SPHERICAL TRIG FORMULAE

1. The cosine formula: $\cos a = \cos b \cos c + \sin b \sin c \cos A$

2. The sine formula:
$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

3. Formula C: $\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A$

4. The four-parts formula: $\cos a \cos C = \sin a \cot b - \sin C \cot B$

FORMULAE for CO-ORDINATE TRANSFORMATIONS

1. **Equatorial system to Horizon System**

   $\cos z = \sin \phi \sin \delta + \cos \phi cos\delta \cos \mathrm{LHA}$

   $\sin z \cos \mathrm{A} = \sin \delta \cos \phi - \cos \delta \sin \phi \cos \mathrm{LHA}$

   $\sin z \sin \mathrm{A} = -\sin \mathrm{LHA} \cos \delta$

2. **Horizon system to Equatorial system**

   $\sin \delta = \sin \phi \cos z + \cos \phi \sin z \cos \mathrm{A}$

   $\cos \delta \cos \mathrm{LHA} = cos\phi \cos z - \sin \phi \sin z \cos \mathrm{A}$

   $\cos \delta \sin \mathrm{LHA} = -\sin \mathrm{A} \sin z$

3. **Equatorial system to Ecliptic system**

   $\sin \beta = \sin \delta \cos \epsilon - \cos \delta \sin \epsilon \sin \alpha$

   $\cos \beta \cos \lambda = \cos \delta \cos \alpha$

   $\cos \beta \sin \lambda = \sin \delta \sin \epsilon + \cos \delta \cos \epsilon \sin \alpha$

4. **Ecliptic system to Equatorial system**

   $\sin \delta = \sin \beta \cos \epsilon + \cos \beta \sin \epsilon \sin \lambda$

   $\cos \delta \cos \alpha = \cos \beta \cos \lambda$

   $\cos \delta \sin \alpha = -\sin \beta \sin \epsilon + \cos \beta \cos \epsilon \sin \lambda$

5. **Equatorial system (1950) to galactic system**

$$\sin b = -cos\delta \sin(\alpha - 282^\circ.25) \sin 62^\circ.6 + \sin\delta \cos 62^\circ.6$$

$$\cos b \cos(l - 33^\circ) = \cos\delta \cos(\alpha - 282^\circ.25)$$

$$\cos b \sin(l - 33^\circ) = \cos\delta \sin(\alpha - 282^\circ.25) \cos 62^\circ.6 + \sin\delta \sin 62^\circ.6$$

6. **Galactic system to Equatorial system (1950)**

$$\sin\delta = \cos b \sin(l - 33^\circ) \sin 62^\circ.6 + \sin b \cos 62^\circ.6$$

$$\cos\delta \cos(\alpha - 282^\circ.25) = \cos b \cos(l - 33^\circ)$$

$$\cos\delta \sin(\alpha - 282^\circ.25) = \cos b \sin(l - 33^\circ) \cos 62^\circ.6 - \sin b \sin 62^\circ.6$$

where

z is the zenith distance

ϕ is the observer's latitude

δ is the declination

LHA is the hour angle

A is the azimuth

λ, β are celestial longitude and latitude

ϵ is the earth's obliquity (23.4°)

b, l are the galactic longitude and latitude

and the galactic longitude of the north celestial pole is 123°.

Some Applications of Spherical Trig in Astronomy

1. When astronomers observe celestial objects to determine their brightnesses, they must take into account the fact that the apparent brightness of a source will depend on the amount of atmosphere through which its light must pass. This amount will increase as z, the zenith distance of a star increases and is referred to as the air mass, X. For small airmasses (or zenith distances), air mass = $\sec z$.

$$\sec z = (\sin\phi \sin\delta + \cos\phi \cos\delta \cos \mathrm{H})^{-1}$$

where ϕ is the observer's latitude and δ, H, the star's declination and hour angle respectively. However, when z becomes large, X and $\sec z$ differ. This is because of the curvature of the earth and its atmosphere. If z is greater than 60° , then

$$\mathrm{X} = \sec z - 0.0018167(\sec z - 1) - 0.002875(\sec z - 1)^2 - 0.0008083(\sec z - 1)^3$$

2. The attraction of the sun and the moon on the earth's equatorial bulge causes the celestial pole to precess around the north ecliptic pole with a period of about 26,000 years. As a result of this, the Vernal Equinox moves westward along the ecliptic. This causes the right ascension α and the declination δ to change and so we must correct for this effect. (A separate sheet which shows the required calculations will be provided.)

3. When the radial velocity of a celestial source is measured, one must allow for the fact that the observed velocity includes a component due to the earth's orbital velocity around the sun. This will change throughout the year and must therefore be accounted for. (A separate sheet showing the required calculations will be provided.)

4. When a source with variable brightness or velocity is observed, one must take into account the fact that the time required for the light to reach the earth will depend on the position of the earth in its orbit. Astronomers deal with this problem by computing the heliocentric time which is the time when the light from the object would reach the sun. A 'heliocentric' time correction is made to the time of the observation and this correction can be as large as 8 minutes and 13 seconds (0.00577553 days) if the source lies on the ecliptic and is on the meridian at midnight. The heliocentric time correction Δt is added to the geocentric time and its value is: $-0.00577R\cos\beta\cos(\odot - \lambda)$ where R is the earth's distance from the sun in astronomical units, λ and β are the celestial longitude and latitude of the source and $\odot$ is the celestial longitude of the sun at the time of the observation.

Exercise for the student

- For an observer at northern latitude ϕ determine how long a star of declination δ will be above the horizon each day.