UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL/MAY 2007 EXAMINATIONS

AST320H1 (Winter 2007)

Duration: 3 hours; Examination aids: Calculator only

Marks: All three questions have equal weight. All subitems of questions have equal weight.
Note: A list of constants is appended.

1. Evolution of a star like the Sun.
   (a) Sketch an HRD with a main sequence. Also sketch the evolutionary track for a star with a mass of $1\,M_\odot$, starting at the zero-age main sequence and ending at the white-dwarf cooling track. Label the various parts.
   (b) Describe in physical terms why the luminosity increases on the main sequence.
   (c) Describe in physical terms why the luminosity increases on the red giant branch.

2. Evolution of the Universe.
   (a) What is the cosmological constant Λ? What equation of state (relation between mass-energy and pressure) does it have? What is its effect on the expansion of the Universe?
   (b) Write down the Friedman equation, and show that for $\Omega_{\rm tot} = 1$, $k = 0$, and ignoring radiation, it can be written as

$$\left(\frac{\dot{R}}{R}\right)^2 = H_0^2\left(\frac{\Omega_{\rm M,0}}{R^3} + \Omega_{\Lambda,0}\right). \tag{1}$$

   (c) The solution to the above version of the Friedman equation is given by,

$$R(t) = \left(\frac{\Omega_{\rm M,0}}{\Omega_{\Lambda,0}}\right)^{1/3} \sinh^{2/3}\left(\frac{3}{2}H_0 t\sqrt{\Omega_{\Lambda,0}}\right). \tag{2}$$

   Consider two cases, one with $\Omega_m = 1$, $\Omega_\Lambda = 0$ and one similar to the currently preferred one, with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$. For both cases, calculate the present age of the Universe, and sketch the evolution of $R(t)$ from the big bang ($R = 0$) to a size twice the present one ($R = 2$). How do the two ages you calculated compare to the ages of the oldest objects known?

3. Type Ia supernovae.
   (a) In a type Ia supernova, all the material in a carbon-oxygen white dwarf is burnt explosively. Describe in what type of progenitor systems type Ia supernovae occur, what triggers nuclear fusion, and why all type Ia supernovae have approximately equal luminosity.
   (b) Give physical arguments why the start of nuclear fusion leads to an explosion and not to steady nuclear burning.
   (c) Type Ia supernovae have been used to infer that the expansion of the universe is accelerating. Describe how was this done. To help your explanation, sketch the expected brightness as a function of z for a flat, matter-dominated ($\Omega_{\rm tot} = \Omega_m = 1$) and for our actual universe ($\Omega_{\rm tot} = 1$, $\Omega_m \simeq 0.3$ and $\Omega_\Lambda \simeq 0.7$).

Physical constants

(`http://physics.nist.gov/cuu/Constants`)

speed of light *in vacuo*	c	=	$2.99792458 \times 10^{10}\,\mathrm{cm\,s^{-1}}$ (exact)
Gravitational constant	G	=	$6.673(10) \times 10^{-8}\,\mathrm{dyne\,cm^2\,g^{-2}}$
Planck's constant	h	=	$6.62606876(52) \times 10^{-27}\,\mathrm{erg\,s}$
$[h/2\pi]$	$\hbar$	=	$1.054571596(82) \times 10^{-27}\,\mathrm{erg\,s}$
Boltzmann's constant	k	=	$1.3806503(24) \times 10^{-16}\,\mathrm{erg\,K^{-1}}$
Stefan-Boltzmann constant $[\frac{1}{60}\pi^2k^4/\hbar^3c^2 = ac/4]$	σ	=	$5.670400(40) \times 10^{-5}\,\mathrm{erg\,s^{-1}\,cm^{-2}\,K^{-4}}$
Avogadro's number	N_A	=	$6.02214199(47) \times 10^{23}\,\mathrm{mol^{-1}}$
Molar gas constant $[kN_\mathrm{A}]$	$\mathcal{R}$	=	$8.314472(15) \times 10^{7}\,\mathrm{erg\,mol^{-1}\,K^{-1}}$
electron mass	m_e	=	$9.10938188(72) \times 10^{-28}\,\mathrm{g}$
proton mass	m_p	=	$1.67262158(13) \times 10^{-24}\,\mathrm{g}$

Other units

atomic mass unit $[\frac{1}{12}m(^{12}C)]$	m_u	=	$1.66053873(13) \times 10^{-24}\,\mathrm{g}$
hydrogen mass	m_H	=	$1.6735525 \times 10^{-24}\,\mathrm{g}$
electric charge	e	=	$4.80320420(18) \times 10^{-10}\,\mathrm{ESU}$
		=	$1.602176462(63) \times 10^{-20}\,\mathrm{EMU}$
electron volt	eV	=	$1.602176462(63) \times 10^{-12}\,\mathrm{erg}$
Ångstrom	Å	=	$10^{-8}\,\mathrm{cm}$

Astronomical units

(Nautical Almanac 1993)

Solar mass	$M_\odot$	=	$1.9891 \times 10^{33}\,\mathrm{g}$
Solar radius	$R_\odot$	=	$6.96 \times 10^{10}\,\mathrm{cm}$
Solar luminosity	$L_\odot$	$\simeq$	$3.826 \times 10^{33}\,\mathrm{erg\,s^{-1}}$ (not official)
Solar temperature	$T_{\mathrm{eff},\odot}$	$\simeq$	$5780\,\mathrm{K}$ (not official)
astronomical unit	AU	=	$1.49597870 \times 10^{13}\,\mathrm{cm}$
parsec	pc	=	$3600 \times 180/\pi\,\mathrm{AU} \simeq 3.086 \times 10^{18}\,\mathrm{cm}$
Julian year	yr	=	$365.25 \times 84600\,\mathrm{s}$ ($\sim\pi\,10^7\,s$)

Some formulae

ideal gas	$P = nkT$, $n = \rho/\mu m_\mathrm{H}$ $c_v = \frac{3}{2}Nk$, $c_p/c_v = 5/3$
non-relativistic degenerate gas	$P = K_1 n_\mathrm{e}^{5/3}$, $n_\mathrm{e} = \rho/\mu_\mathrm{e} m_\mathrm{H}$, $K_1 = \frac{1}{5}(3\pi^2)^{2/3}(\hbar^2/m_\mathrm{e})$ $K_1/m_\mathrm{H}^{5/3} = 9.91 \times 10^{12}$ (cgs)
relativistic degenerate gas	$P = K_2 n_\mathrm{e}^{4/3}$, $n_\mathrm{e} = \rho/\mu_\mathrm{e} m_\mathrm{H}$, $K_2 = \frac{1}{4}(3\pi^2)^{1/3}\hbar c$ $K_2/m_\mathrm{H}^{4/3} = 1.231 \times 10^{15}$ (cgs)
scale height	$\mathcal{H} = kT/\mu m_\mathrm{H} g$