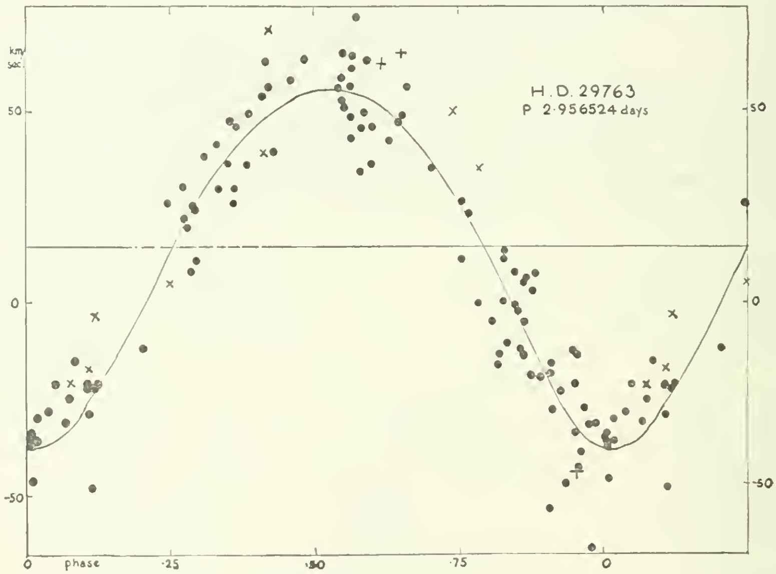


residuals was reduced by a quarter. Because of the large scatter of the observations, not much improvement could be made in the period, although the Ottawa observations covered a two-year interval.

Recourse was had to eight velocities given by Frost, Barrett and Struve, *Ap. J.*, vol. 64, p. 1, 1926, obtained from 1903 to 1922. The number of cycles elapsed between the last Ottawa plate and the single plate of 1922 was doubtful, so three plates were taken here to help fix the period:

J.D. 2432602.523	Velocity −44	(Ca II +22)	km./sec.
604.486	+61		
607.537	+65	(Ca II +25)	

By assuming the period to be constant, an estimate of 2.956524 ±0.000050 days was derived.



The present orbit is based on the Ottawa observations only. Parker's weights were adhered to, and his 104 plates were grouped into 9 normal places, using the first plate as origin, as follows:

Phase	Velocity	Weight
.1446	+22.50	0.4
.2444	+ 2.28	1.2
.2925	−17.47	1.4
.3893	−37.55	1.6
.5158	−23.41	1.0
.6823	+20.81	0.8
.7501	+33.75	0.7
.8192	+52.68	0.9
.9764	+53.73	2.2

The uneven distribution of weights results largely from the nearness of the period to three days, which makes the observations fall into three groups.

A couple of solutions by the Wilsing-Russell method suggested the preliminary elements: phase of periastron = 0.4110, γ = 14.5 km./sec., K = 46.6 km./sec., e = .13, ω = 175°. A least-squares solution led to the following final elements; the original elements are given on the right for comparison.

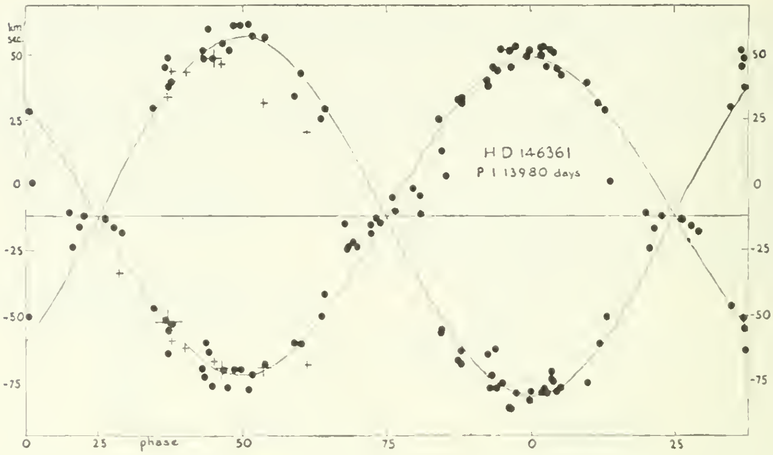
Revised Elements	Previous Elements
P 2.956524 ± 0.000050 days (not varied)	1.5047
γ + 14.56 ± 2.75 km./sec.	13.55
K 46.72 ± 1.73 km./sec.	44.34
e 0.128 ± 0.040	0.087
ω 172° ± 18°	243°
T_0 2417898.451 ± 0.020 J.D.	
$a \sin i$ 1.0 × 10^6 km.	0.9 × 10^6
mass function 0.03 ⊙	0.0135 ⊙
mean error single plate ± 12.3 km./sec.	± 16

It will be seen that the shape of the curve is somewhat altered by the change of period. Because of the drastic grouping, the elements are possibly even less reliable than the mean errors would suggest. In the diagram the solid circles represent Ottawa observations; ×'s, Yerkes observations; and +'s, Dunlap observations.

Revised Orbit, H.D. 146361, σ^2 CrB

Sixty-seven measures are given in *Pub. D.A.O.* vol. 3, p. 232, including four Mount Wilson plates; a further five plates appear in *Pub. D.A.O.*, vol. 6, p. 234. A strong positive correlation with unit slope indicated P_2. Dr. Pearce of Victoria kindly furnished a couple of corrections to the dates given in volume 3, as well as the velocity for an out of the meridian plate. All of these confirm the short period. With these data, the revised period is estimated at 1.13980 ± 0.00001 days; the observations cover more than 6000 revolutions.

The present orbit is based entirely on these 73 plates. As a preliminary step γ and the mass-ratio, r, were determined by the methods of Zurhellen, Paddock and O. C. Wilson (*Bulletin L.O.*, vol. 8, p. 156; *Ap. J.*, vol. 93, p. 30) applied to the 47 plates showing both spectra; this much reduces the subsequent labour in the least-squares determination of the remaining elements.



A circular orbit gave a good fit; preliminary elements were: $K_1 = 60$ km./sec., $K_2 = 67$, km./sec., $e = 0$, $T_0 = 2423869.113$ together with $\gamma = -11.87$ km./sec. ($r = 1.12$). The observations assembled on the period mentioned above were grouped into 10 normal places as follows, the weights being roughly proportional to the number of plates in the group, and phases reckoned from T_0:

Phase	V_1 km./sec.	V_2	Weight
.03312	48.64	−76.95	3
.12200	32.07	−61.77	1
.24856	−13.72		3
.37320	−54.14	39.58	1
.44937	−68.93	53.07	2
.51115	−72.13	57.38	2
.62500	−52.85	31.82	1
.74886	−12.96		4
.87137	27.38	−61.75	2
.95360	45.11	−77.06	3

γ and P as above were accepted; a least-squares solution by Sterne's method was made for the remaining elements, including both K's as a check. It may be noted in passing that the forms of the equations of condition for double-line binaries as given in *Pub. D.O.*, vol. 1, p. 327 and *Pub. D.A.O.*, vol. 7, no. 17, p. 291, are at first sight a little misleading; each observation gives rise to *two* equations of condition, e.g. for Sterne's method,

$$\delta V_1 = \delta\gamma + \cos L_1 \delta K_1 + \sin L_1 K_1 \mu \delta T_0 + \\ \cos 2L_1 K_1\, e \cos \omega_1 + \sin 2L_1 K_1\, e \sin \omega_1$$

$$\delta V_2 = \delta\gamma - \cos L_1 \delta K_2 - \sin L_1 K_2 \mu \delta T_0 - \cos 2L_1 K_2\ e \cos \omega_1 - \sin 2L_1 K_2\ e \sin \omega_1$$

For markedly unequal components, Paddock's method, which leads to a single equation for each pair of measures would be preferable.

The solution resulted in the following elements:

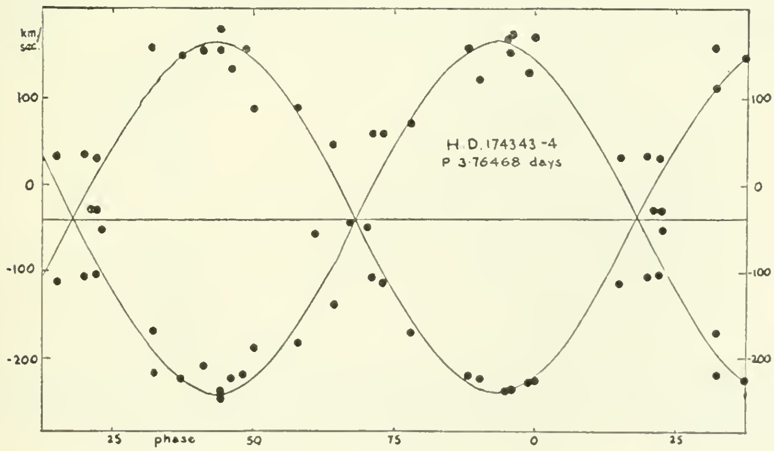
	Revised Elements	Previous Elements
P	1.13980 + 0.00001 days	7.975
T_0	2423869.1110 ± .0016 J.D.	
K_1	60.12 ± 0.77 km./sec.	60.12
K_2	68.18 ± 0.77 km./sec.	68.77
e	0.0166 ± 0.011	0.081
ω	94° ± 29°	90°
γ	11.87 ± 0.50 km./sec. (Wilson's method)	−10.63
mean error single plate	± 4.7 km./sec.	about 10
$a_1 \sin i$	9.42×10^5 km.	6.57×10^6
$a_2 \sin i$	10.68×10^5 km.	7.52×10^6
K_1/K_2	1.13 ±.02 (cf. 1.12 by Wilson's method)	
$m_1 \sin^3 i$	0.133 ⊙	0.94 ⊙
$m_2 \sin^3 i$	0.117 ⊙	0.82 ⊙

Eight more measures were made available April 1948, after the orbit had been completed.

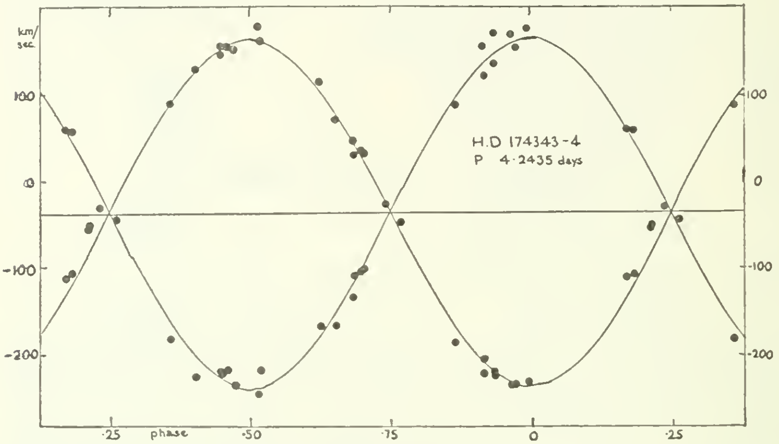
By revising the period to $1.139789 \pm 7 \times 10^{-6}$, the plates, marked +, fit the curve, as shown above. T_0 should be revised to 3869.105; the other changes in the orbit are less than the mean errors of the elements.

Revised Orbit, H.D. 174343–4, 205 Draconis

The correlation in this case was positive, but one could not be



certain from the figure whether ${}_1P_2$ or ${}_2P_2$ was indicated. The lines of the two components are so alike that no reliable indication of phase can be drawn from them. On trial, ${}_2P_2 = 4.24$ days was found to give the greater improvement in fit. Only 105 cycles are covered by the observations, so that the period is not fixed with great precision. $P = 4.2435 \pm .003$ days was finally adopted.



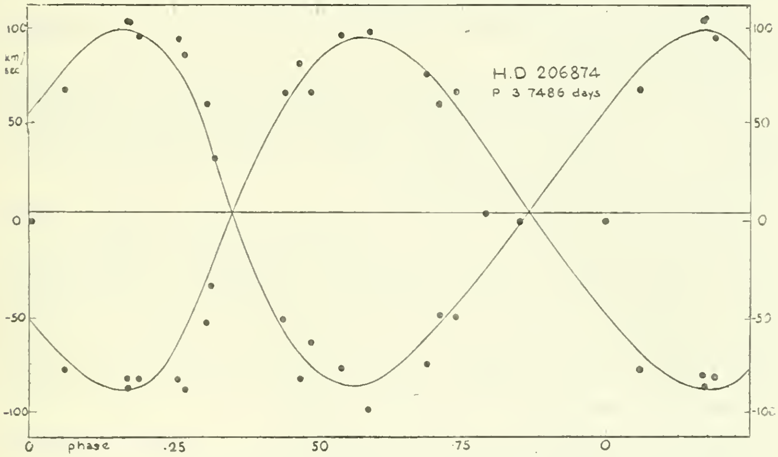
In view of this uncertainty only the best circular orbit was sought. Five measures not used in the original solution were included as they now fit the curves tolerably well. Six plates nearer the γ-axis were omitted as before. The orbit then rests on 24 pairs of observations. Preliminary elements: $K_1 = K_2 = 98$ km./sec., $\gamma = -19$ km./sec., T_0 = J.D. 2,422,160.050, $e = 0$. A least-squares solution for the best values of K_1, K_2, γ and T_0 gave the following final elements with their mean errors:

	Revised Elements	Previous Elements
P	4.2435 ± 0.0030 days	3.76468
γ	−18.6 ± 0.8 km./sec.	−18.8
K_1	101.0 ± 1.5 km./sec.	98.3
K_2	100.2 ± 1.5 km./sec.	97.7
T_0	2422160.044 ± .009 J.D.	
mean error single plate	± 5.7 km./sec.	± 7.0
$a_1 \sin i$	5.89×10^6 km.	5.09×10^6
$a_2 \sin i$	5.85×10^6 km.	5.06×10^6
$m_1 \sin^3 i$	1.72 $\odot$	1.47
$m_2 \sin^3 i$	1.73 $\odot$	1.48

The changes in the elements, except for the period, are seen to be trifling. The component designated I, happens to have the larger amplitude.

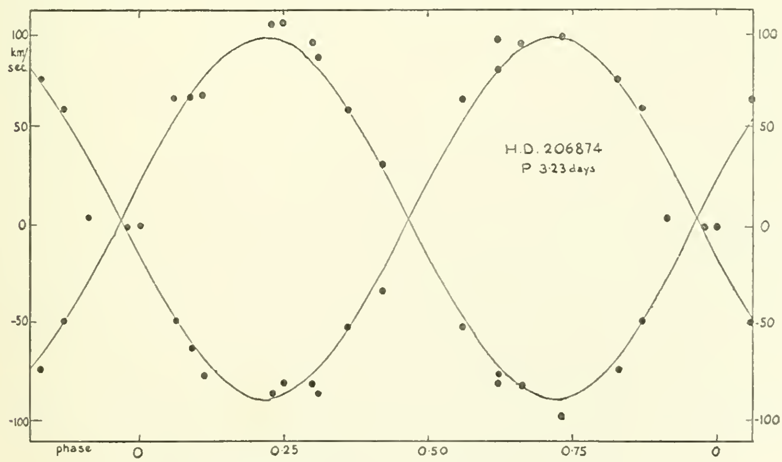
H.D. 206874, Boss 5591

This orbit appears in *Ap. J.*, vol. 53, p. 218, based on only 19 plates. The spectra are indistinguishable; the probable error of



Published velocity curve of H. D. 206874

6 km./sec. is a little large. Because of the sparseness of the observations, no conclusion can be drawn from a correlation diagram. The phase distribution obtained with the published period seemed unsatisfactory, and a thorough trial of alternative periods was made. None of the four simplest spurious periods showed any improvement.



Suggested alternative unrelated velocity curve.

The distribution of observations in time seemed to allow the possibility of an alternative *unrelated* period, and after several trials, 3.23 days was found to give a rather good fit to a circular orbit. The residuals are reduced by about one-half.

Further observations are desirable to establish with certainty the period suggested.

Diagrams of the representations obtained with the two periods are given. In the lower figure is shown the 3.23 day period; the curves are for symmetric circular orbits. Above are shown the same observations on the original period.

The writer's thanks are due to Dr. F. S. Hogg, director of the David Dunlap Observatory, for several suggestions of basic importance to this inquiry, and for guidance throughout. Acknowledgment is made also to Drs. J. A. Pearce and R. M. Petrie of Victoria, who provided material on some of the Victoria stars, and to Dr. R. F. Sanford of Mt. Wilson, who supplied information on Moore 278 and gave permission to revise the orbit.

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Richmond Hill, Ontario,
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