Contents

1 Syllabus

Lectures M4, R12, AB 113, from March 12 to April 5.

Lecturer Marten van Kerkwijk, MP 1203B, 416-946-7288, mhvk@astro.utoronto.ca

Office hours Drop by my office, or by appointment

Web page `http://www.astro.utoronto.ca/~mhvk/TRANSMINI2/`

notes pdf

1.1 Synopsis

Transient stars sometimes appear on the sky. In this course, we will discuss "transient observables," the mechanisms by which transients release the radiation we observe. It is a follow up of the mini-course on the transient physics required to understand known types of transients – novae, kilonovae, supernovae, hypernovae, X-ray and γ-ray bursts – and their aftermath.

1.1.1 Transient Observables

- Known causes: mass ejection, interaction with shells, power from nuclear decay or other sources;
- Expected lightcurves, focussing on semi-analytic approximations;
- Nature of different power sources;
- Evolution of remnants.

1.1.2 Course texts

There is no specific book, though I will make use of *Supernovae and nucleosynthesis*, by Arnett (Princeton Univ. Press, 1996), which is partially based on his initial set of papers on semi-analytic approximations [links to come], as well as other review articles.

1.1.3 Evaluation

- One problem set (30%)
- Investigation of a particular type of transient, and discussing this with the instructor (40%).
- Presentation to the class about the transient (30%)

2 Introduction

2.1 What determines how a transient evolves?

For cases where mass is ejected due to strong energy deposition (dealt with in previous mini-course), the evolution usually includes the following stages:

1. Shock break-out;

2. Expansion (and thus cooling) with radiative losses (what we see!), possibly compensated with heating, e.g., by radioactive decay, recombination, a possible central power source, and/or internal shocks;

3. Interaction with surrounding matter (and thus reheating), e.g., a companion, left-over accretion disks, previously ejected shells, or simply the interstellar medium.

Exceptions to the above include transients where matter is confined. E.g.,

- Accretion disk instabilities (mostly confined gravitationally);
- X-ray bursts (idem);
- Magnetar flare afterglows (pair plasma confined magnetically).

2.2 Basics

At maximum brightness, a supernova has a typical luminosity of $\sim 10^{11}\,L_\odot$ and temperatures of $\sim\!10^4$\,K. Thus the effective radius of the emitting region is $\sim(10^5/4)\,R_\odot \simeq 2\times10^{15}\,\mathrm{cm} \simeq 100\,\mathrm{AU}$.

The typical velocities are 2 to 10 Mm/s, implying large kinetic energy $\sim 1\,\mathrm{B}\,(M/M_\odot)(v/10\,\mathrm{Mm/s})^2$ (recall, $1\,\mathrm{B} \equiv 10^{51}\,\mathrm{erg}$). It also implies an expansion time of $\sim 2\times10^6\,\mathrm{s}$, similar to the observed durations.

For this duration, the total radiated energy is $\sim 0.1\,\mathrm{B}$, i.e., large, but quite a bit smaller than the kinetic energy.

Since the radius is substantially larger than even the largest possible progenitors (say a red supergiant with a radius of $\sim 10^3\,R_\odot$), the gas must have cooled quite a bit. Indeed, for a small progenitor such as a white dwarf, essentially no thermal energy would be left: thus there must be a longer-lived power source such as radio-active decay.

More generally, one can conclude that to obtain a transient as bright as a supernova, one needs to either

1. Start from a large star;
2. Have an internal energy source (lasting for of order weeks); or
3. Have external matter to interact with (at $\sim 100\,\mathrm{AU}$).

2.3 Timescales

2.3.1 Expansion

$$t_{\rm exp} = \frac{R}{v} \simeq 1\,{\rm d}\,\left(\frac{R}{10^{15}\,{\rm cm}}\right)\left(\frac{v}{10^{9}\,{\rm cm/s}}\right)^{-1}.$$

Typically, the velocity v will soon approach a constant, and the radius R and thus the expansion timescale will increase linearly with time.

2.3.2 Diffusion

$$t_{\rm diff} = \frac{\kappa M}{\beta c R} \simeq 0.6\,{\rm yr}\,\left(\frac{\kappa}{0.4\,{\rm cm^2/g}}\right)\left(\frac{M}{M_\odot}\right)\left(\frac{R}{10^{14}\,{\rm cm}}\right)^{-1},$$

where β is a factor that depends on the density distribution (13.8 for constant density, used above).

One can derive the scaling from radiative diffusion:

$$\frac{L}{4\pi r^2} = -\frac{1}{3}c\ell_{\rm mfp}\nabla U = \frac{c}{3\kappa\rho}\frac{\partial a T^4}{\partial r} \simeq \frac{c\mathcal{E}}{3\kappa M},$$

where $\mathcal{E} = \int_V aT^4 d^3x \simeq aT^4 V$ is the total energy in radiation, we used that $M \simeq \rho V$, and one identifies the diffusion time with $\mathcal{E}/L$.

2.3.3 Heating

$$t_{\rm heat} = \frac{E}{\epsilon M}.$$

For nuclear decay with some half-time $t_{1/2}$, the heating time equal to the decay time for an energy of 1 B, mass of 1 $M_\odot$, and decay energy of 0.5 MeV/nucleon.

2.4 Overall evolution

Given the timescales, without heating, one would expect a transient to be at its brightest when the diffusion and expansion times match. With heating, as it becomes transparent, the luminosity will start to track the input heating rate.

3 Arnett's semi-analytic evolution

We follow A96 (which in turn follows Arnett `1979ApJ...230L..37`, `1980ApJ...237..541A`, `1982ApJ...253..785A`; a nice recent write-up for SN I – which have little effects od recombination – is in `2017ApJ...846...33A`) and consider a ball of gas with initial radius R_0 that is homologously expanding at constant velocity $v_{\rm sc}$, and has an initial thermal energy $\mathcal{E}_0$. The first law of thermodynamics can be written as,

$$\dot{E} + P\dot{V} = \epsilon M - L,$$

where E is the total energy, $V \equiv \frac{4\pi}{3}R^3$ is the volume, P the pressure, ϵ the energy generation rate (by radioactive decay) per unit mass, M the ejecta mass, and L the luminosity.

Assume the energy and pressure are dominated by radiation, i.e., $E \simeq \mathcal{E}$, and $P \simeq \frac{1}{3}\mathcal{E}/V$. Dividing by $\mathcal{E}$ on both sides and using homology, one finds,

$$4\frac{\dot{T}}{T} + 3\frac{\dot{R}}{R} + \frac{1}{3}3\frac{\dot{R}}{R} = 4\left(\frac{\dot{T}}{T} + \frac{\dot{R}}{R}\right) = \frac{1}{\tau_{\rm heat}} - \frac{1}{\tau_{\rm diff}},$$

where the heating timescale $\tau_{\rm h} = \mathcal{E}/\epsilon M$ and the diffusion timescale, which leads to a lumunosity $L_{\rm diff} = \mathcal{E}/\tau_{\rm diff}$, is also given by,

$$\tau_{\rm diff} = \frac{\kappa M}{\beta c R} = \tau_{\rm diff,0}\frac{R_0}{R},$$

where in the second equality one implicitly assumes constant opacity. For a constant density ball, $\beta = 13.8$.

The above suggests to consider the evolution of the product $(TR)^4$. We assume its spatial ($x \equiv r/R$) and time dependence can be split,

$$R^4(t)T^4(x,t) = R_0^4 T_0^4 \phi(t)\Psi(x).$$

For constant density $\rho = M/\frac{4}{3}\pi R^3$ and constant opacity κ,

$$\Psi(x) = \frac{\sin(\pi x)}{\pi x}.$$

In terms of these functions, the thermal energy can be written as,

$$\mathcal{E} = \int_0^R aT(r,t)^4 4\pi r^2\,dr = 4\pi R^3 aT(0,t)^4 \int_0^1 \Psi(x)x^2\,dx = \frac{4}{\pi}R_0^3 a T_0^4 \frac{R_0}{R}\phi(t) = \mathcal{E}_0\frac{R_0}{R}\phi(t)$$

where we used that $\int_0^1 \Psi(x)x^2\,dx = 1/\pi^2$. The factor R_0/R accounts for adiabatic expansion and $\phi(t)$ for radiation loss and radioactive heating. Given this, the luminosity is given by

$$L = \frac{\mathcal{E}}{\tau_{\rm diff}} = \frac{\mathcal{E}_0\frac{R_0}{R}\phi(t)}{\tau_{\rm diff,0}\frac{R_0}{R}} = L_0\phi(t).$$

Supposing the initial thermal energy is of order the kinetic energy, i.e., $\mathcal{E}_0 \simeq \frac{1}{2}Mv_{\rm sc}^2$, the initial lumonosity $L_0 = \mathcal{E}_0/\tau_{\rm diff,0} \propto v_{\rm sc}^2 R/\kappa$ is independent of mass, but proportional to radius. Faster ejections (larger energy) from larger stars (faster diffusion) give more luminous transients.

3.1 Diffusion and heating

With just diffusion and heating, one has

$$\frac{\frac{d}{dt}(TR)^4}{(TR)^4} = \frac{\dot\phi}{\phi} = \frac{1}{\tau_{\rm heat}} - \frac{1}{\tau_{\rm diff}} \qquad \Leftrightarrow \qquad \frac{\dot\phi}{\phi} = \left[\frac{\epsilon/\epsilon_0}{\tau_{\rm heat,0}\phi} - \frac{1}{\tau_{\rm diff,0}}\right]\frac{R}{R_0},$$

where we tried to write in terms of ratios on the right-hand side, with ϵ/ϵ_0 capturing the time dependence of the heating process (and where we again implicitly assumed constant opacity).

Ignoring heating, an analytic solution is possible. Using that $\tau_{\rm diff} = \tau_{\rm diff,0}(R_0/R) = \tau_{\rm diff}/(1 + v_{\rm sc}t/R_0)$, and defining an expansion timescale $\tau_{\rm exp} = R/v_{\rm sc}$, one finds

$$\phi = \exp\left(-\frac{t}{\tau_{\rm diff,0}} - \frac{t^2}{2\tau_{\rm exp}\tau_{\rm diff,0}}\right).$$

Generally, $\tau_{\rm diff,0} \gg \tau_{\rm exp}$, and thus for $t > \tau_{\rm exp}$, the lightcurve is essentially a Gaussian, with a timescale that is the geometric mean of the expansion and diffusion times scales, $\tau_{\rm lc} = \sqrt{\tau_{\rm exp}\tau_{\rm diff,0}} \propto \sqrt{\kappa M/v_{\rm sc}}$. Slower, more massive ejections lead to longer transients.

Including heating, the integration needs to be done numerically. However, generally, one expects maximum to occur when $\dot\phi = 0$, i.e., when $1/\tau_{\rm heat} = 1/\tau_{\rm diff}$ (of course, if heating is too small, this maximum after explosion never happens). From their definitions, the timescales match when $L = \epsilon M$. Thus, maximum luminosity gives a measure of the total amount of radioactive decay – “Arnett’s rule.” (This will be an underestimate if the opacity is decreasing with time – or if this is happening effectively due to recombination.)

3.2 Including recombination

At some temperature T_i, material will recombine and become essentially transparent. If this happens inside the cloud, then this will effectively be at optical depth zero, and the photosphere would be at $T_{\rm eff}^4 \simeq 2T_i^4$. As more matter recombines, the photosphere will move in, with recombination and advection ("freed" radiation) giving additional sources of luminosity. At this time, one will have,

$$L_{\rm diff} + L_{\rm adv} + L_{\rm rec} = L_{\rm min} = 4\pi R_i^2 \sigma 2 T_i^4,$$

where $R_i = x_i R$ is the radius of the recombination front, and where we used the subscript "min" as a reminder that the luminosity cannot be lower than this value for this radius.

The luminosity due to recombination is

$$L_{\rm rec} = -4\pi R_i^2 \dot{R}_i \rho Q = -3x_i^2 \dot{x}_i \frac{4\pi}{3} R^3 \rho Q = -3x_i^2 \dot{x}_i M Q,$$

where Q is the energy release per unit mass due to recombination.

For the advection and diffusion terms, the results depend on whether the front moves slow or fast compared to the time to adjust the overall temperature structure. Generally, though, $L_{\rm diff} = \mathcal{E}/\tau_{\rm diff}$ and,

$$L_{\rm adv} = -\dot{x}_i \frac{\partial \mathcal{E}}{\partial x_i}$$

but the total thermal energy $\mathcal{E}$ and diffusion timescale $\tau_{\rm diff}$ may now depend on x_i. In consequence, not only the differential equation for ϕ has to be solved, but also one for the recombination front position x_i. The latter can be derived from the constraint that the additional luminosity $L_{\rm rec} + L_{\rm adv}$ has to match the excess luminosity $L_{\rm min} - L_{\rm diff}$, or

$$-\dot{x}_i \left[3x_i^2 M Q + \frac{\partial \mathcal{E}}{\partial x_i} \right] = 4\pi R^2 x_i^2 2\sigma T_i^4 - \frac{\mathcal{E}}{\tau_{\rm diff}}$$

Below, we will also use the timescale on which the initial energy would be radiated at an effective temperature of $2^{1/4} T_i$,

$$\tau_{\rm i,0} \equiv \frac{\mathcal{E}_0}{L_{\rm min,0}} = \frac{\frac{4}{\pi} R_0^3 a T_0^4}{4\pi R_0^2 2 \frac{ac}{4} T_i^4} = \frac{4 R_0}{\pi^2 c} \frac{T_0^4}{2 T_i^4}.$$

3.2.1 Slow recombination front

If the recombination front moves slowly, photon diffusion inside it will ensure the temperature structure adjusts to its new outer boundary, $R_i = x_i R$, with the same spatial structure ($[T(x)/T(0)]^4 = \Psi(x)$). Thus, the total thermal energy will be

$$\mathcal{E} = 4\pi R^3 a T(0,t)^4 \int_0^{x_i} \Psi(x/x_i) x^2 dx = \mathcal{E}_0 \frac{R_0}{R} \phi(t) x_i^3,$$

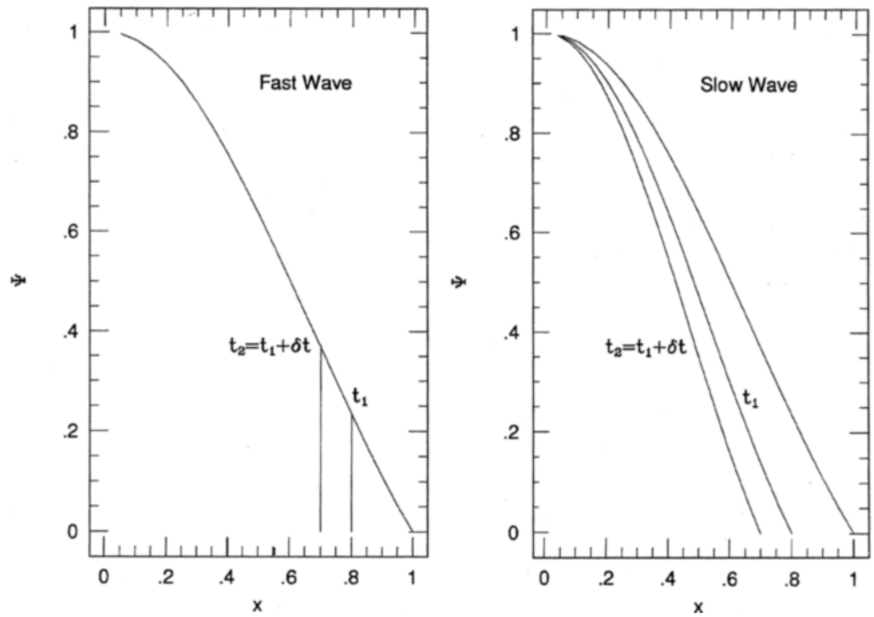

Figure 1: Fast and slow approximation to a recombination wave. From A96, his Fig.~13.7.

where $\phi(t)$ accounts for changes in central properties due to the recombination wave and associated energy loss. Given this, the advection luminosity is given by,

$$L_{\rm adv} = -\dot{x}_i \frac{\partial \mathcal{E}}{\partial x_i} = -3x_i^2 \dot{x}_i \mathcal{E}_0 \frac{R_0}{R} \phi(t).$$

Since the size is decreasing, the luminosity due to photon diffusion also changes, becoming

$$L_{\rm diff} = \frac{\mathcal{E}}{\tau_{\rm diff}} = \frac{\mathcal{E}_0}{\tau_{\rm diff,0}} \phi(t) x_i,$$

where we used that $\tau_{\rm diff} = \tau_{\rm diff,0}(R_0/R)x_i^2$, with the dependence on x_i^2 reflecting the dependence of $\tau_{\rm diff}$ on M/R (for constant density the mass enclosed within the recombination front scales with x_i^3). The differential equations to be solved thus become,

$$\frac{\dot{\phi}}{\phi} = \frac{\epsilon M}{\mathcal{E}_0 \phi x_i^3} \frac{R}{R_0} - \frac{1}{\tau_{\rm diff,0} x_i}{}^2 \frac{R}{R_0},$$

$$-3x_i^2 \dot{x}_i \left[MQ + \mathcal{E}_0 \frac{R_0}{R} \phi \right] = 4\pi R^2 x_i^2 2\sigma T_i^4 - \frac{\mathcal{E}_0}{\tau_{\rm diff,0}} \phi x_i.$$

Simplifying,

$$\frac{\dot{\phi}}{\phi} = \left[\frac{\epsilon/\epsilon_0}{\tau_{\rm heat,0}\phi x_i^3} - \frac{1}{\tau_{\rm diff,0} x_i^2}\right]\frac{R}{R_0},$$

$$-3x_i^2\dot{x}_i = \frac{\dfrac{x_i^2}{\tau_{\rm i,0}}\left(\dfrac{R}{R_0}\right)^2 - \dfrac{\phi x_i}{\tau_{\rm diff,0}}}{\dfrac{MQ}{\mathcal{E}_0} + \dfrac{R_0}{R}\phi}$$

3.2.2 Fast recombination front

For a fast-moving recombination front, the temperature structure inside will not react to the fact that the outer parts are being chopped off. The total thermal energy inside the recombination wave is,

$$\mathcal{E}_{x<x_i} = 4\pi R^3 a T(0,t)^4 \int_0^{x_i}\Psi(x)x^2\,dx = \mathcal{E}_0\frac{R_0}{R}\phi(t)\,\pi^2\int_0^{x_i}\Psi(x)x^2\,dx,$$

and thus the advection luminosity is given by,

$$L_{\rm adv} = -\dot{x}_i\frac{\partial\mathcal{E}_{x<x_i}}{\partial x_i} = -3x_i^2\dot{x}_i\frac{\pi^2}{3}\Psi(x_i)\mathcal{E}_0\frac{R_0}{R}\phi(t).$$

The luminosity due to photon diffusion from the inside now changes only because we are evaluating it at a different position, becoming

$$L_{\rm diff} = L^0_{\rm diff}\frac{\left.|-x^2\partial\Psi/\partial x\right|_{x_i}}{\left.|-x^2\partial\Psi/\partial x\right|_1} = \frac{\mathcal{E}_0}{\tau_{\rm diff,0}}\phi(t)\left|-x^2\frac{\partial\Psi}{\partial x}\right|_{x_i} = \frac{\mathcal{E}_0}{\tau_{\rm diff,0}}\phi(t)\pi^2 I(x_i).$$

where $L^0_{\rm diff}$ is the diffusion luminosity we would obtain ignoring the recombination wave, and where we have used that $[-x^2\partial\Psi/\partial x]_{x_i} = (1/\pi)\sin(\pi x_i) - x_i\cos(\pi x_i) = \pi^2 I(x_i)$ (where $\pi^2 I(x_i) = \pi^2\int_0^{x_i}\Psi(x)x^2dx$ is the normalised integral).

The differential equations to be solved now become,

$$\frac{\dot{\phi}}{\phi} = \frac{\epsilon M}{\mathcal{E}_0\phi\pi^2 I(x_i)}\frac{R}{R_0} - \frac{1}{\tau_{\rm diff,0}}\frac{R}{R_0},$$

$$-3x_i^2\dot{x}_i\left[MQ + \mathcal{E}_0\frac{R_0}{R}\phi\frac{\pi^2}{3}\Psi(x_i)\right] = 4\pi R^2 x_i^2 2\sigma T_i^4 - \frac{\mathcal{E}_0}{\tau_{\rm diff,0}}\phi\pi^2 I(x_i).$$

Simplifying,

$$\frac{\dot{\phi}}{\phi} = \left[\frac{\epsilon/\epsilon_0}{\tau_{\rm heat,0}\pi^2 I(x_i)} - \frac{1}{\tau_{\rm diff,0}}\right]\frac{R}{R_0},$$

$$-3x_i^2\dot{x}_i = \frac{\dfrac{x_i^2}{\tau_{\mathrm{i},0}}\left(\dfrac{R}{R_0}\right)^2 - \dfrac{\phi\pi^2 I(x_i)}{\tau_{\mathrm{diff},0}}}{\dfrac{MQ}{\mathcal{E}_0} + \dfrac{R_0}{R}\phi\dfrac{\pi^2}{3}\Psi(x_i)}$$

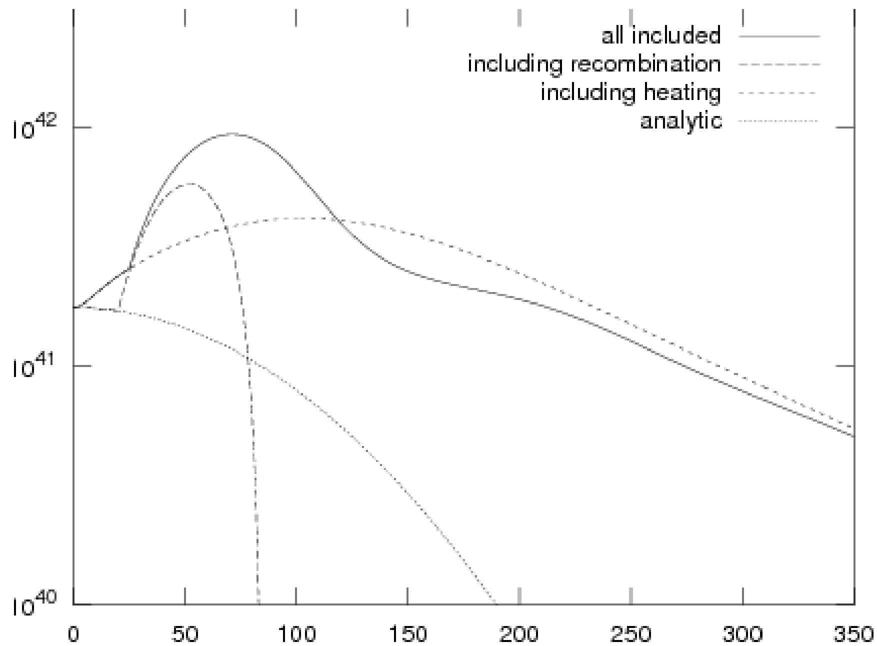

Figure 2: Comparison of explosions with and without recombination and heating by radioactive decay. Note that I could not reproduce all curves in A96 in detail, in particular not for the "slow" case. Still, the general trends are clear and should be correct.

4 Supernova remnants

Three phases can be distinguished before the ejecta become subsonic relative to the surrounding circum- or interstellar medium and a supernova remnant merges in with it:

- **Free expansion**, which lasts until the mass of the circum- or interstellar medium swept up by the ejecta is roughly the ejecta mass. In this phase, mass and kinetic energy are roughly conserved, and the velocity is thus roughly constant.

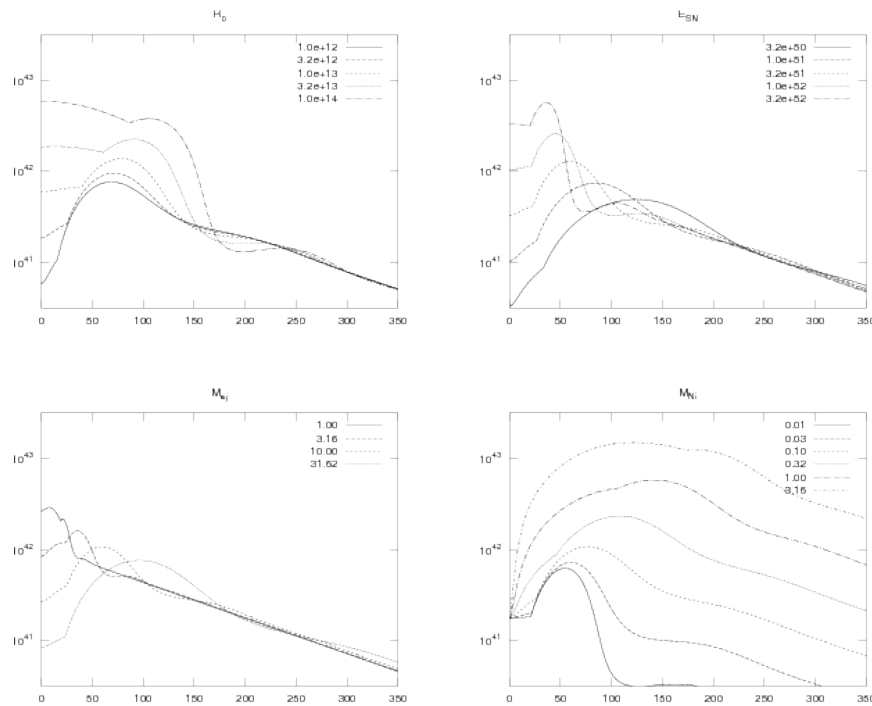

Figure 3: Semi-analytic lightcurves for supernovae with varying properties. Those not varied are held fixed at those inferred for SN 1987A by A96 (his Table 13.2): $M_{\rm ej} = 15\,M_\odot$, $E_{\rm SN} = 1.7\,{\rm B}$, $R_0 = 3 \times 10^{12}\,{\rm cm}$, $\kappa = 0.2\,{\rm cm}^2\,{\rm g}^{1}$, $M_{\rm Ni} = 0.075\,M_\odot$, $T_{\rm ion} = 4500\,{\rm K}$, $Q_{\rm ion} = 13.6\,{\rm eV\,nucleon}^{-1}$. Ignored is losses of gamma rays, and hence the luminosity at late times is overestimated.

- **Blastwave** (Sedov-Taylor), in which radiative losses from shocked ejecta and interstellar mediais not significant, so that the while the mass increases, total energy is roughly conserved.

- **Snow plow**, in which radiate energy losses are signficant, but momentum remains conserved.

4.1 Sedov-Taylor phase

A full self-similar solution is possible for this phase, but the expected scaling can be seen from a one-zone model and energy conservation:

$$\frac{1}{2}M_{\rm tot}v_{\rm s}^2 = E_0,$$

where $M_{\rm tot}$ is the total mass and $v_{\rm s} = dR_{\rm s}/dt$ the shock velocity. Assuming the mass is dominated by swept up material with constant density ρ,

$$M_{\rm tot} = \frac{4\pi}{3}R_{\rm s}^3\rho,$$

one finds,

$$\frac{1}{2}\frac{4\pi}{3}\rho R_{\rm s}^3\left(\frac{dR_{\rm s}}{dt}\right)^2 = E_0,$$

which can be integrated to give,

$$R_{\rm s} = \left(\frac{75}{8\pi}\frac{E_0}{\rho}\right)^{1/2} t^{2/5}.$$

A more detailed calculation gives $R_{\rm s} \simeq (2.026E_0/\rho)^{1/5}t^{2/5}$. For the shock velocity, one finds,

$$v_{\rm s} = \frac{2R}{5t}.$$

4.2 Snow-plow phase

Assuming the energy from the shock is immediately radiated away, all that happens is that matter is slowed down as momentum is shared, i.e., $M_{\rm tot}v_{\rm s}$ is constant. Inserting $M_{\rm tot}$ as above, one finds,

$$R_{\rm s} \propto t^{1/4},$$

where the constant of proportionality depends on when the transition from the Sedov-Taylor solution happens.