Problem Set I: Big Bang Nucleosynthesis *due 3 Apr 2023*

We will try to understand the expected abundances of Helium and Deuterium in a little more detail than done in the textbook, and check the statement that no heavier elements are produced (do read the BBN section first!). We will need the initial neutron mass fraction, $X_{\mathrm{n},0} = n_{\mathrm{n}}/n_{\mathrm{B}} = 0.158$, the neutron half life, $t_{\mathrm{n},1/2} = 617\,\mathrm{s}$, and some experimentally determined cross sections (valid around $T = 9 \times 10^8\,\mathrm{K}$):

$$\langle\sigma v\rangle_{\mathrm{n+p\to D+\gamma}} \simeq 4 \times 10^{-26}\ \mathrm{m^3\,s^{-1}}, \tag{I.1}$$

$$\langle\sigma v\rangle_{\mathrm{D+D\to {}^3He+n}} \simeq 2 \times 10^{-23}\ \mathrm{m^3\,s^{-1}}, \tag{I.2}$$

$$\langle\sigma v\rangle_{\mathrm{D+D\to T+p}} \simeq 2 \times 10^{-23}\ \mathrm{m^3\,s^{-1}}, \tag{I.3}$$

$$\langle\sigma v\rangle_{\mathrm{D+p\to {}^3He+\gamma}} \simeq 5 \times 10^{-28}\ \mathrm{m^3\,s^{-1}}. \tag{I.4}$$

For our purposes, the temperature dependence can be ignored, since it is very weak ($\nu \sim 0.2$ for $\mathrm{n+p}$, $\nu \sim 1.2$ for $\mathrm{D+D}$, and $\nu \sim 1.4$ for $\mathrm{D+p}$), and the density varies much more strongly than the temperature as a function of time in the early universe ($n_{\mathrm{B}} \propto R^{-3} \propto T^3$).

Helium

1. Use the reaction rates to derive the Deuterium mass fraction $X_{\mathrm{D}} = 2n_{\mathrm{D}}/n_{\mathrm{B}}$ at which Deuterium production (from neutrons and protons) balances destruction by fusion to heavier elements. Assume $X_{\mathrm{n}} = 0.1$.
2. Use the Saha equation Eq. 16.8 to calculate the temperature where one expects $X_{\mathrm{D}}/X_{\mathrm{n}}X_{\mathrm{p}} \simeq 0.2$, for $\eta = 6 \times 10^{-10}$ (the current best fit from CMB measurements) as well as for values of η a factor of 100 lower and higher. (*Hint: changing from number densities to mass fractions in Eq. 16.8 leaves you with a factor n_{B} – write this in terms of η and T using Eq. 16.1; then, find the temperatures by iteration – two significant digits is good enough. Finally, if you fail here, assume $T = 8 \times 10^8$ K below but do tell whether temperature would increase or decrease for increasing η.*)
3. Show that the ages of the Universe for the three temperatures inferred using Eq. 16.5 are $t = (343,\ 257,\ 183)$ s. Next, calculate Helium abundances assuming that all neutrons that have not decayed by this time end up in Helium nuclei.
4. Comparing your results to Fig. 16.2, you should find your predictions agree well for $\eta_{10} = \eta/10^{-10} = 6$ and 600, but not for $\eta_{10} = 0.06$. What could be wrong? As a guide, calculate for a neutron the mean time $\tau_{\mathrm{n:p}}$ that elapses before it meets a proton and forms Deuterium. Compare these interaction times with the ages. Correct (roughly) the Helium abundances found earlier, and again compare to Fig. 16.2.

Deuterium

We continue with looking at the expected abundance of Deuterium, for three cases of the baryon-to-photon ratio, $\eta = (0.06, 6, 600) \times 10^{-10}$ (and corresponding ages of $t = (343, 257, 183)\,\mathrm{s}$ at which deuterium becomes present in significant amounts).

1. Calculate the Deuterium mass fraction X_{D} for which the time scale $\tau_{\mathrm{D:D}}$ for one Deuteron to fuse with another Deuteron is the same as the ages for the three values of η. Compare with Fig. 16.2.
2. This time the estimates fail worst for $\eta_{10} = 600$. To see why this might be the case, calculate for a Deuteron the timescale $\tau_{\mathrm{D:p}}$ for it to meet and fuse with a proton. Does this help to understand the discrepancy?

Carbon

The textbook states that no significant heavy elements like carbon are produced. We confirm that this is the case.

1. Use Eq. 7.21 to show that the the mass fraction of carbon formed during nucleosynthesis, for $\eta = 6 \times 10^{-10}$, is of order $X_{\mathrm{C}} \simeq 3 \times 10^{-21}$.
2. Given your result, can Carbon be ignored for, say, the first stars? Check by comparing the energy generation rates by the p-p and CNO cycles at $7 \times 10^{7}\,\mathrm{K}$ (which is the temperature where we found the first stars could generate C themselves).