Mini-Problem Set II: Halting the collapse of a cloud *due 12 Jan 2009*

Consider a cloud that has just started to collapse.

1. Use scaling arguments to show that for an isothermal gas, once collapse starts the gravitational term in the equation of hydrostatic equilibrium increases more rapidly than the pressure gradient, so that the free-fall approximation is reasonable.
   Solution. *The pressure term $P/R \propto \rho/R \propto R^{-4}$; the gravitational term $GM\rho/R^2 \propto R^{-5}$, i.e., it increases faster.*
2. What happens once the gas changes from isothermal to adiabatic? Does one indeed expect the collapse to halt eventually for this case?
   Solution. *Yes. For the adiabatic case, the pressure term $P/R \propto \rho^{5/3}/R \propto R^{-6}$, which increases faster with decreasing radius than the gravitational term ($\propto R^{-5}$).*
3. Suppose the cloud originally was permeated by a magnetic field, with magnetic pressure $\frac{1}{8\pi}B^2$ below the gas pressure? Could the increase in magnetic field strength halt the collapse? *Hint: assume conservation of magnetic flux, BR^2.*
   Solution. *To conserve magnetic flux, $B \propto R^{-2}$; thus, $P_{\mathrm{mag}}/R \propto B^2/R \propto R^{-5}$. Therefore, magnetic pressure will remain as unimportant as it was at the start..*
4. Suppose the cloud originally were rotating slowly (with a velocity below the Keplerian one). Could the spin-up during contraction halt the collapse? *Hint: assume conservation of angular momentum.*
   Solution. *Conservation of angular momentum implies $v \propto R^{-1}$. On the other hand, the Kepler velocity scales as $v_{\mathrm{Kep}} \propto R^{-1/2}$. Thus, at some point the cloud will be spinning at the Keplerian velocity, and this will influence the collapse.*