Cosmological expansion

due 24 Mar 2023

The Universe evolves differently depending on which components are present. We discuss radiation and a cosmological constant, assuming the current best-fit model of the universe holds, i.e., that it is flat ($k = 0$, $\Omega_{\rm tot} = 1$), with $\Omega_{\rm M,0} = 0.3$, $\Omega_{\Lambda,0} = 0.7$, and $H_0 = 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$.

The influence of radiation

We will derive when radiation was dominant, but more carefully than done in CO (it may help to first read CO 29.2). We use that for relativistic bosons (like photons) the energy density *per spin state* is $u_{\rm B} = \frac{1}{2}aT^4$ and that for relativistic fermions (electrons, neutrinos), it is $u_{\rm F} = \frac{7}{16}aT^4$. Furthermore, for relativistic particles, the entropy density is generally $s = (\rho c^2 + P)/T = (u + u/3)/T = \frac{4}{3}u/T$.

1. What is $\rho_{\rm rad,0}$, the current contribution of radiation to the total energy density of the Universe? And what is $\Omega_{\rm rad,0}$? How will this contribution increase with redshift (or R) and when will it be equal to the matter component?
2. Show that as the Universe expands, the *energy* in a constant co-moving volume is not conserved, but that the *entropy* is conserved. Explain why this makes sense (a one-liner should suffice!).
3. In item 1, we ignored neutrinos, which is incorrect. We turn first to an earlier phase, with temperature of $\sim 10^{12}$ K, when the energy density was dominated by photons (two spin states), electrons and positrons (two spin states each), and three types of neutrinos and anti-neutrinos (one spin state each). At this time, adding all contributions up, what was the total entropy density? (easiest to give in units of $\frac{4}{3}aT^3$.)
4. At some point the neutrinos decoupled, and their temperature T_ν continued the normal scaling with R. Later, the electrons and positrons annihilated, producing photons and raising the temperature of the photons. Use entropy conservation to show that the photon temperature T_γ was raised by the factor $(11/4)^{1/3}$ given in CO eq. 29.73. Given this, what is the current neutrino temperature?
5. Re-estimate the scale factor (and redshift) at which the energy density in relativistic species equals that in matter. Does this match the $R = 3.05 \times 10^{-4}$ given on the bottom of p. 1174 in CO? (Ignore small differences due to the choice of $\Omega_{\rm M}$.)
6. Above, we saw that the photons created by electron-positron annihilation made a difference for our calculations. Photons are also created when electrons combine with protons and alpha particles to form Hydrogen and Helium atoms. Why can this contribution to the entropy density be safely ignored?
7. The neutrino number density (for each species) is given by $n_\nu = \frac{3}{2}\zeta(3)4\pi(kT_\nu/hc)^3$, with $\zeta(3) \simeq 1.202$. Evaluate this for the present. Why is this much larger than the baryon number density? What limit on the (average) neutrino mass (in eV) can we set from the fact that they do not make a major contribution to the total mass in the Universe?
8. ***Bonus:*** The expression for n_ν above is valid when $kT \gg m_\nu c^2$. Why can we nevertheless use it even with the present-day temperature?

The cosmological constant

The dark energy component, which causes an acceleration of the expansion of the Universe, can be interpreted as a cosmological constant Λ, with a corresponding contribution $\Omega_{\Lambda,0} = \Lambda c^2/3H_0^2$. The cosmological constant induces an acceleration between two points given by $\frac{1}{3}\Lambda c^2 r$, where r is the separation (see CO Eq. 29.109).

1. What is the value of Λ implied by $\Omega_{\Lambda,0} = 0.7$? Compare the acceleration this introduces between the Earth and the Sun with the gravitational acceleration. Repeat for the Sun and the Galactic centre.
2. Write out a simplified Friedmann equation for our case of having significant contributions only from $\Omega_{\mathrm{M},0}$ and $\Omega_{\Lambda,0}$, and use this to find at what scale factor the expansion starts to accelerate rather than to decelerate (*Hint: take the square root, and then the time derivative to find* $\ddot{R}$). What redshift does this correspond to?
3. The solution to this simplified version of the Friedman equation is given by Eq. 14.8. From this, how old is the Universe? How does this compare with what you get for a matter-dominated, $\Omega_{\mathrm{M},0} = 1$, $\Omega_{\Lambda,0} = 0$ Universe? What value of $\Omega_{\Lambda,0}$ do you infer from the fact that the oldest globular clusters are about 12 Gyr old? (Here, continue to assume that $\Omega_{\mathrm{tot}} = 1$.)
4. The measured values of of $\Omega_{\mathrm{M},0}$ and $\Omega_{\Lambda,0}$ seem quite close. To see whether we appear to live in "a special time," use the simplified Friedman equation to calculate the values of Ω_{M} and Ω_Λ for when the scale factor was $R = 0.1$ and $R = 10$.
5. What does the presence of the cosmological constant imply for the night sky in the very distant future? Consider stars in our Galaxy and other galaxies separately.