Assignment I: Due Friday Oct 3rd 10AM

You can either hand it in to me in class or slip it under my office door (MP1210). You can collaborate as a group to work out the problems but you should write up the answers independently.

1. When the Milky Way galaxy collides with the Andromeda galaxy in a few billion years, estimate how many stars will encounter phyiscal collision with another star? assume all stars are sun-like in mass and radius. You can safely ignore enhancement of the cross section due to mutual gravity (gravitational focussing), given the high velocity of impact.

2. What is the closest approach of a star to the Solar system, over the latter's lifetime? Currently the nearest star is situated at ~ 1 pc $\sim 10^5$ AU. In the past, however, some stars could have passed much closer and could have potentially influenced the architecture of the solar system. Estimate by assuming that the velocity dispersion in the solar neighbourhood to be ~ 30 km/s, and all stars are sun-like with a local stellar mass density of $\sim 0.1 M_\odot \mathrm{pc}^{-3}$.

3. (BT problem 2.7) Astronauts orbiting an unexplored planet find that (i) the surface of the planet is precisely spherical and centered on $r = 0$; and (ii) the potential exterior to the planetary surface is $\Phi = -GM/r$ exactly, that is, there are no non-zero multipole moments other than the monopole. Can they conclude from these observations that the mass distribution in the interior of the planet is spherically symmetric? If not, give a simple example of a non-spherical mass distribution that would reproduce the observations. *Hint: use BT equation (2.95).*

4. (BT problem 3.6) A star orbiting in a spherical potential suffers an arbitrary instantaneous velocity change while it is at pericenter. Show that the pericenter distance of the ensuing orbit cannot be larger than the initial pericenter distance. *Hint: my way is to show that the radial velocity $\dot{r}$ is non-zero, or $\dot{r}^2 > 0$, when the star returns to the original periaps. There may be other ways.*

5. Consider a galaxy that has a surface brightness profile of $I(R) = I_0 \exp^{-R/R_d}$, where R_d is the disk exponential length and R the distance from the center. This problem will require some simple programming and the ability to make simple plots.

1. If this disk is a razor-thin disk and is viewed face-on, obtain the rotational velocity for a circular orbit at radius R. To invert from surface brightness, $I(R)$, to surface density, $\Sigma(R)$, you may assume that the mass-to-light ratio is a constant. Plot your answer from $R = 0.01 R_d$ to $10 R_d$.

2. Compare the circular velocity you obtain above to that of a system which has the same mass within radius R, $M(R) = \int_0^R \Sigma(R') 2\pi R' dR'$, but the mass is distributed spherically.

3. Compare the circular velocity you obtain above to that of a thick disk which has the same radial profile in surface brightness, but which has a finite vertical scale height z_0, or, $\rho(R, z) \propto \frac{1}{2z_0} \exp^{-|z|/z_0}$. Perform the comparison for orbits that lie in the disk midplanes.

4. Return to our razor-thin disk. Now assume all light from inside of $R = 0.2R_d$ comes from dust reflection, namely, a mass-to-light ratio of zero inside this radius. What is the circular velocity profile? If the system potential is spherical, the particle at this radius should not feel any more gravity, according to Gauss theorem, and should have a circular velocity of 0. What is the difference in a disk potential, and why?

5. Now instead of truncate the disk inward of a radius, we truncate it outward of, say, $R = 1.0R_d$. How is this reflected in the circular velocities? and why?

6. Epicycles. Consider an orbit in the razor-thin disk that has a small vertical and radial velocity components. Calculate the frequencies of epicycles for the two directions sparately and compare them in a plot against that of the orbital frequencies, Ω.

7. If galactic astronomy is on-going in this galaxy, explain how by measuring the rotation curves and the epicyclic motions of the stars, one is able to infer the shape of the galaxy potential.