

Midterm AST320, 9 March 2007

Examination aids: Calculators only.

Note: The four parts have equal weight. Answers can be brief and in point-form, but be sure that you give physical arguments for answers and that derivations can be followed. A list of constants is appended.

1. Basics of a star's life. Consider a young star with a mass like that of the Sun, but starting with a radius and a central temperature too cool for nuclear fusion.
 - (a) What happens to the total, potential, and kinetic energy of the star as it radiates into space? And, as a result, what happens to the radius and central temperature? At some point, the radius and temperature stop changing: the star has reached the main sequence. What has happened?
 - (b) Now imagine you took a star with the same mass and initial large radius, but you replaced all the hydrogen with helium. Also for this star, at some point the radius and temperature will stop changing. At that point, will its central temperature, radius, and luminosity be smaller, similar, or larger than would be the case for a star with lots of hydrogen? Be sure to explain why. (*Note: bonus points for an order-of-magnitude estimate for R .*)

2. Nuclear fusion in the Sun.

- (a) Write down the pp1 cycle. Order the steps from slow to fast, and give the physical reasons for this ordering.
- (b) The energy released for every He nucleus produced is $\varepsilon_0 = 26.20 \text{ MeV} = 4.2 \times 10^{-5} \text{ erg}$. How many neutrinos are produced for every helium nucleus? Given that, how many neutrinos does the Sun emit every second? And how many will pass through you during this midterm?

Physical constants

(<http://physics.nist.gov/cuu/Constants>)

speed of light <i>in vacuo</i>	c	=	$2.99792458 \times 10^{10} \text{ cm s}^{-1}$ (exact)
Gravitational constant	G	=	$6.673(10) \times 10^{-8} \text{ dyne cm}^2 \text{ g}^{-2}$
Planck's constant	h	=	$6.62606876(52) \times 10^{-27} \text{ erg s}$
	$[h/2\pi] \hbar$	=	$1.054571596(82) \times 10^{-27} \text{ erg s}$
Boltzmann's constant	k	=	$1.3806503(24) \times 10^{-16} \text{ erg K}^{-1}$
Stefan-Boltzmann constant			
	$[\frac{1}{60}\pi^2 k^4 / \hbar^3 c^2 = ac/4] \sigma$	=	$5.670400(40) \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$
Avogadro's number	N_A	=	$6.02214199(47) \times 10^{23} \text{ mol}^{-1}$
Molar gas constant	$[kN_A] \mathcal{R}$	=	$8.314472(15) \times 10^7 \text{ erg mol}^{-1} \text{ K}^{-1}$
electron mass	m_e	=	$9.10938188(72) \times 10^{-28} \text{ g}$
proton mass	m_p	=	$1.67262158(13) \times 10^{-24} \text{ g}$

Other units

atomic mass unit $[\frac{1}{12}m(^{12}\text{C})]$	m_u	=	$1.66053873(13) \times 10^{-24} \text{ g}$
hydrogen mass	m_H	=	$1.6735525 \times 10^{-24} \text{ g}$
electric charge	e	=	$4.80320420(18) \times 10^{-10} \text{ ESU}$ $= 1.602176462(63) \times 10^{-20} \text{ EMU}$
electron volt	eV	=	$1.602176462(63) \times 10^{-12} \text{ erg}$
Ångstrom	Å	=	10^{-8} cm

Astronomical units

(Nautical Almanac 1993)

Solar mass	M_\odot	=	$1.9891 \times 10^{33} \text{ g}$
Solar radius	R_\odot	=	$6.96 \times 10^{10} \text{ cm}$
Solar luminosity	L_\odot	\simeq	$3.826 \times 10^{33} \text{ erg s}^{-1}$ (not official)
Solar temperature	$T_{\text{eff},\odot}$	\simeq	5780 K (not official)
astronomical unit	AU	=	$1.49597870 \times 10^{13} \text{ cm}$
parsec	pc	=	$3600 \times 180/\pi \text{ AU} \simeq 3.086 \times 10^{18} \text{ cm}$
Julian year	yr	=	$365.25 \times 84600 \text{ s} (\sim \pi 10^7 \text{ s})$

Some formulae

ideal gas	$P = nkT, n = \rho/\mu m_H$ $c_v = \frac{3}{2}Nk, c_p/c_v = 5/3$
non-relativistic degenerate gas	$P = K_1 n_e^{5/3}, n_e = \rho/\mu_e m_H, K_1 = \frac{1}{5}(3\pi^2)^{2/3}(\hbar^2/m_e)$ $K_1/m_H^{5/3} = 9.91 \times 10^{12} \text{ (cgs)}$
relativistic degenerate gas	$P = K_2 n_e^{4/3}, n_e = \rho/\mu_e m_H, K_2 = \frac{1}{4}(3\pi^2)^{1/3}\hbar c$ $K_2/m_H^{4/3} = 1.231 \times 10^{15} \text{ (cgs)}$
scale height	$\mathcal{H} = kT/\mu m_H g$