Assignment III: Due Nov. 14th

You can hand it in by slipping it under my office door (MP1210). You can collaborate as a group to work out the problems but you should write up the answers independently.

1. (BT problem 4.8 with an add-on) Consider a spherical system with distribution function $f(E, L)$, where E is the energy and L the total angular momentum. Let $N(E, L)\, dE\, dL$ be the fraction of stars with E and L in the ranges $(E, E + dE)$ and $(L, L + dL)$.

   - Show that
   $$N(E, L) = 8\pi^2 L\, f(E, L)\, T_r(E, L), \tag{1}$$
   where T_r is the radial period defined by equation BT (3.17). *Hint: take a read around BT equation (4.81).*
   - A spherical system of test particles with ergodic distribution function surrounds a point mass. Show that the fraction of particles with eccentricities in the range $(e, e + de)$ is $2e\, de$. Or, $dN/de = 2e$.
   - Binary star systems in the solar neighbourhood (field binaries) exhibit probability distribution in eccentricity that scales as $dN/de = 2e$. So do binaries in globular clusters that suffer extensive interactions with single stars (Heggie, 1975, MNRAS, 173, 729, Fig. 1-3). What can one infer about the dynamical past of field binaries?

2. (BT problem 7.13) Using equation (7.173) for the evaporation time of soft binaries, estimate the maximum semi-major axis of a primordial soft binary that could survive for 10Gyr in the solar neighborhood. Assume that the distribution function in the solar neighborhood is isotropic and Maxwellian, with rms velocity 50 km s^{-1}, that all stars have mass $1M_\odot$, and that the stellar density is $\rho = 0.04M_\odot$ pc^3 (from Tables 1.1 and 1.2). After this estimate, look up what is the widest field binary known.

3. (BT problem 7.16) A tidal-capture binary is formed as a result of a close encounter of two stars of equal mass m. The minimum separation during the encounter is $d_{\min}$, and the orbital energy dissipated in the encounter is $\Delta E \ll Gm^2/d_{\min}$. Once the binary has formed, more energy is dissipated in each successive orbit, until eventually the binary orbit is circularized. If the spin angular momentum of the stars is negligible compared to the orbital angular momentum, show that the radius of the final circular orbit is $2d_{\min}$.[1]

[1] Incidentally, we used this property (Wu & Murray 2002, Wu & Lithwick 2011) to explain the presence of hot jupiters around other stars. The trick part is how the planet orbits become so eccentric to warrant close encounters with their host stars.