The first generation of stars *due 6 March 2023*

We will derive properties of the first generation of stars and compare these with normal ones. For the first generation of stars, only primordial hydrogen and helium are available (i.e., there is zero metallicity). Below, use $X = 0.77$, $Y = 0.23$ and $Z = 0$.

Equation of state, opacity, fusion

Describe in what way stars with zero metallicity might be different from stars with solar abundances ($X = 0.708$, $Y = 0.273$, $Z = 0.019$), as follows:

1. Show that the equation of state is not influenced much by a change in metallicity. Consider separately ideal gas, degenerate matter, and radiation-dominated matter.
2. Discuss the differences in opacity between the two cases, considering separately electron scattering, free-free and bound-free opacities. *(Note: use expressions from the notes – the CO expression for free-free Kramers opacity is wrong, while the KWW expression for bound-free is useful near the photosphere only.)* On the main sequence, will high-mass or low-mass stars be more affected by these changes (i.e., would it affect their luminosities, convection zones, etc.)?
3. Which hydrogen fusion processes are available in solar-abundance and zero-metallicity stars? Again discuss whether it is low or high-mass stars that are affected most, and thus in which way zero-metallicity stars will be different from solar-metallicity ones.

Properties of main-sequence stars

We now try to see whether, given the differences between solar-metallicity and zero metallicity stars, we can understand the properties of zero-metallicity main-sequence stars shown in the Figure.

1. Read off the radius and luminosity for a (near)zero metallicity $1\,M_\odot$ star from the figures. From your discussion above, what causes the differences from the Sun?
2. Use hydrostatic equilibrium to (re-)derive that for homologous stars with an ideal-gas equation of state, $T_\mathrm{c} \propto M/R$. Next, use the equation of radiative energy transport to derive a second scaling relations for T_c in terms of other stellar properties, and combine the two to show that one expects, $L \propto M^3$.
3. Now also assume that fusion occurs in a fixed mass fraction of the star. Derive a second scaling relation for L from the energy balance equation (with the power-law approximation for the energy generation rate, $\epsilon \propto \rho T^\nu$) and use it to show that $R \propto M^{(\nu-1)/(\nu+3)}$.
4. In the dependence of radius on mass, insert values for ν appropriate for the p-p and CNO cycles. Compare this to the parts between 1 and $13\,M_\odot$ and above $13\,M_\odot$ in the Figure. Which result is expected and which is surprising?
5. Now use the dependence of radius on mass to determine how T_c scales with mass. Again insert values of ν, and scale to solar units by using $T_\mathrm{c} \simeq 1.5 \times 10^7\,\mathrm{K}$ for a $1\,M_\odot$ zero-metallicity star.
6. Why is there a bend in the mass-radius relation at $M \simeq 13\,M_\odot$? (Address not only why there is a bend, but also show that it makes sense it happens at approximately this mass.)
7. *Bonus question:* What could cause the downturn in the mass-radius relation at masses less than $\sim 1\,M_\odot$?

Properties of giants

In the figure, the evolutionary tracks for solar-mass stars of different metallicities go up to the end of the giant branch, when the helium flash happens.

1. What is the structure of a giant star? What could cause the higher metallicity tracks to be further to the red?
2. What determines the luminosity of a giant star? Why is the luminosity at the end of the track much lower for the near-zero metallicity case?

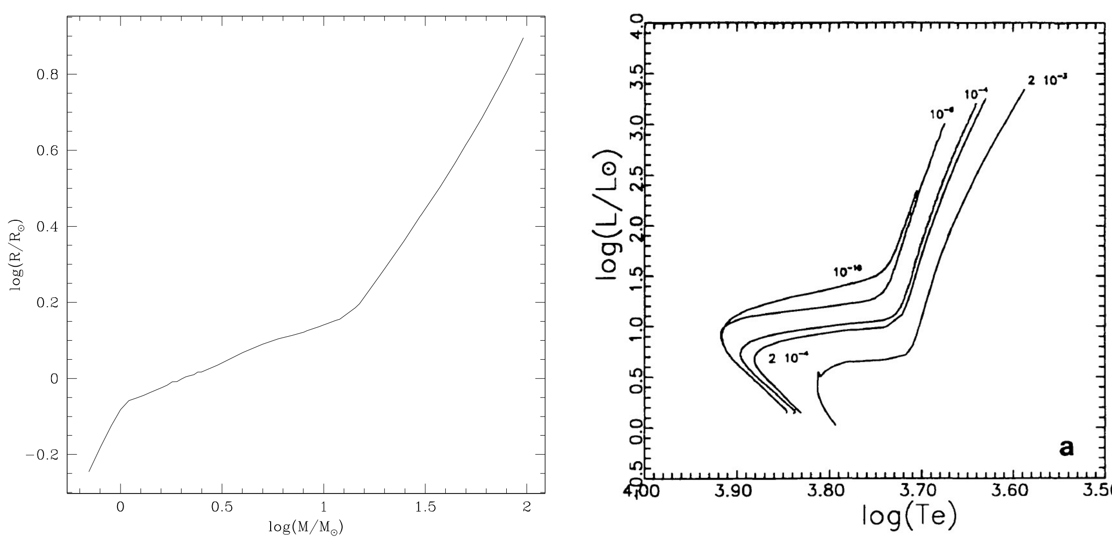

Fig. .1. *Left:* Mass-radius relation for zero-metallicity stars. From Marigo et al., 2001, A&A 371, 152. *Right:* Evolutionary tracks for $1\,M_\odot$ stars of different metallicities; the left-most is for essentially zero metallicity ($Z = 10^{-10}$), while the right-most is for one-tenth solar ($Z = 2 \times 10^{-3}$). From Cassisi & Castellani, 1993, ApJS 88, 509. *Right:* Mass-radius relation for zero-metallicity stars. From Marigo et al., 2001, A&A 371, 152.