# UNIVERSITY OF TORONTO <br> Faculty of Arts and Science <br> APRIL/MAY 2015 EXAMINATIONS <br> AST320H1S 

Duration: 3 hours
Examination Aids: Calculator Only
Marks: All three questions have equal weight. All subitems of questions have equal weight.

1. Convection.
(a) What is convection and under what conditions does it occur? And what is semiconvection and when does it occur? (Be sure to describe both in physical terms, in one or two sentences, and feel free to add a small sketch.)
(b) We encounter convection zones in the envelopes of low-mass stars, but not in the envelopes of high-mass stars. What is the reason for this difference? And why do we encounter convection zones in the cores of high-mass stars, but not in those of low-mass stars?
(c) The convection zones in the cores of high-mass stars decrease in extent as the stars evolve on the main sequence. Why would this be?
2. The differences between the first stars ( $X=0.77, Y=0.23, Z=0$ ), and current ones (solar abundances: $X=0.708, Y=0.273, Z=0.019$ ). For each type of star mentioned below, describe how the abundance differences would affect the appropriate equation of state, opacity, and nuclear fusion process, and discuss how these differences affect basic properties such as luminosity, radius, central temperature, central density, location of the convection zones, etc.
(a) Low-mass main-sequence stars.
(b) High-mass main-sequence stars.
(c) Low-mass red giants (consider both core and envelope).
3. Evolution of the Universe.
(a) What is the cosmological constant $\Lambda$ ? What equation of state (relation between massenergy and pressure) does it have? What is its effect on the expansion of the Universe?
(b) Write down the Friedman equation, and show that for $\Omega_{\mathrm{tot}}=1, k=0$, and ignoring radiation, it can be written as

$$
\begin{equation*}
\left(\frac{\dot{R}}{R}\right)^{2}=H_{0}^{2}\left(\frac{\Omega_{\mathrm{M}, 0}}{R^{3}}+\Omega_{\Lambda, 0}\right) . \tag{1}
\end{equation*}
$$

(c) The solution to the above version of the Friedman equation is given by,

$$
\begin{equation*}
R(t)=\left(\frac{\Omega_{\mathrm{M}, 0}}{\Omega_{\Lambda, 0}}\right)^{1 / 3} \sinh ^{2 / 3}\left(\frac{3}{2} H_{0} t \sqrt{\Omega_{\Lambda, 0}}\right) \tag{2}
\end{equation*}
$$

Consider two cases, one with $\Omega_{m}=1, \Omega_{\Lambda}=0$ and one similar to the currently preferred one, with $\Omega_{m}=0.3, \Omega_{\Lambda}=0.7$. For both cases, calculate the present age of the Universe, and sketch the evolution of $R(t)$ from the big bang $(R=0)$ to a size twice the present one ( $R=2$ ). How do the two ages you calculated compare to the ages of the oldest objects known?

## Physical constants

(http://physics.nist.gov/cuu/Constants)

| speed of light in vacuo | $c$ | $=2.99792458 \times 10^{8} \mathrm{~m} \mathrm{~s}^{-1}$ (exact) |
| :--- | :--- | :--- |
| Gravitational constant | $G$ | $=6.673(10) \times 10^{-11} \mathrm{~N} \mathrm{~m}^{2} \mathrm{~kg}^{-2}$ |
| Planck's constant | $h$ | $=6.62606876(52) \times 10^{-34} \mathrm{~J} \mathrm{~s}$ |
|  | $[h / 2 \pi]$ | $\hbar$ |
| Boltzmann's constant | $k$ | $=1.054571596(82) \times 10^{-34} \mathrm{~J} \mathrm{~s}$ |
| Stefan-Boltzmann constant |  |  |
| $\quad\left[\frac{1}{60} \pi^{2} k^{4} / \hbar^{3} c^{2}=a c / 4\right]$ | $\sigma$ | $=5.670400(40) \times 10^{-8} \mathrm{~W} \mathrm{~m}^{-2} \mathrm{~K}^{-4}$ |
| Avogadro's number | $N_{\mathrm{A}}$ | $=6.02214199(47) \times 10^{23} \mathrm{~mol}^{-1}$ |
| Molar gas constant $\quad\left[k N_{\mathrm{A}}\right]$ | $\mathcal{R}$ | $=8.314472(15) \mathrm{J} \mathrm{mol}^{-1} \mathrm{~K}^{-1}$ |
| electron mass | $m_{\mathrm{e}}$ | $=9.10938188(72) \times 10^{-31} \mathrm{~kg}$ |
| proton mass | $m_{\mathrm{p}}$ | $=1.67262158(13) \times 10^{-27} \mathrm{~kg}$ |

Other units

| atomic mass unit $\left[\frac{1}{12} m\left({ }^{12} C\right)\right]$ | $m_{\mathrm{u}}$ | $=1.66053873(13) \times 10^{-27} \mathrm{~kg}$ |
| :--- | :--- | :--- |
| hydrogen mass | $m_{\mathrm{H}}$ | $=1.6735525 \times 10^{-27} \mathrm{~kg}$ |
| electric charge | $e$ | $=1.602176462(63) \times 10^{-19} \mathrm{C}$ |
| electron volt | eV | $=1.602176462(63) \times 10^{-19} \mathrm{~J}$ |
| Angstrom | $\AA$ | $=10^{-10} \mathrm{~m}$ |

## Astronomical units

(Nautical Almanac 1993)
Solar mass
Solar radius
Solar luminosity
Solar temperature
astronomical unit

$$
\begin{array}{ll}
M_{\odot} & =1.9891 \times 10^{30} \mathrm{~kg} \\
R_{\odot} & =6.9551(3) \times 10^{8} \mathrm{~m} \\
L_{\odot} & =3.839(5) \times 10^{26} \mathrm{~W} \text { (not official) } \\
T_{\text {eff }, \odot} & =5777(2) \mathrm{K}(\text { not official }) \\
\mathrm{AU} & =1.49597870 \times 10^{11} \mathrm{~m} \\
\mathrm{pc} & =3600 \times 180 / \pi \mathrm{AU}=3.0856776 \times 10^{16} \mathrm{~m} \\
\mathrm{yr} & =365.25 \times 84600 \mathrm{~s}\left(\sim \pi 10^{7} \mathrm{~s}\right)
\end{array}
$$

Julian year

## Some formulae

ideal gas

$$
\begin{aligned}
& P=n k T, n=\rho / \mu m_{\mathrm{H}} \\
& c_{v}=\frac{3}{2} N k, c_{p} / c_{v}=5 / 3
\end{aligned}
$$

non-relativistic degenerate gas $\quad P=K_{1} n_{\mathrm{e}}^{5 / 3}, n_{\mathrm{e}}=\rho / \mu_{\mathrm{e}} m_{\mathrm{H}}, K_{1}=\frac{1}{5}\left(3 \pi^{2}\right)^{2 / 3}\left(\hbar^{2} / m_{\mathrm{e}}\right)$ $K_{1} / m_{\mathrm{H}}^{5 / 3}=9.91 \times 10^{6}(\mathrm{SI})$
relativistic degenerate gas
$P=K_{2} n_{\mathrm{e}}^{4 / 3}, n_{\mathrm{e}}=\rho / \mu_{\mathrm{e}} m_{\mathrm{H}}, K_{2}=\frac{1}{4}\left(3 \pi^{2}\right)^{1 / 3} \hbar c$ $K_{2} / m_{\mathrm{H}}^{4 / 3}=1.231 \times 10^{10}(\mathrm{SI})$
scale height
$\mathcal{H}=k T / \mu m_{\mathrm{H}} g$

