UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL/MAY 2014 EXAMINATIONS

AST320H1S

Duration: 3 hours

Examination Aids: Calculator Only

Marks: All three questions have equal weight. All subitems of questions have equal weight.

1. Star formation and evolution. Low-mass pre-main-sequence stars are well-described as completely convective objects, and in a mini problem set we found that their radius is approximately $R = 0.85\,R_\odot\,(M/0.1\,M_\odot)^{2/3}(T_{\rm eff}/3000\,{\rm K})^{-4/3}(t/1\,{\rm Myr})^{-1/3}$ and their central temperature $T_{\rm c} = 0.88 \times 10^6\,{\rm K}\,(M/0.1\,M_\odot)^{1/3}(\mu/0.6)(T_{\rm eff}/3000\,{\rm K})^{4/3}(t/1\,{\rm Myr})^{1/3}$.

   (a) Show how the scalings of these relations arise, assuming $T_{\rm eff}$ is constant during the contraction. (*Note: you do* not *have to reproduce the actual numbers, just that* $R \propto M^{2/3}T_{\rm eff}^{-4/3}t^{-1/3}$ *and* $T_{\rm c} \propto M^{1/3}\mu T_{\rm eff}^{4/3}t^{1/3}$.)

   (b) Given the above, how does the density scale with mass and central temperature? For a completely convective star, $\rho_{\rm c} = 5.99\overline{\rho}$. Use this and the results above to calculate $\rho_{\rm c}$ at $T_{\rm c} = 10^6\,{\rm K}$ for stars with masses of 1, 0.2, and $0.04\,M_\odot$ (assume $\mu = 0.6$ and $T_{\rm eff} = 3000\,{\rm K}$). Draw these points in the equation of state diagramme on the next page. Next, for each star, sketch how the central density and temperature will evolve as the star contracts. For the different masses, what eventually prevents the star from contracting further? (Be sure that what you write is consistent with the tracks you sketched.)

   (c) We now turn to the further evolution. For the $1\,M_\odot$ star only, what evolutionary stages will it go through? Sketch the evolution of central density and temperature in the equation of state diagramme, labelling your sketch with the various stages and events.

2. After the collapse of a stellar core, a proto-neutron star is formed, which is in hydrostatic equilibrium. We will assume it has mass $1.4\,M_\odot$ and radius 20 km, and use that models give a temperature between $kT = 20$ and 50 MeV.

   (a) Show that the temperature is (roughly) consistent with the Virial Theorem.

   (b) Neutrinos with a thermal distribution have average energy $\overline{E}_\nu = 3kT$, and the interaction cross-section with nucleons is roughly $\sigma(E_\nu) \sim 4 \times 10^{-46}(E_\nu/10\ {\rm MeV})^2\ {\rm m}^2$. Calculate the mean-free path inside the proto-neutron star (assuming constant density, and $kT = 35$ MeV), and estimate the time it requires neutrinos to diffuse out.

   (c) Use the thermal content and the timescale from above to calculate the neutrino luminosity. From the luminosity and the radius, calculate an "effective temperature" (here we'll ignore differences between photons and neutrinos). Use the temperature to estimate the average energy of emitted neutrinos and show that the neutrino emission rate is $\sim 1.3 \times 10^{57}\,{\rm s}^{-1}$.

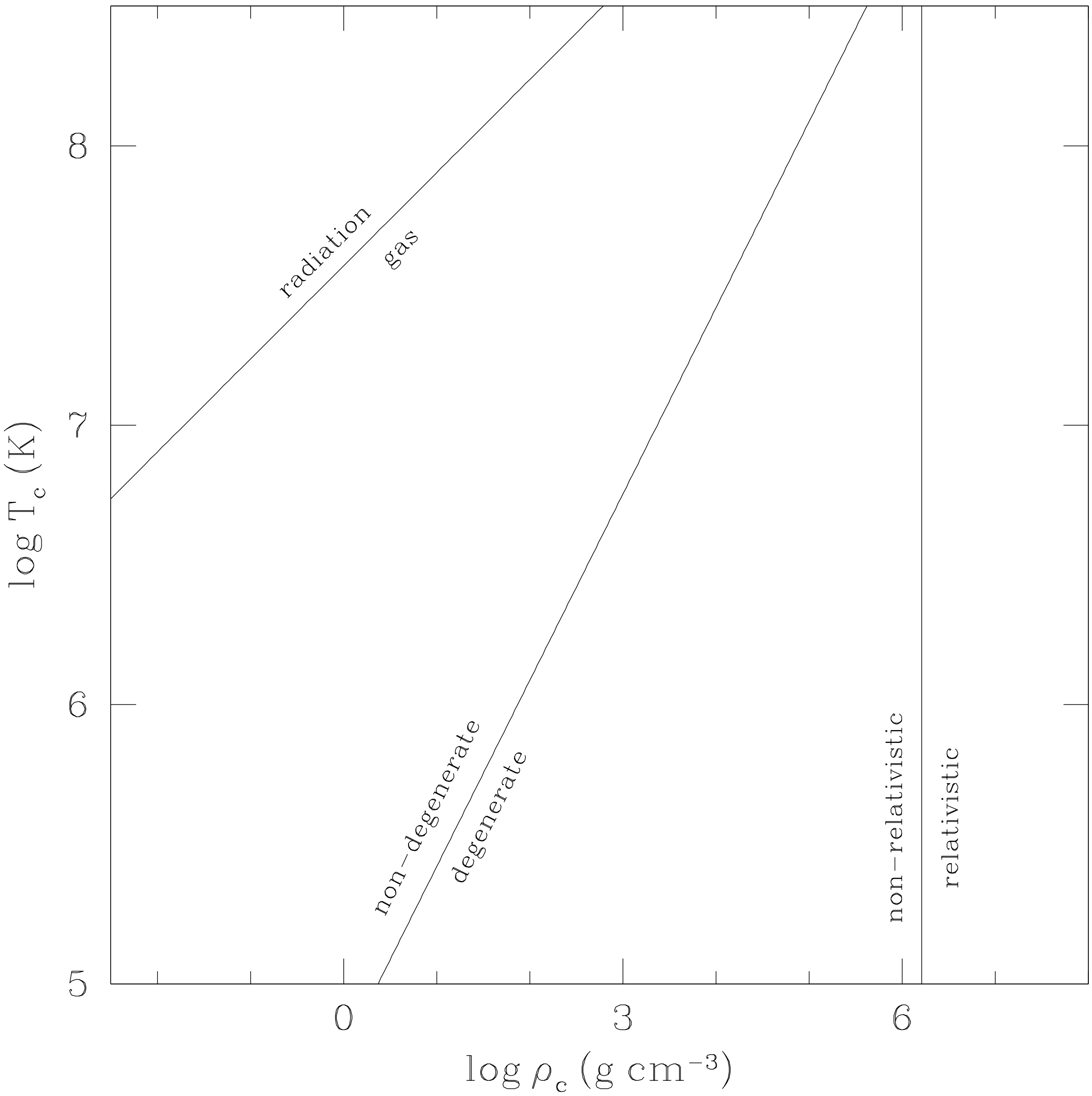

Figure 1: The T, ρ diagram for $X = 0.7$ and $Z = 0.02$, with the areas indicated where matter behaves as an ideal gas ($P \propto nT$), non-relativistic degenerate gas ($P \propto n_e^{5/3}$), relativistic degenerate gas ($P \propto n_e^{4/3}$), or radiation-dominated gas.

3. Nuclear fusion in the Early Universe.

   (a) Write down the reactions by which Helium is formed in the Early Universe.

   (b) How do the conditions differ from those in cores of stars, and how does this influence the chain of reactions that is used?

   (c) In Fig. 2 below, predicted abundances of the light elements are shown as function of baryon-to-photon ratio η. Describe briefly why the Helium abundance has only a slow dependence on η over much of the range shown, but a rapid decline towards low values of η.

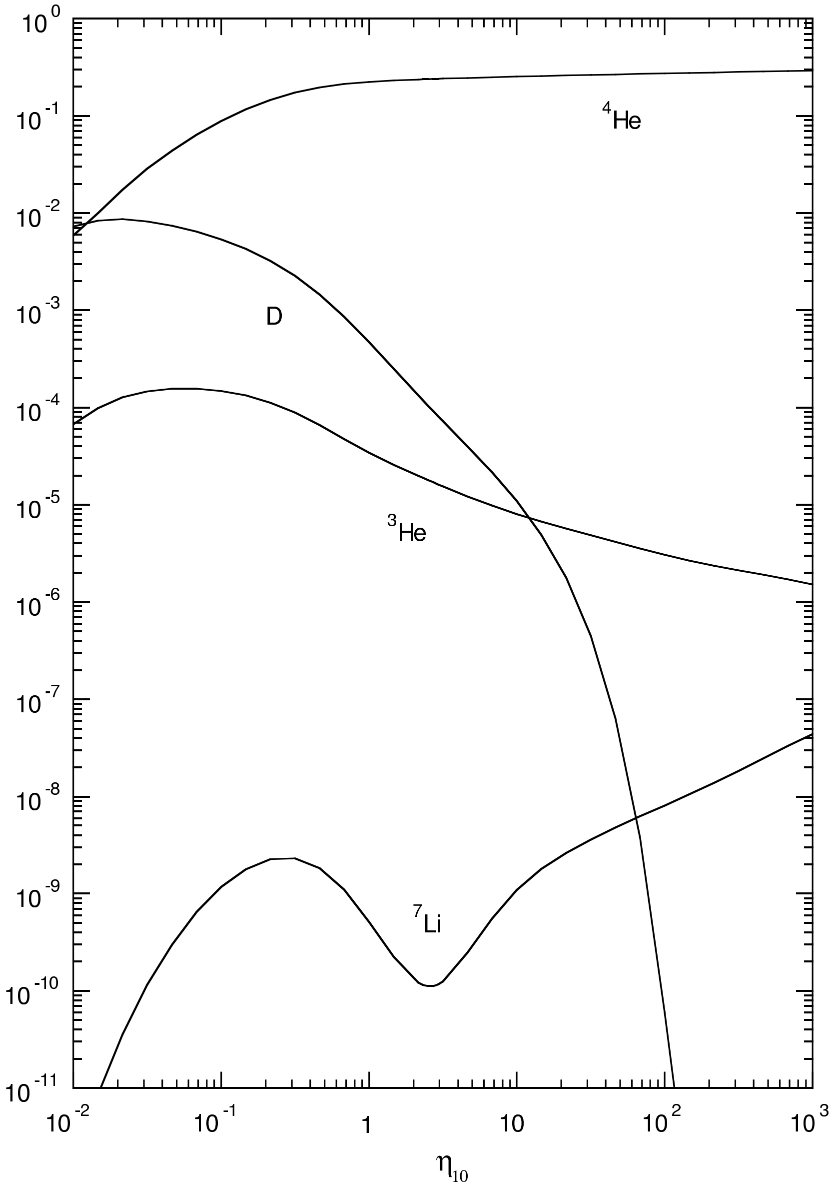

Figure 2: Expected abundances for the light elements as a function of baryon to photon ratio (in units of 10^{-10}). From Mukhanov (2003, astro-ph/0303073).

Physical constants

(`http://physics.nist.gov/cuu/Constants`)

speed of light *in vacuo*	c	=	$2.99792458 \times 10^{8}\,\mathrm{m\,s^{-1}}$ (exact)
Gravitational constant	G	=	$6.673(10) \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Planck's constant	h	=	$6.62606876(52) \times 10^{-34}\,\mathrm{J\,s}$
$[h/2\pi]$	$\hbar$	=	$1.054571596(82) \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann's constant	k	=	$1.3806503(24) \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Stefan-Boltzmann constant $[\frac{1}{60}\pi^2 k^4/\hbar^3 c^2 = ac/4]$	σ	=	$5.670400(40) \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Avogadro's number	N_{A}	=	$6.02214199(47) \times 10^{23}\,\mathrm{mol^{-1}}$
Molar gas constant $[kN_{\mathrm{A}}]$	$\mathcal{R}$	=	$8.314472(15)\,\mathrm{J\,mol^{-1}\,K^{-1}}$
electron mass	m_{e}	=	$9.10938188(72) \times 10^{-31}\,\mathrm{kg}$
proton mass	m_{p}	=	$1.67262158(13) \times 10^{-27}\,\mathrm{kg}$

Other units

atomic mass unit $[\frac{1}{12}m(^{12}C)]$	m_{u}	=	$1.66053873(13) \times 10^{-27}\,\mathrm{kg}$
hydrogen mass	m_{H}	=	$1.6735525 \times 10^{-27}\,\mathrm{kg}$
electric charge	e	=	$1.602176462(63) \times 10^{-19}\,\mathrm{C}$
electron volt	eV	=	$1.602176462(63) \times 10^{-19}\,\mathrm{J}$
Ångstrom	Å	=	$10^{-10}\,\mathrm{m}$

Astronomical units

(Nautical Almanac 1993)

Solar mass	$M_{\odot}$	=	$1.9891 \times 10^{30}\,\mathrm{kg}$
Solar radius	$R_{\odot}$	=	$6.9551(3) \times 10^{8}\,\mathrm{m}$
Solar luminosity	$L_{\odot}$	=	$3.839(5) \times 10^{26}\,\mathrm{W}$ (not official)
Solar temperature	$T_{\mathrm{eff},\odot}$	=	$5777(2)\,\mathrm{K}$ (not official)
astronomical unit	AU	=	$1.49597870 \times 10^{11}\,\mathrm{m}$
parsec	pc	=	$3600 \times 180/\pi\,\mathrm{AU} = 3.0856776 \times 10^{16}\,\mathrm{m}$
Julian year	yr	=	$365.25 \times 84600\,\mathrm{s}$ ($\sim\pi\,10^{7}\,\mathrm{s}$)

Some formulae

ideal gas	$P = nkT$, $n = \rho/\mu m_{\mathrm{H}}$ $c_v = \frac{3}{2}Nk$, $c_p/c_v = 5/3$
non-relativistic degenerate gas	$P = K_1 n_{\mathrm{e}}^{5/3}$, $n_{\mathrm{e}} = \rho/\mu_{\mathrm{e}} m_{\mathrm{H}}$, $K_1 = \frac{1}{5}(3\pi^2)^{2/3}(\hbar^2/m_{\mathrm{e}})$ $K_1/m_{\mathrm{H}}^{5/3} = 9.91 \times 10^{6}$ (SI)
relativistic degenerate gas	$P = K_2 n_{\mathrm{e}}^{4/3}$, $n_{\mathrm{e}} = \rho/\mu_{\mathrm{e}} m_{\mathrm{H}}$, $K_2 = \frac{1}{4}(3\pi^2)^{1/3}\hbar c$ $K_2/m_{\mathrm{H}}^{4/3} = 1.231 \times 10^{10}$ (SI)
scale height	$\mathcal{H} = kT/\mu m_{\mathrm{H}} g$