Problem Set I: Seasons; Image of a Candidate Exo-Planet *due 19 Sep 2008*

General comment about problem sets: things are usually simpler than you think (no trick questions!). More specifically, (i) you should not need more space than what is provided below the questions (OK, assuming neat, fairly small handwriting ;-); (ii) unless explicitly stated, you should not need information other than the lectures and Carroll & Ostlie; (iii) unstated complications should be ignored; for instance, in I.1.1, do not worry about the atmosphere, and assume we're interested in the flux received on a 1 square meter piece of ground that is perfectly horizontal, and in I.2 ignore the orientation of the orbit (of course, it does not hurt your mark to mention that you ignore such complications and thus show that you are aware of possible pitfalls).

Also, a request: please start on the problem set well before the due date (due at the start of class on the above date), and preferably come to office hours if you have questions (we do answer e-mails, but only by the time of the next lecture; hence, e-mails the day before the due date will likely not be answered in time).

Before you start this problem set, you may find it useful to read CO §1.3, "Daily and seasonal changes," §2.3, "Kepler's laws derived," and §3.2, "The magnitude scale."

I.1. Solar System Seasons

In principle, two effects could cause seasons: a changing distance from the Sun in an eccentric orbit, and changing amount of daytime and height of the Sun in the sky if a planets rotation axis is inclined relative to its orbit. For Earth, the latter effect dominates. Here, we derive this and look at what to expect for other planets in the solar system. (*For planetary data, see CO, App. C.*)

1. Write down a general expression of the flux f received from the Sun as a function of luminosity of the Sun $L_\odot$, distance from the Sun r and the zenit distance z (the angle between the zenit and the line of sight to the Sun).
2. Calculate the fractional change in f due to the variations in r for the Earth (i.e., the ratio between f at perihelium and f at apohelium, for the same z). Also calculate the fractional change in f due to variation in z at noon (i.e., for the days when the Sun is highest and lowest in the sky when it crosses the meridian; keep r fixed and assume Northern latitude 45°). Are your results consistent with the statement above that the inclination of the Earth's axis to the orbit is more important for causing seasons?
3. Now repeat the above calculation for Mercury, Mars, Jupiter, and Uranus. Which effect dominates for each of these planets?
4. While teaching "astronomy for poets" (AST 101), I often encountered the misconception that seasons are due to one hemisphere of the Earth being closer to the Sun than the other. Show that this has negligible effect.

I.2. Direct Image of a Planet to a Solar-Mass Star

As mentioned in class, your Professor's Summer was exciting, in including the discovery of a possible planetary mass companion to a solar-mass star (see Lafrenière, Jayawardhana, and Van Kerkwijk, submitted to *Astrophysical Journal (Letters)* [link on course website]). Unlike planets found around others stars, we see this candidate companion directly in images. This is possible since the star is only about 5 Myr old, and hence there has been little time for a planet to cool down. Furthermore, it is at a relatively large angular separation of 2.2 arcsec, which corresponds to 330 AU at the distance of 150 pc inferred for the association to which the star belongs.

Indeed, as we will see during the course, finding a planet at such a large distance is very surprising. We believe it is a companion nevertheless because it is unlikely to find something not associated so close to the star. But in the future we hope to confirm this by verifying that the two objects move together on the sky. Below, we will derive what we should expect.

1. Assuming the planet is in a circular orbit, what is its expected orbital velocity [$\mathrm{km\,s^{-1}}$]? And what the expected orbital period [yr]? (Assume the host star has mass $1\,M_\odot$, and neglect the effect of the $\sim 0.008\,M_\odot$ mass of the planet [but check you understand why this is OK!])
2. Still assuming a bound orbit, but not necessarily a circular one, what are the lowest and highest velocity the object could have? And what the shortest and longest possible orbital periods? Do explain your reasoning, e.g., using a sketch with orbits, or arguments based on energetics.
3. The primary has measurable *proper motion*, i.e., its position on the sky changes, by about 30 milli-arcsec per year. What physical velocity does this correspond to? Given your calculations above, what is the expected range in proper motion of the companion? Do you conclude that it will be similar or that it can be very different?

I.3. Magnitudes

The planetary mass candidate companion of the young, 5 Myr-old star mentioned above has an apparent magnitude in the J-band (centred at 1.2 μm) of $m_J = 17.9$ and an inferred temperature of about 1800 K. We use this to derive its mass and test whether a similarly massive planet could still hide in the outskirts of the solar system.

1. We used the so-called DUSTY models of Chabrier et al. (2000, *Astroph. J.*, **542**, 464) to infer a mass from the temperate and age. Download the models [link on course web site], and verify that the candidate companion must have a mass of $\sim 0.008\,M_\odot$. What is this in Jupiter masses? And how does the predicted radius compare to that of Jupiter?
2. Show that our observed, apparent J-band magnitude is consistent with that the absolute J-band magnitude predicted by the models.
3. Suppose the Sun – which is about 5 Gyr old – also had a $0.008\,M_\odot$ companion at a separation of 330 AU. How bright do the models predict the source would appear to us in the J-band? (Since this planet would be much colder than appropriate for the DUSTY models, you now need to use the COND models of Baraffe et al. 2003, *Astron. & Astroph.*, **402**, 701 [link at course web site]). The best sky survey in J is the so-called 2-Micron All-Sky Survey (2MASS), which is sensitive down to $J = 15.8$. Would it have detected the source? *Note: from our work, this scenario seems unlikely: we found only such companion among about 80 stars.*