Midterm AST320, 2 March 2009

Examination aids: Calculators only.

Note: The six parts have equal weight. Answers can be brief and in point-form, but be sure that you give physical arguments for answers and that derivations can be followed. A list of constants is appended.

1. Luminosity of a radiative star.
   (a) Use the stellar structure equations to show that for a star supported by thermal pressure and with an opacity κ that is independent of density and temperature (e.g., electron scattering), the luminosity scales as,
   $$L \propto \frac{\mu^4}{\kappa} M^3. \qquad \text{(M.1)}$$
   (b) In the above, you did not have to consider how energy is generated. Why? What will happen to a star if nuclear fusion does not provide enough energy to balance the energy loss given by Eq. M.1, and, conversely, what will happen if fusion provides more energy than is lost?
   (c) From binaries, we know that the luminosities of real (main-sequence) stars between 1 and $10\,M_\odot$ depend somewhat steeper on mass than derived above, with $L \propto M^4$. Which assumption we made above is likely wrong? Would a better approximation change the result in the right direction?

2. Mass-radius relation for white dwarfs.
   (a) Use the equation of state for non-relativistic, completely degenerate matter and numbers for the appropriate polytropic model (see Table M.1) to show that for a completely degenerate white dwarf, the mass-radius relation is given by (you can assume pure Carbon composition)

$$R = 9 \times 10^6 \left(\frac{M}{M_\odot}\right)^{-1/3} \text{ m.} \tag{M.2}$$

   (b) Describe briefly in physical terms why the radius decreases with increasing mass, and why there is a maximum mass the white dwarf can have.
   (c) Calculate the maximum mass for a white dwarf (again, you can assume a composition of pure Carbon).

Table M.1. Constants for the Lane-Emden functions

n	ξ_1	$-\xi^2 \frac{d\theta_n}{d\xi}\Big\vert_{\xi_1}$	$\frac{\rho_c}{\overline{\rho}}$	$K \frac{R^{(n-3)/n}}{GM^{(n-1)/n}}$	$\frac{P_c}{GM^2/R^4}$
0.0	2.4494	4.8988	1.0000	...	0.119366
0.5	3.7528	3.7871	1.8361	2.270	0.26227
1.0	3.14159	3.14159	3.28987	0.63662	0.392699
1.5	3.65375	2.71406	5.99071	0.42422	0.770140
2.0	4.35287	2.41105	11.40254	0.36475	1.63818
2.5	5.35528	2.18720	23.40646	0.35150	3.90906
3.0	6.89685	2.01824	54.1825	0.36394	11.05066
3.5	9.53581	1.89056	152.884	0.40104	40.9098
4.0	14.97155	1.79723	622.408	0.47720	247.558
4.5	31.83646	1.73780	6189.47	0.65798	4922.125
5.0	∞	1.73205	∞	∞	∞

Taken from Chandrasekar, 1967, Introduction to the study of stellar structure (Dover: New York), p. 96

Physical constants

(`http://physics.nist.gov/cuu/Constants`)

speed of light *in vacuo*	c	=	$2.99792458 \times 10^{8}\,\mathrm{m\,s^{-1}}$ (exact)
Gravitational constant	G	=	$6.673(10) \times 10^{-11}\,\mathrm{N\,m^{2}\,kg^{-2}}$
Planck's constant	h	=	$6.62606876(52) \times 10^{-34}\,\mathrm{J\,s}$
$[h/2\pi]$	$\hbar$	=	$1.054571596(82) \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann's constant	k	=	$1.3806503(24) \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant			
$[\frac{1}{60}\pi^2 k^4/\hbar^3 c^2 = ac/4]$	σ	=	$5.670400(40) \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Avogadro's number	N_{A}	=	$6.02214199(47) \times 10^{23}\,\mathrm{mol^{-1}}$
Molar gas constant $[kN_{\mathrm{A}}]$	$\mathcal{R}$	=	$8.314472(15)\,\mathrm{J\,mol^{-1}\,K^{-1}}$
electron mass	m_{e}	=	$9.10938188(72) \times 10^{-31}\,\mathrm{kg}$
proton mass	m_{p}	=	$1.67262158(13) \times 10^{-27}\,\mathrm{kg}$

Other units

atomic mass unit $[\frac{1}{12}m(^{12}C)]$	m_{u}	=	$1.66053873(13) \times 10^{-27}\,\mathrm{kg}$
hydrogen mass	m_{H}	=	$1.6735525 \times 10^{-27}\,\mathrm{kg}$
electric charge	e	=	$1.602176462(63) \times 10^{-19}\,\mathrm{C}$
electron volt	eV	=	$1.602176462(63) \times 10^{-19}\,\mathrm{J}$
Ångstrom	Å	=	$10^{-10}\,\mathrm{m}$

Astronomical units

(Nautical Almanac 1993)

Solar mass	$M_{\odot}$	=	$1.9891 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_{\odot}$	=	$6.9551(3) \times 10^{8}\,\mathrm{m}$
Solar luminosity	$L_{\odot}$	=	$3.839(5) \times 10^{26}\,\mathrm{W}$ (not official)
Solar temperature	$T_{\mathrm{eff},\odot}$	=	$5777(2)\,\mathrm{K}$ (not official)
astronomical unit	AU	=	$1.49597870 \times 10^{11}\,\mathrm{m}$
parsec	pc	=	$3600 \times 180/\pi\,\mathrm{AU} = 3.0856776 \times 10^{16}\,\mathrm{m}$
Julian year	yr	=	$365.25 \times 84600\,\mathrm{s}$ $(\sim\pi\,10^{7}\,s)$

Some formulae

ideal gas	$P = nkT,\ n = \rho/\mu m_{\mathrm{H}}$ $c_v = \frac{3}{2}Nk,\ c_p/c_v = 5/3$
non-relativistic degenerate gas	$P = K_1 n_{\mathrm{e}}^{5/3},\ n_{\mathrm{e}} = \rho/\mu_{\mathrm{e}} m_{\mathrm{H}},\ K_1 = \frac{1}{5}(3\pi^2)^{2/3}(\hbar^2/m_{\mathrm{e}})$ $K_1/m_{\mathrm{H}}^{5/3} = 9.91 \times 10^{6}$ (SI)
relativistic degenerate gas	$P = K_2 n_{\mathrm{e}}^{4/3},\ n_{\mathrm{e}} = \rho/\mu_{\mathrm{e}} m_{\mathrm{H}},\ K_2 = \frac{1}{4}(3\pi^2)^{1/3}\hbar c$ $K_2/m_{\mathrm{H}}^{4/3} = 1.231 \times 10^{10}$ (SI)
scale height	$\mathcal{H} = kT/\mu m_{\mathrm{H}} g$