Computer projects. More so than for usual problem sets, it will be useful to work as a group to complete these tasks. The MESA documentation paper (arXiv:1009:1622) may prove useful.

1. Origin of rotation for a neutron star. Neutron stars, when born, typically spin at a period ~ 0.5 sec. Here, we study whether this amount of angular momentum arises naturally from the progenitor stars of neutron stars.

Take a $15 M_\odot$ star with solar metallicity and no mass loss, evolve it with MESA to the end of nuclear burning. Assume the star starts with an equatorial surface rotational velocity of 200 km/s, and that the eventual neutron star (with a final radius of 10 km) has a mass of $2 M_\odot$.

- Short of a detailed investigation, we can consider two extreme scenarii for angular momentum evolution (see Maeder & Meynet, ARA&A, 2000). One case is to assume that the star maintains uniform rotation throughout its evolution; the other that each mass shell inside the star retains its original angular momentum till the end.

  Calculate for each of the above scenario what the final spin of the neutron star is. If you think carefully about it, the calculation is really rather simple and shouldn't involve a lot of coding. However, take care to make sure that your star never at any point rotates faster than break-up.

- Spruit & Phinney (Nature, 1998) proposed a different solution. They argue that neither solid body rotation nor specific angular momentum conservation applies throughout the stellar life. In fact, if magnetic field is what couples the rotation of different mass shells, this coupling is effective when the star evolves slower than its own rotation rate, but ineffective when the star evolves faster than the rotation rate. Now define stellar evolution rate as the rate of nuclear fuel exaustion in the core. Continue your solid body rotation calculation till the two timescales meet, and then switch to conservation of specific angular momentum. This gives a final answer for the neutron star spin, if its angular momentum comes purely from the progenitor's in-born spin.

- This value, if it concurs with the Spruit & Phinney result, is slower than the observed values. Those authors therefore propose that neutron star spin comes during the supernova event when the neutron star also receives a mysterious (off-center) kick.

  Given your knowledge of massive star evolution (pre-supernova stages), what do you think are the caveats that may change your above result, and which way would the result go when you include these caveats.

If you are curious about the final rotational fate of our Sun, you can repeat the same procedure for a $1 M_\odot$ star . Observations show that many white dwarfs typically spin with periods of a day, but some spin with periods upward of a century.

2. Inflating hot jupiters. Hot jupiters have oftenly been observed to be bigger than their thermal evolution dictates, ranging upward to $1.8R_J$ at an age of a billion years. A number of suggestions have been raised. We use MESA to explore all of them. You will have to modify some standard inputs (see your-mesa-directory/star/test/inlist_standard for available options) or even modify the source code to be able to perform these tasks.

First evolve a $1M_J$ planet from an early age to when the surface temperature has risen to its maximum $T_{\rm eff} \sim 1400$ K (this should occur somewhere between $10^4 - 10^5$ yrs). Record this model (see Technical Issues) as a starting point for the following evolution.

There are five routes we would be attempting. So divide the task among yourselves. Each person is responsible for writing up one route and making individual presentation on that route.

- tidal energy, see, e.g., Ibgui & Burrows (2009, ApJ, 700, 192). To approximate the tidal energy input using MESA, insert a 'fake' nuclear power generation of ~ 0.1 erg/g/s throughout the entire planet for one million years. Then watch the planet size evolution for one billion years after the tidal heating is switched off. Adjust heating rate, keeping the total energy deposition constant, and look for the heating rate that has a maximum influence on planet's radius at late times. Now if instead you start the one million years of tidal heating at different epochs during evolution, say at 10^6 yrs, 10^7 yrs, 10^8 yrs, 10^9 yrs?

- flash heating, see this problem set. Let the heating (say, tidal heating) be instananeous and large so that part of the planetary convection zone can be turned radiative. For instance, let the total heat deposition be of order 10^{31} erg/s (again for 10^6 yrs) and is concentrated in a shell between 60% and 70% of the radius. Does this cause enough entropy inversion so that convection is shut off? What is the influence on planet radius? what if you vary the deposition rate and the location/thickness of the shell? can the planet be totally disrupted?

- mechanical energy input by wind, see, e.g., Guillot & Showman (2002, A&A,385, 156). Let the 'fake' nuclear generation rate be concentrated near the surface, say, the total energy input is the amount the planet received from the star at 0.03 AU, and the heat deposition starts from the photosphere downward with the volume deposition rate (erg/cm^3/s) proportional to $1/p$. Let the planet relax to a new thermal equilibrium. How does this influence the planetary radius? What if you vary the heating profile, say, as p^α, with $\alpha = -2, -1, 0, 1, 2$?

- semi-convection, see Chabrier & Baraffe (2007, ApJ, 661, 81). Let the chemical gradient inside the planet be so steep that efficient convection is forbidden. If you simply reduce the mixing-length parameter, what happens? Now, instead, insist the code to use the semi-convection option, what happens? Summarize your experiments regarding what is needed to keep the hot jupiters inflated.

- Super-solar metallicity. The luminosity of a fully convective body is limited by the radiative bottle-neck in the atmosphere. It has been suggested that increasing metallicity there can slow down the planetary contraction. First experiment with increasing the metallicity of the overall model. What happens? Now, enhance artificially the opacity by a factor f. How high does your f have to be to keep the planet inflated?

For your final presentation, please explain your results physically, and please try to use as much analytical scalings as possible.

Extra Bonus. There is a 6th route: thermal insolation, see, e.g., Arras & Bildsten (2006, ApJ). The searing of the planet atmosphere by stellar insolation can be modelled by insisting a constant temperature at the surface (say, a pressure of 10 bar), regardless of its actual luminosity. Let this temperature be that of a blackbody sitting at 0.03 AU away from the star. What if this temperature is varied between 500 K and 3000 K? In MESA, the atmosphere boundary condition is set at optical depth $\tau = 100$. —- this is a bonus question because I have not yet found a satisfactory way to implement this in MESA. If someone succeeds, I would be very interested to know the results and method. And you get some bonus points for your course grade.

Simple Technical Issues

If you start a run from pre-main-sequence, and want to record the results at a certain point, let's say, at model 75, in your inlist file, you should have,

```
create_pre_main_sequence_model = .true.
save_model_number = 75
save_model_filename = '75.mod'
```

For a jupiter-mass object, you probably don't want to generate models starting from pre-main-sequence, but want to use a pre-calculated model, like,

```
!create_pre_main_sequence_model = .true.
load_saved_model = .true.
saved_model_name = 'your-mesa-dir/data/star_data/very_low_mass_models/0.001Msun.mod'
save_model_number = 75
save_model_filename = '75.mod'
```

and when wanting to restart from model 75.mod,

```
!create_pre_main_sequence_model = .true.
load_saved_model = .true.
saved_model_name = '75.mod'
```

The relevant MESA source codes are in, e.g., atm/private, mlt/private, etc. Every time you change the MESA source code, do './install' in your-mesa-dir, as well as do './mk' in your working directory, before running.