Midterm AST320, 3 March 2014

Examination aids: Calculators only.

Note: The four parts have equal weight. Answers can be brief and in point-form, but be sure that you give physical arguments for answers and that derivations can be followed. A list of constants is appended.

1. Stellar timescales
   (a) Write down the equations for the hydrodynamic (free-fall) and thermal (Kelvin-Helmholtz) timescales. Evaluate them for the Sun.
   (b) For each of the following objects, tell whether it is in hydrostatic equilibrium and whether it is in thermal equilibrium. If either is broken, describe what will happen as a result *(note: be brief; for quite a few, just a word like "contracts" or "collapses" suffices)*.
   - Molecular cloud well above the Jeans mass.
   - Pre-main sequence star.
   - Brown dwarf.
   - Main-sequence star.
   - Helium core of star just after hydrogen is exhausted.
   - Young white dwarf.
   - Iron core of star just before supernova explosion.
   - White dwarf in which carbon has been reignited.

2. Nuclear fusion in the Sun.
   (a) Write down the pp1 cycle. Number the steps, order them by their speed, and and give the physical reasons for this ordering.
   (b) The energy released for every He nucleus produced is $\varepsilon_0 = 26.20\,\mathrm{MeV} = 4.2 \times 10^{-12}\,\mathrm{J}$. How many neutrinos are produced for every helium nucleus? Given that, how many neutrinos does the Sun emit every second? And how many will pass through you during this midterm?

Physical constants

(http://physics.nist.gov/cuu/Constants)

speed of light *in vacuo*	c	=	$2.99792458 \times 10^{8}\,\mathrm{m\,s^{-1}}$ (exact)
Gravitational constant	G	=	$6.673(10) \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Planck's constant	h	=	$6.62606876(52) \times 10^{-34}\,\mathrm{J\,s}$
$[h/2\pi]$	$\hbar$	=	$1.054571596(82) \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann's constant	k	=	$1.3806503(24) \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant $[\frac{1}{60}\pi^2 k^4/\hbar^3 c^2 = ac/4]$	σ	=	$5.670400(40) \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Avogadro's number	N_{A}	=	$6.02214199(47) \times 10^{23}\,\mathrm{mol^{-1}}$
Molar gas constant $[kN_{\mathrm{A}}]$	$\mathcal{R}$	=	$8.314472(15)\,\mathrm{J\,mol^{-1}\,K^{-1}}$
electron mass	m_{e}	=	$9.10938188(72) \times 10^{-31}\,\mathrm{kg}$
proton mass	m_{p}	=	$1.67262158(13) \times 10^{-27}\,\mathrm{kg}$

Other units

atomic mass unit $[\frac{1}{12}m(^{12}C)]$	m_{u}	=	$1.66053873(13) \times 10^{-27}\,\mathrm{kg}$
hydrogen mass	m_{H}	=	$1.6735525 \times 10^{-27}\,\mathrm{kg}$
electric charge	e	=	$1.602176462(63) \times 10^{-19}\,\mathrm{C}$
electron volt	eV	=	$1.602176462(63) \times 10^{-19}\,\mathrm{J}$
Ångstrom	Å	=	$10^{-10}\,\mathrm{m}$

Astronomical units

(Nautical Almanac 1993)

Solar mass	$M_{\odot}$	=	$1.9891 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_{\odot}$	=	$6.9551(3) \times 10^{8}\,\mathrm{m}$
Solar luminosity	$L_{\odot}$	=	$3.839(5) \times 10^{26}\,\mathrm{W}$ (not official)
Solar temperature	$T_{\mathrm{eff},\odot}$	=	$5777(2)\,\mathrm{K}$ (not official)
astronomical unit	AU	=	$1.49597870 \times 10^{11}\,\mathrm{m}$
parsec	pc	=	$3600 \times 180/\pi\,\mathrm{AU} = 3.0856776 \times 10^{16}\,\mathrm{m}$
Julian year	yr	=	$365.25 \times 84600\,\mathrm{s}$ ($\sim\pi\,10^{7}\,\mathrm{s}$)

Some formulae

ideal gas	$P = nkT$, $n = \rho/\mu m_{\mathrm{H}}$ $c_v = \frac{3}{2}Nk$, $c_p/c_v = 5/3$
non-relativistic degenerate gas	$P = K_1 n_{\mathrm{e}}^{5/3}$, $n_{\mathrm{e}} = \rho/\mu_{\mathrm{e}} m_{\mathrm{H}}$, $K_1 = \frac{1}{5}(3\pi^2)^{2/3}(\hbar^2/m_{\mathrm{e}})$ $K_1/m_{\mathrm{H}}^{5/3} = 9.91 \times 10^{6}$ (SI)
relativistic degenerate gas	$P = K_2 n_{\mathrm{e}}^{4/3}$, $n_{\mathrm{e}} = \rho/\mu_{\mathrm{e}} m_{\mathrm{H}}$, $K_2 = \frac{1}{4}(3\pi^2)^{1/3}\hbar c$ $K_2/m_{\mathrm{H}}^{4/3} = 1.231 \times 10^{10}$ (SI)
scale height	$\mathcal{H} = kT/\mu m_{\mathrm{H}} g$