-1. Exercises in §2.16 of HKV look suspiciously similar to the ones you will be asked in your qualify exam. They contain a lot of basic concepts in stellar astronomy that should also be a part of your mental vocabulary. I suggest you to go over them by yourself.

0. The following two problems are adapted from my undergraduate course which are useful for enforcing concepts on radiative transfer. You do not have to turn these ones in.

1) Human BMR. Estimate the blackbody luminosity [erg/s] from the surface of your body. Compare your answer with the basal metabolic rate (BMR) of an average human, ~ 1500 Kcal/day (this is the energy one spends to maintain basic bodily functions – including temperature – and is typically measured at room temperature). Can you explain the difference?

2) Greenhouse Effect. The Sun heats the Earth. One can derive an equilibrium temperature of an air-less Earth (T_p). This has to be modified when an atmosphere exists. The earth atmosphere is optically thin (transparent) in the visible wavelengths so the solar radiation are not intercepted and hits the ground directly. However, it is optically thick (opaque) in the infra-red wavelengths and absorbs heat from the ground black-body (with temperature $T = T_g$) radiation. This heat is then lost to the vacuum outside as the atmosphere radiates with a photosphere (top) temperature $T = T_p$ (think why).

Let the optical depth of the atmosphere measured at the ground-level to be τ_g ($\tau_g = \int_{\text{ground}}^{\infty} \kappa \rho dr$), while that at the photosphere to be 2/3 (see definition of photosphere). Use the radiative diffusion equation to show that

$$T_g^4 = T_p^4 \left[1 + \frac{3}{4}\left(\tau_g - \frac{2}{3}\right)\right]. \tag{1}$$

At the current epoch, $T_g = 288K > T_p = 255K$. Calculate the current atmospheric optical depth τ_g. What is the corresponding value on Venus?

the following problems are to be handed in, to my office, MP 1210, by 9AM, Oct. 8th. You can slip them in under the door if I am not in.

Homework policy: you are encouraged to collaborate, but you have to be independent in writing up the answers..

1. Photons inside the Sun.

1) Nuclear reactions in the solar center release photons of energy $\sim Mev$ (million electron volts, typical energy for reactions involving nucleus). At the surface, the Sun shines roughly as a blackbody with a temperature of $5600K$. Assume surface photons are emitted with wavelength at the peak of the black-body. For each photon released at the center, how many are emitted at the surface? What processes are responsible for splitting the high energy photons and down-converting them in energy?

2) Consider these photons as they random walk through the Sun. Opacity values inside the star are given at Fig. 4.6 of HKV , take a value that is most representative for the bulk of the star. Estimate the photon diffusion time across the Sun.

3) Your above answer ought to be much longer than the thermal time of the Sun (Kelvein-Helmholtz time). Can you explain why it takes much longer to cool down the Sun than the time it takes for photons to leak out?

4) Now consider diffusion of a helium nucleus. Do you think that we can witness hydrogen core burning by observing a rise in the surface abundance of helium, as the star ages? The dominant process to stop a fast moving helium nucleus to shoot out of the star is Coulomb scattering with other nuclei (mostly hydrogen). (hint: start by estimating the collisional cross section due to Coulomb scattering, for an ion moving at sound speed).

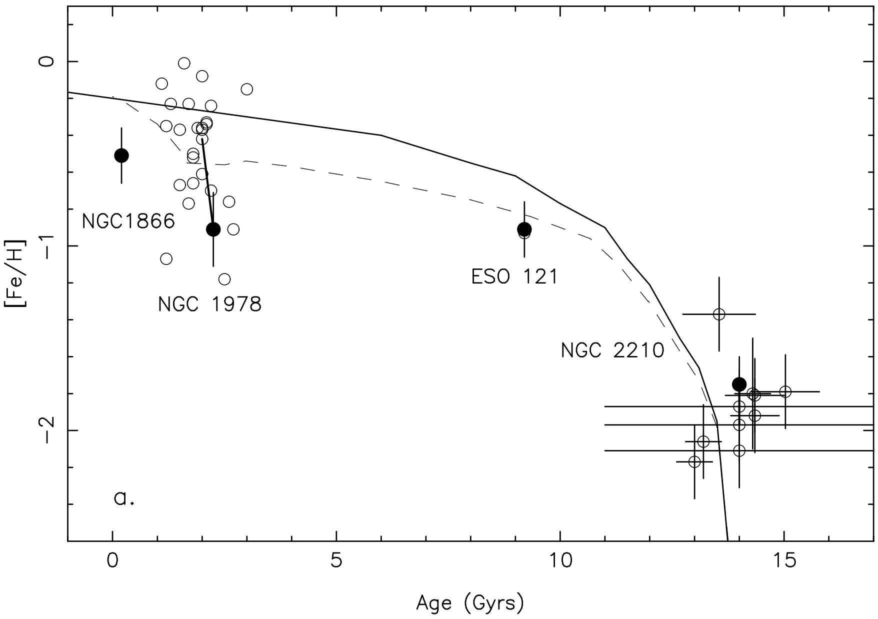

Figure 1: Age-metallicity relationship for a number of globular clusters in the Large Megellanic Clouds, (Hill et al, 2000, Astronomy & Astrophysics, 364,L19). The vertical axis ([Fe/H]) measures logarithmic metal abundance in these stars relative to our Sun. The horizontal axis indicates the age of these clusters. The open and solid circles are real data, with error-bars overplotted, The solid and dashed curves are analytical results for continuous and bursty star formation, respectively.

2. Pollution of the galaxy.

Hydrogen ($\sim 74\%$) and helium ($\sim 26\%$) were the two dominant elements made by the Big-Bang. The Sun (age ~ 4.6 Giga-years), on the other hand, is formed out of gases that has $\sim 2\%$ metals. These metals are produced by preceding generation of stars through nucleosynthesis. They are essential for forming terrestrial planets and likely, forming the giant planets. Younger systems typically exhibits higher metallicities (the so-called age-metallicity relation, see Fig. 1). We make two scaling models to quantitatively account for this behaviour.

1) Assume most preceding stars were born at a burst ~ 14 Giga-years ago (corresponding to the oldest clusters in Figure 1). These stars die after a life-time (approximated by their main-sequence life-time) of nucleosynthesis and eject high metallicity gas that enriches the galaxy. The metal content of the galaxy should rise with time as more and more stars die. Derive a scaling that describes how metallicity in the galaxy rises with time (i.e., $Z \propto t^n$, find the value of n). Assume that this burst forms stars of different masses, satisfying the Salpeter initial-mass-function: the number of stars formed in the mass interval $(M, M+dM)$ is dN with $dN/dM \propto M^{-2.35}$. Moreover, assume the amount of metal put out by a star scales with the stellar mass.

2) Instead, assume that stars in the galaxy are formed at a uniform rate (the current rate is $\sim 1 M_\odot$ per year) over the history of the galaxy. What is the value of n?

3) Compare these two scalings with results for globular clusters, shown in Fig. 1, assuming clusters formed at a time t have the galactic metallicity Z at that time. Could you decide on your favourite model? either yes or not, present your arguments. There has been much progress in the field since Hill et al(2000), it is educational to look up ADS citations of this paper.

3. Ionization and equation of state Partial ionization regions (of hydrogen and/or helium), in many types of stars, spatially coincides with the convection zones. We explore the reasons here. §3.7 and Figs. 3.11, 4.6 of HKV are relevant for this discussion.

1) In the partial ionization region, $\nabla_{\rm ad} \equiv (\partial \ln T/\partial \ln P)_{\rm ad}$ drops. Can you evaluate $\nabla_{\rm ad}$ at the zone of hydrogen half ionization? After doing so, can you give physical arguments for the decrement in $\nabla_{\rm ad}$? You would need to know the temperature when ionization happens. Saha equation predicts that this occurs when $kT \ll 13.6$ev (the ionization energy of hydrogen). Why is this so?

2) In the partial ionization region, $\nabla_{\rm rad}$ rises. This is related to the large opacity bump in this region (Fig. 4.6 of HKV), which, in turn, arises because the bound-free opacity, the relevant opacity, has the form $\kappa \propto \rho T^{-3.5}$ (also called the Kramer's opacity).[1] This relatively steep temperature dependence explains why low mass stars are likely convective in their envelopes. We derive the temperature dependence here. Ionization cross section for photons with energies above the ionization threshold goes as $\sigma \propto 1/\nu^3$, where ν is the photon frequency. We integrate over photon frequency to obtain the Rosseland mean opacity with the form $\kappa = \kappa(\rho, T)$. Show that $\kappa \propto T^{-3.5}$.

3) So partial ionization leads to convection, because $\nabla_{\rm ad}$ lies below $\nabla_{\rm rad}$, or, radiation is incapable of transporting away the stellar luminosity even at the steepest temperature gradient (that is critically stable). Why is the CNO burning core of a massive star convective?

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[1] The free-free opacity has a similar dependence, but for a different reason.