Problem Set I: Seasons; the Black Widow Pulsar; Magnitudes *due 22 Sep 2017*

General comment about problem sets: things are usually simpler than you think (no trick questions!). More specifically, (i) you should not need more space than what is provided below the questions (OK, assuming neat, fairly small handwriting ;-); (ii) unless explicitly stated, you should not need information other than the lectures and Carroll & Ostlie; (iii) unstated complications should be ignored; for instance, in I.1.1, do not worry about the atmosphere, and assume we're interested in the flux received on a 1 square meter piece of ground that is perfectly horizontal.

Also, a request: please start on the problem set well before the due date (due at the start of class on the above date), and preferably come to office hours if you have questions (we do answer e-mails, but only by the time of the next lecture; hence, e-mails the day before the due date will likely not be answered in time).

Before you start this problem set, you may find it useful to read CO §1.3, "Daily and seasonal changes," §2.3, "Kepler's laws derived," and §3.2, "The magnitude scale."

I.1. Solar System Seasons

In principle, two effects could cause seasons: a changing distance from the Sun in an eccentric orbit, and changing amount of daytime and height of the Sun in the sky if a planets rotation axis is inclined relative to its orbit. For Earth, the latter effect dominates. Here, we derive this and look at what to expect for other planets in the solar system. (*For planetary data, see CO, App. C.*)

1. Write down a general expression of the flux f received from the Sun as a function of luminosity of the Sun $L_{\odot}$, distance from the Sun r and the zenit distance z (the angle between the zenit and the line of sight to the Sun).
2. Calculate the fractional change in f due to the variations in r for the Earth (i.e., the ratio between f at perihelium and f at apohelium, for the same z). Also calculate the fractional change in f due to variation in z at noon (i.e., for the days when the Sun is highest and lowest in the sky when it crosses the meridian; keep r fixed and assume Northern latitude $45°$). Are your results consistent with the statement above that the inclination of the Earth's axis to the orbit is more important for causing seasons?
3. Now repeat the above calculation for Mercury, Mars, Jupiter, and Uranus. Which effect dominates for each of these planets?
4. While teaching "astronomy for poets" (AST 101), I often encountered the misconception that seasons are due to one hemisphere of the Earth being closer to the Sun than the other. Show that this has negligible effect.

I.2. The Black Widow Pulsar

A favourite binary of your professor is PSR B1957+20, nicknamed the "Black Widow Pulsar," consisting of a radio pulsar and a companion with a very low mass, well in the brown-dwarf regime. The pulsar strongly irradiates its companion, causing it to be much hotter on the side facing the pulsar. (It is certainly not required to answer this problem set, but if you are interested, you can read my contribution on this pulsar binary: Van Kerkwijk, Breton, & Kulkarni, 2011, *Astroph. J.*, **728**, 95.) *For units and constants such as the solar mass and radius, see CO, App. A.*

1. From radio timing of the pulsar, the two stars are in a circular orbit with a period $P = 0.3819666$ d. The pulsar orbit's projected semi-major axis $a_{\rm P} \sin i = 0.0892253$ lt-s (light seconds), where the inclination i is the angle between the line of sight and the orbit normal). For the companion, I measured velocities along the line of sight, with amplitude $K = 353 \pm 4\ {\rm km\,s^{-1}}$. Given the above, what is is the total projected semi-major axis (in light seconds)? Show that the mass ratio is $M_{\rm P}/M_{\rm C} = 69.2 \pm 0.8$.?
2. We first assume that we view the system edge-on, i.e., $i = 90°$, and we will ignore uncertainties. Use the general version of Kepler's law to show that the system's total mass is 1.81 $M_\odot$. What are the corresponding pulsar and companion masses? Show how the masses scale with inclination, and that for the inclination inferred from modelling of the irradiated companion lightcurve, of $i \simeq 65°$, one infers $M_{\rm P} = 2.40\ M_\odot$. (*Note: This mass is substantially higher than can be understood for some models of neutron stars. Those would thus be excluded if this can be confirmed.*)
3. The radio emission from the pulsar disappears for 40 minutes around *superior conjuction*, when the pulsar is behind the companion. How large would the companion have to be to cause this eclipse? The maximum radius the companion can have without mass flowing to the pulsar is given by the *Roche lobe radius*, $R_{\rm L} \simeq 0.46a(M_{\rm C}/(M_{\rm P}+M_{\rm C}))^{1/3}$ (this approximation is valid for $M_{\rm C} < M_{\rm P}$). What is its value? Given your answers, are you surprised that it is thought the eclipse is *not* due to the companion, but rather to outflows? (Indeed, this is partially where the nickname comes from: the pulsar seems to induce outflows that slowly destroy the companion.)

I.3. Magnitudes

The companion of the pulsar mentioned above varies greatly in brightness as we see more or less of its heated side. When that side is in front, the apparent visual magnitude is $m_V = 20.6$ and the temperature is very similar to that of the Sun, $\sim$5800 K. We use this, as well as the fact that the distance to the pulsar is estimated to be 2 kpc, to infer its radius.

1. To prepare, first consider a G2 V main-sequence star like the Sun. Look up its absolute visual magnitude M_V in CO, Appendix G. If this star were placed at the 2 kpc, what would be its apparent magnitude, m_V?
2. In the previous question, we did not correct for the extinction of star light by interstellar dust. From other measurements, it has been found that, in the visual wavelength band, only 50% of the light emitted by stars in the cluster reaches Earth. What correction in magnitudes does this correspond to? Apply this to your answer above. *(You should not need it, but if you are confused, read the part on interstellar extinction in CO, §12.1)*
3. You should have found that m_V for a star like the Sun at 2 kpc is different from the value of m_V given above for the pulsar companion (*If you could not answer the preceding questions, assume $m_V = 17$ for a star like the Sun*). Given that the temperature is similar, what do you conclude about the size of the companion, is it larger or smaller than the Sun? By what factor? Does it makes sense with the maximum radius you found in I.2.3?