Physics of Transients (2018) – problem set – due 2018 Jan 29

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1 Mass transfer, mass loss, tidal synchronisation, supernova explosions

Many stars are binaries, and their evolution can be affected by mass transfer, mass loss, and supernova explosions. We consider their effect on the orbit. For a full analysis, one should go further, considering the effect on the size of the Roche lobe, as well as on the star's radius.

Note: all you really need here is the basic properties of circular orbits. Remember in particular that the total angular momentum is $L = m_1 v_1 a_1 + m_2 v_2 a_2 = \mu v a = m_1 m_2 \sqrt{Ga/M}$ (where the reduced mass is $\mu = m_1 m_2 / M$, the total mass $M = m_1 + m_2$, total semi-major axis $a = a_1 + a_2$ and total orbital velocity $v = v_1 + v_2$).

1.1 Mass transfer and loss

1. Show that when mass is transferred from star 1 to star 2 at a rate $\dot{m} = -\dot{m}_1 = \dot{m}_2$, the orbital separation will change as

$$\frac{\dot{a}}{a} = 2\frac{\dot{m}(m_2 - m_1)}{m_1 m_2}$$

2. Show that if star 1 is losing mass at a rate $\dot{m}_1$ in a spherically symmetric wind, the orbital separation a will evolve as

$$\frac{\dot{a}}{a} = -\frac{\dot{m}_1}{M}$$

1.2 tidal synchronization

Generally, tides will tend to make stars rotate synchronously, but for a small enough companion mass, this is not possible. Consider a binary in which star 1 rotates just a little slower than synchronous (and in which the rotational angular momentum of star 2 can be neglected). Derive how angular momentum transfer at a rate $\dot{L}$ from the orbit to star 1 will influence the orbital period $P_{\rm orb}$ and the spin period P_1, and use this to show that to be able to bring star 1 back to synchronicity requires

$$I_{\rm orb} > 3I_1.$$

1.3 Supernova explosions

Consider that star 1 explodes, ejecting mass $m_{\rm env}$.

1. For simplicity, assume the envelope and its associated momentum disappear instantaneously. For what $m_{\rm env}$ (in terms of m_1 and m_2) will the binary unbind?

2. What is the maximum velocity one can reasonable obtain? In particular, consider a B9 main sequence star (with a companion of choice), and compare your result with the $850\,{\rm km\,s^{-1}}$ found by Brown et al. (`http://adsabs.harvard.edu/abs/2005ApJ...622L..33B`) for what they call a "hypervelocity" star (which they argue arose from a binary interacting with the supermassive black hole in the Galactic centre).