Problem Set I: Seasons; Magnitudes; Pulsar Binary *due 21 Sep 2007*

General comment about problem sets: things are usually simpler than you think (no trick questions!). More specifically, (i) you should not need more space than what is provided below the questions (OK, assuming neat, fairly small handwriting ;-); (ii) unless explicitly stated, you should not need information other than the lectures and Carroll & Ostlie; (iii) unstated complications should be ignored; for instance, in I.1.1, do not worry about the atmosphere, and assume we're interested in the flux received on a 1 square meter piece of ground that is perfectly horizontal (of course, it does not hurt your mark to mention that you ignore such complications and thus show that you are aware of possible pitfalls).

Also, a request: please start on the problem set well before the due date, and preferably come to office hours if you have questions (we do answer e-mails, but only by the time of the next lecture; hence, e-mails the day before the due date will likely not be answered in time).

Before you start this problem set, you may find it useful to read CO §1.3, "Daily and seasonal changes," §2.3, "Kepler's laws derived," and §3.2, "The magnitude scale."

I.1. Solar System Seasons

In principle, two effects could cause seasons: a changing distance from the Sun in an eccentric orbit, and changing amount of daytime and height of the Sun in the sky if a planets rotation axis is inclined relative to its orbit. For Earth, the latter effect dominates. Here, we derive this and look at what to expect for other planets in the solar system. (*For planetary data, see CO, App. C.*)

1. Write down a general expression of the flux f received from the Sun as a function of luminosity of the Sun $L_\odot$, distance from the Sun r and the zenit distance z (the angle between the zenit and the line of sight to the Sun).
2. Calculate the fractional change in f due to the variations in r for the Earth (i.e., the ratio between f at perihelium and f at apohelium, for the same z). Also calculate the fractional change in f due to variation in z at noon (i.e., for the days when the Sun is highest and lowest in the sky when it crosses the meridian; keep r fixed and assume Northern latitude $45°$). Are your results consistent with the statement above that the inclination of the Earth's axis to the orbit is more important for causing seasons?
3. Now repeat the above calculation for Mercury, Mars, Jupiter, and Uranus. Which effect dominates for each of these planets?
4. While teaching "astronomy for poets" (AST 101), I often encountered the misconception that seasons are due to one hemisphere of the Earth being closer to the Sun than the other. Show that this has negligible effect.

I.2. A Bizarre Binary

Your professor was involved in a study of a rather bizarre binary, PSR J1740−5340, consisting of a radio pulsar and a more normal companion, which is part of the globular cluster NGC 6397. (It is certainly not required to answer this problem set, but if you are interested, you can read my small contribution on this system: Orosz & Van Kerkwijk, 2003, *Astron. & Astroph.*, **397**, 237.) *For units and constants such as the solar mass and radius, see CO, App. A (inside front cover).*

1. From radio timing of the pulsar, the two stars are in a circular orbit with a period $P = 1.35$ d. The projected semi-major axis of the pulsar's orbit is $a_{\rm P} \sin i = 1.65$ lt-s (light seconds). Assuming that the pulsar has mass of 1.4 solar masses ($M_{\rm P} = 1.4\ M_{\odot}$; a 'canonical' neutron-star mass), and that the inclination $i = 90°$ (i.e., that we view the system from the side), use the general version of Kepler's third law (CO, Eq. 2.37) to show that the mass of the companion is $M_{\rm C} = 0.19\ M_{\odot}$. (*Hint: first use Eq. 2.23 to rewrite Kepler's law in terms of $a_{\rm P}$, P, $M_{\rm P}$ and $M_{\rm C}$.*) Suppose we had reasons to believe the companion mass was $1\,M_{\odot}$ (see next question for why we might – erroneously – believe so), what would this imply? (i.e., how could we change the assumptions above to get a consistent result?)
2. Using the above masses and the period, calculate the semi-major axis a of the orbit. Write your result in units of solar radii ($R_{\odot}$).
3. The radio emission from the pulsar disappears for 13 hours around *superior conjuction*, when the pulsar is behind the companion. To see whether this eclipse could be caused by the companion blocking the radio emission, first estimate the maximum radius the companion could have without matter flowing to the pulsar (see CO, §18.1, in particular Fig. 18.2 and below). What maximum eclipse duration would this correspond to? Given your answer, are you surprised that the discoverers of this system think the radio emission is absorbed not by the companion, but by a wind emanating from it?

I.3. Magnitudes

The companion of the pulsar mentioned above has an apparent visual magnitude $m_V = 16.8$ and a temperature of 5400 K. We use this, as well as the fact that it is in the globular cluster NGC 6397, to estimate its radius.

1. First, let us assume the companion is a normal "main-sequence" star like the Sun. If so, given its temperature, use the table in CO, Appendix G, to estimate what would be its mass, radius, and absolute magnitude, M_V.
2. For the absolute magnitude you found, what would be apparent magnitude, m_V? Here, use that our system is in the globular cluster NGC 6397: This cluster is at a distance of 2.7 kpc and suffers from some "interstellar extinction" by interstellar dust, which means that, in the visual wavelength band, only 60% of the light emitted by stars in the cluster reaches Earth. *(You do not need to, but you may want to read the part on interstellar extinction in CO, § 12.1)*
3. You should have found that m_V for a main-sequence star with the same temperature as the companion is different from the observed value of $m_V = 16.8$ (*If you could not answer the previous question, assume $m_V = 18$ for the main-sequence star*). What do you conclude about the companion; is it intrinsically more or less luminous than a main-sequence star with the same temperature? By what factor? Is it therefore bigger or smaller? By what factor? What final radius estimate does this imply?