Problem Set II: Tides; Cores of Stars; Mass Loss in Binaries *due 7 Oct 2017*

For general comments about problem sets, see problem set I. For this specific one, you may find it useful to read CO §19.2, "Tidal forces," and 10.2, "Pressure equation of state."

II.1. Tides

1. What is the tidal acceleration between your head and your toes due to the Moon? And what due to the Earth? How do these compare to the acceleration due to the Earth's gravity?
2. Suppose you are old and as a last act want to see for yourself what it is to cross an event horizon (knowing you cannot come back). To get there, though, you have to survive the tidal forces. Given this, should you jump into a $10\,M_\odot$ stellar-mass black hole such as the one in Cygnus X-1, or are you better off at the $3\times10^6\,M_\odot$ black hole in the centre of our Galaxy? (To estimate this, calculate the tidal acceleration at the *event horizon* – CO, § 17.3, esp. Eq. 17.27 – and compare your results again to Earth's gravity; use the Newtonian estimate, even though this obviously will not be entirely correct.)
3. Now consider a more astrophysical example, of a star like the Sun falling towards a supermassive black hole in the centre of a galaxy. At some point, the star will start to be torn apart; for what black hole mass $M_{\rm BH,crit}$ does this happen at the event horizon? Would you expect to observe anything from such events for black holes with masses above or below this limit? (Events like this have found only relatively recently, with the first convincing case presented in 2006 by Gezari et al. (*Astroph. J.*, 653, L25).) Repeat the above for a white dwarf and a neutron star.

II.2. Cores of Stars

What are typical densities ρ_c, pressures P_c, and temperatures T_c in cores of stars and how does this depend on mass? And what processes dominate the pressure?

Note: In case you want to script this problem set, data from Appendix G of the book are available electronically at the book's web site.

1. Look up CO, App. G, which gives stellar data. Get the masses and radii for main-sequence stars of spectral type B0, A0, F0, the Sun, K0, and M0, and write these in a table. Add columns for ρ_c, P_c, and T_c, and look up the values for the Sun in CO, § 11.1. Estimate values for the other stars using scaling relations (assuming the structure of all main-sequence stars is the same).
2. Use the values from your table to estimate how R scales with M (i.e., assuming $R \propto M^\alpha$, what is α?). Here, first graph R as a function of M using logarithmic axes, so the dependence is a straight line, and then get α either graphically or from a fit with a programme. Given your result, how do ρ_c, P_c, and T_c scale with M?
3. In example 10.2.1, CO show that for the Sun radiation pressure is not important. Given the scalings you derived, for what mass does radiation pressure become important? (*If you could not derive the scalings, use the numbers in your table to make a guess*).
4. In class, we mentioned that the ideal gas law broke down when particle wavelengths overlap. To estimate whether this happens in the Sun, first calculate the mean particle spacing in the centre of the Sun, $r = n^{-1/3}$ (feel free to assume pure hydrogen composition). Effectively, this implies we "know" particles' positions to this accuracy, and this means we cannot "know" their momentum, to a limit given by Heisenberg's uncertainty principle (CO, Eq. 5.19). Use this to estimate the uncertainty in momentum for both a proton and an electron, and convert that to an uncertainty in their energy. Compare that energy with the thermal energy appropriate for the Sun's core. You should find that it considerably smaller. Given your scalings, for what mass would the energy given by the uncertainty relation become comparable to that inferred from the temperature? Which one is first, the electron or the proton?
   Bonus: What is the astrophysical relevance of this mass?

II.3. Mass loss in binary stars

Many stars are in binaries. Some stars have strong mass loss, at least for parts of their lives. Here, we consider the general effects on the orbit, and apply it to the Earth-Sun "binary."

1. Show that if star 1 is losing mass at a rate $\dot{m}_1$, the orbital separation a will evolve as $\dot{a}/a = -\dot{m}_1/M$ (where $M = m_1 + m_2$ is the total mass). *Note: You may want to revisit "cannibalism in close binary stars."*
2. How much do you expect the Earth-Sun system to expand over the 10 Gyr main-sequence lifetime of the Sun, given that the Sun is losing mass in the form of light? (This was actually considered by Jeans, 1924, *MNRAS*, **85**, 2). And how much given the mass-loss rate of $3 \times 10^{-14}\,M_\odot\,\mathrm{yr}^{-1}$ associated with the "solar wind" (see CO, ex. 11.2.1)? Finally, as a giant, the Sun will loose mass much faster, ending its life as a $0.6\,M_\odot$ white dwarf. At what distance will this leave the Earth?