Physical Ingredients for Constructing a Star (or Planet)

1. Support against gravity

   Pressure: hydrostatic equilibrium
   equation of state

   Energy: virial theorem

2. Source of energy

   Contraction

   Nuclear power

   Photon propagation/emission

Nuclear Fusion

Einstein: $E = m\,c^2$

atomic unit: $u = 1.660540 \times 10^{-27}$ kg (mass of ^{12}C/12)

proton: $m_p = 1.672623 \times 10^{-27}$ kg $= 1.0072765\ u$ = **938.79 MeV**

neutron: $m_n = 1.674929 \times 10^{-27}$ kg $= 1.0086653\ u$

electron: $m_e = 9.109390 \times 10^{-31}$ kg $= 0.0054858\ u$

Hydrogen atom: $m_H = 1.007825\ u = m_p + m_e$ − electro-static/2

13.6 eV binding energy of H atom

Helium nucleus: $m_{He} = 4.00151\ u = 2\,m_p + 2\,m_n - \Delta m$

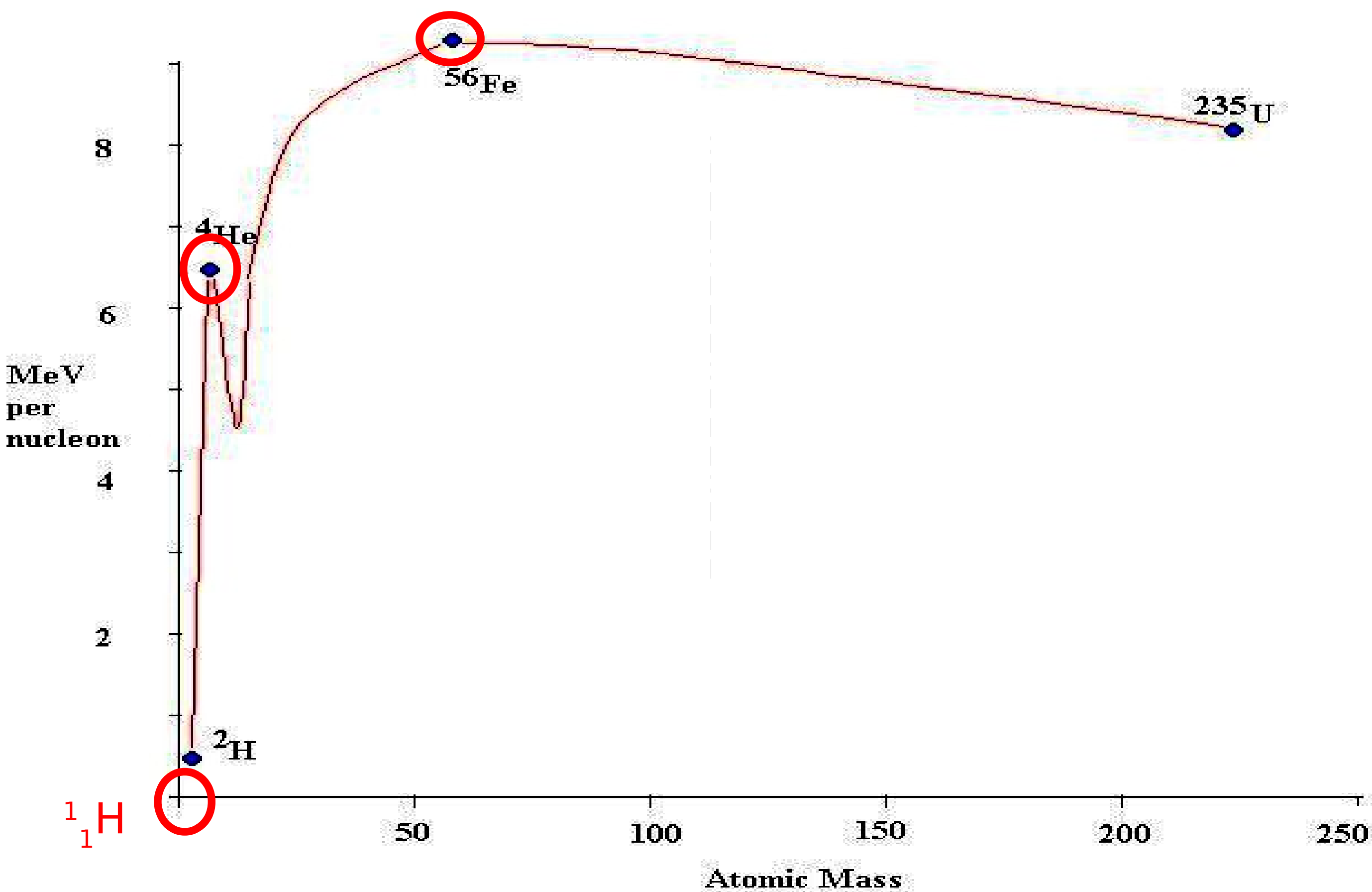
$\Delta m = 0.03037\ u \sim 0.7\%\ (4\,m_H) \sim 28$ MeV

Fusion 4 p → He releases ~28 MeV

Binding Energy of a Nucleus

--- energy released when formed
--- energy required to unbind

${}^{1}_{1}H$	$E_b = 0$		
${}^{4}_{2}He$	$\Delta\, m\, c^2$ = 28.03 MeV, hence	7.08 MeV/nucleon	
${}^{16}_{8}O$	$(Z\, m_p + (A\text{-}Z)\, m_n - m_{nucleus})\, c^2/A =$	7.97 MeV/nucleon	
${}^{56}_{26}Fe$		8.79 MeV/nucleon	Most stable nucleus easily made in stars.
$({}^{238}_{92}U$		7.3 MeV/nucleon)	



Basic concepts in nuclear physics

1) Binding energy of an element: ${}^{A}_{Z}X$: $E_b = (Z\, m_p + (A\text{-}Z)\, m_n - m_{nucleus})\, c^2$

2) attractive **strong force** binds protons & neutrons together against repulsive Coulomb force

Four basic forces in nature
long-range forces: gravity, electro-magnetism (force carriers have no mass, F ~$1/r^2$)
short-range forces: strong force, weak force (force carries have mass,
strength falls off dramatically beyond nuclear dimension ~1 fm = 10^{-15} m)

3) For A less than a critical value (~56),
strong force increases with A faster than Coulomb repulsion
$\Rightarrow$ binding/nucleon increases with A
Beyond that, nucleus too large $\Rightarrow$ binding/nucleon decreases A

---- energetically favourable to fuse H to He ..... to Fe, but not past Fe
---- past Fe, fission energetically favourable; fusion becomes endothermic

Nuclear Energy Yield: H burning to Fe yields ~ 9 MeV/nucleon

${}^{1}_{1}\mathrm{H}$ ${}^{4}_{2}\mathrm{He}$ ${}^{16}_{8}\mathrm{O}$ ${}^{56}_{26}\mathrm{Fe}$

7.08 MeV/nucleon ~0.7% $m_p c^2$

7.97 MeV/nucleon, ~0.8% $m_p c^2$

8.79 MeV/nucleon, ~1% $m_p c^2$

Fusion: Each proton can maximally yield ~1% $m_p c^2$ ~ 9 MeV ~ 10^{-12} J

(1 g of Hydrogen fusion ~ annual energy consumption of one Canadian)

Fusion energy available for Sun: $E \sim 10^{-12}$ J x $M_\odot/m_p \sim 10^{45}$ J >> $GM_\odot^2/R_\odot$

Nuclear Timescale: $t_{nuc} \sim E/L_\odot \sim 10^{45}\ \mathrm{J}/L_\odot \sim 10^{11}$ yr

Actual lifespan ~10^{10} years (now about half-way)

1) L increases in later life;
2) not all H burned;
3) not burned to Fe.

Compare: dynamical time: $t_{dyn} \sim 30$ min

thermal time: $t_{KH} \sim 10^{7}$ yr

Reminder: Periodic Table of the Elements

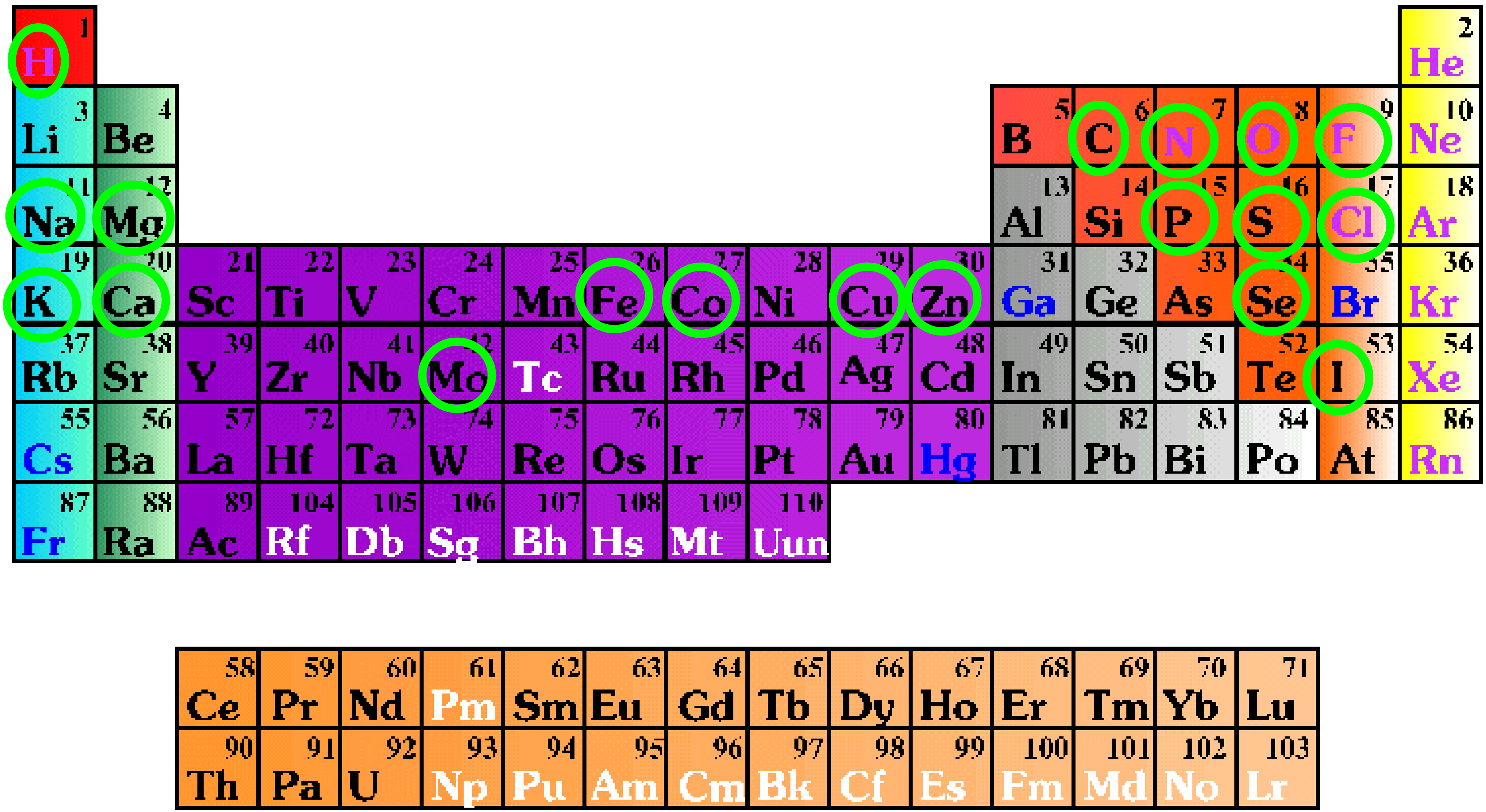
(from http://www.chemicool.com)

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Uun								

58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

Uranium & Plutonium

Elements used by the Human body



How to set the Nuclear Fire?

(or why we don't yet have clean fusion power on Earth)

$4\ {}^{1}_{1}H \rightarrow {}^{4}_{2}He$ + energy release of ~7 MeV/nucleon

Problem 1: **Coulomb barrier**

protons have to **overcome the electrostatic repulsion** between them and reach the realm of strong force (~1 fm = 10^{-15} m, also the size of a nucleus)

(e^{-} too far out to be relevant, ~10^{-11} m)

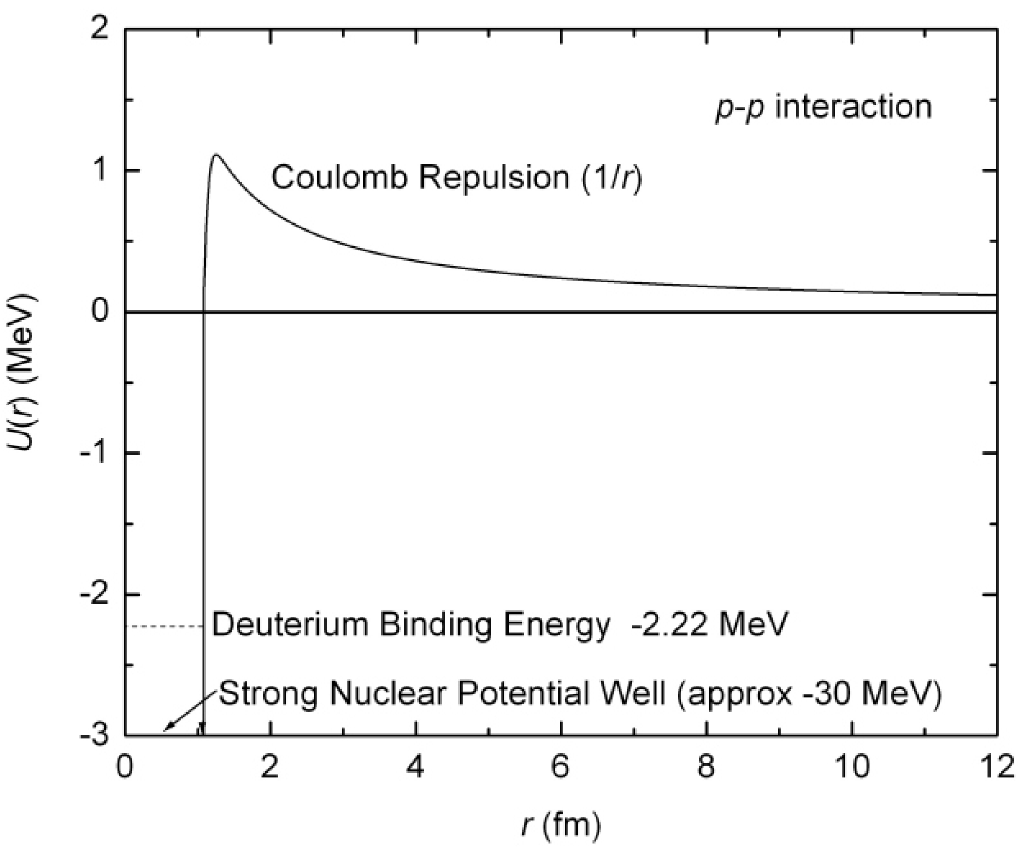
Classical solution: thermal motion of the nuclei

Two protons need to get as close as ~1 fm for strong force attraction to set in

Coulomb Force is a potential force (like gravity, but can repel)

Coulomb potential $U_C = \frac{1}{4\pi\epsilon_0}\frac{e_1 e_2}{r}$ *(1.4 MeV for $r=1\,fm$)*

As two protons approach each other, total energy is conserved:



$$\frac{1}{2}m_p v_\infty^2 + U_c(\infty) = U_c(1\,fm)$$

$$\frac{1}{2}m_p v_\infty^2 = \frac{3}{2}kT \geq \frac{e^2}{4\pi\epsilon_0(1\,fm)}$$

$$T \geq T_{classical} = \frac{e^2}{6k\pi\epsilon_0(1\,fm)} \sim 10^{10}K$$

But from Virial Theorem,
we know that Sun has $T_c \sim 10^7$ K

Sir Arthur Eddington (1882-1944)

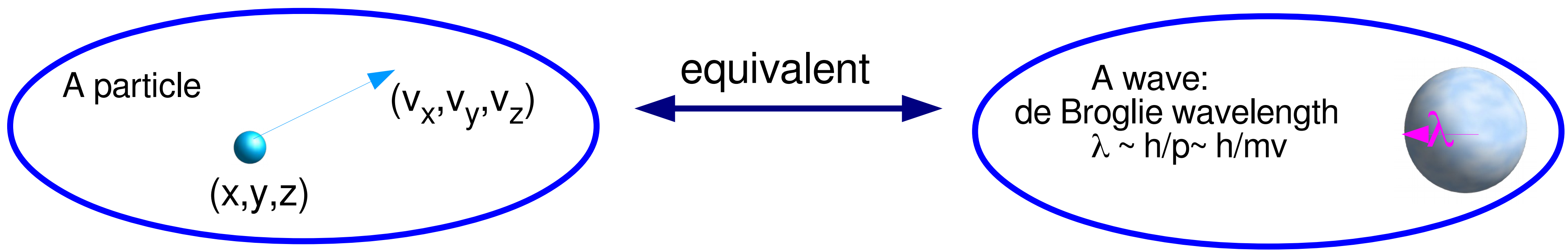
Arthur Eddington thought that nuclear processes must be involved to account for the radiant energy of the sun, but was criticized because the temperature was seen to be not hot enough when considered by classical physics alone.

His tongue-in-cheek reply to his critics: "*I am aware that many critics consider the stars are not hot enough. The critics lay themselves open to an obvious retort; we tell them to go and find a hotter place*."

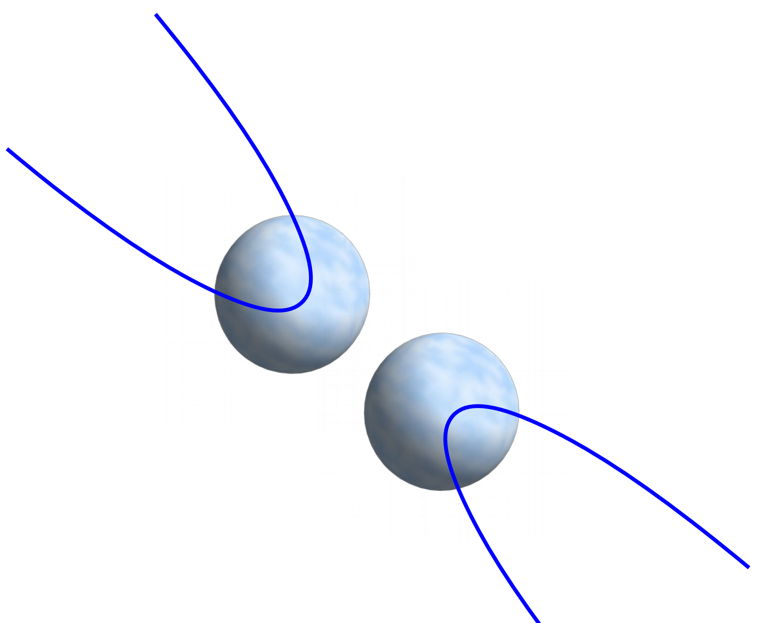
Vindicated by quantum mechanics.
Protons are 'BIG'.

Quantum mechanical solution: Wave-Particle Duality

(size of a particle depends on its momentum)



Use *Quantum Tunneling:* if a proton gets within one de Broglie wavelength of another, there is a certain probability that they “are” closer than ~1 fm



$$U_c=\frac{e^2}{4\pi\epsilon_0\lambda}\le\frac{1}{2}m_p v_\infty^2=\frac{1}{2}\frac{p^2}{m_p}\sim\frac{1}{2m_p}\left(\frac{h}{\lambda}\right)^2$$

$$\lambda\le\frac{4\pi\epsilon_0 h^2}{2e^2 m_p}\sim10^{-13}m\sim100\,fm$$

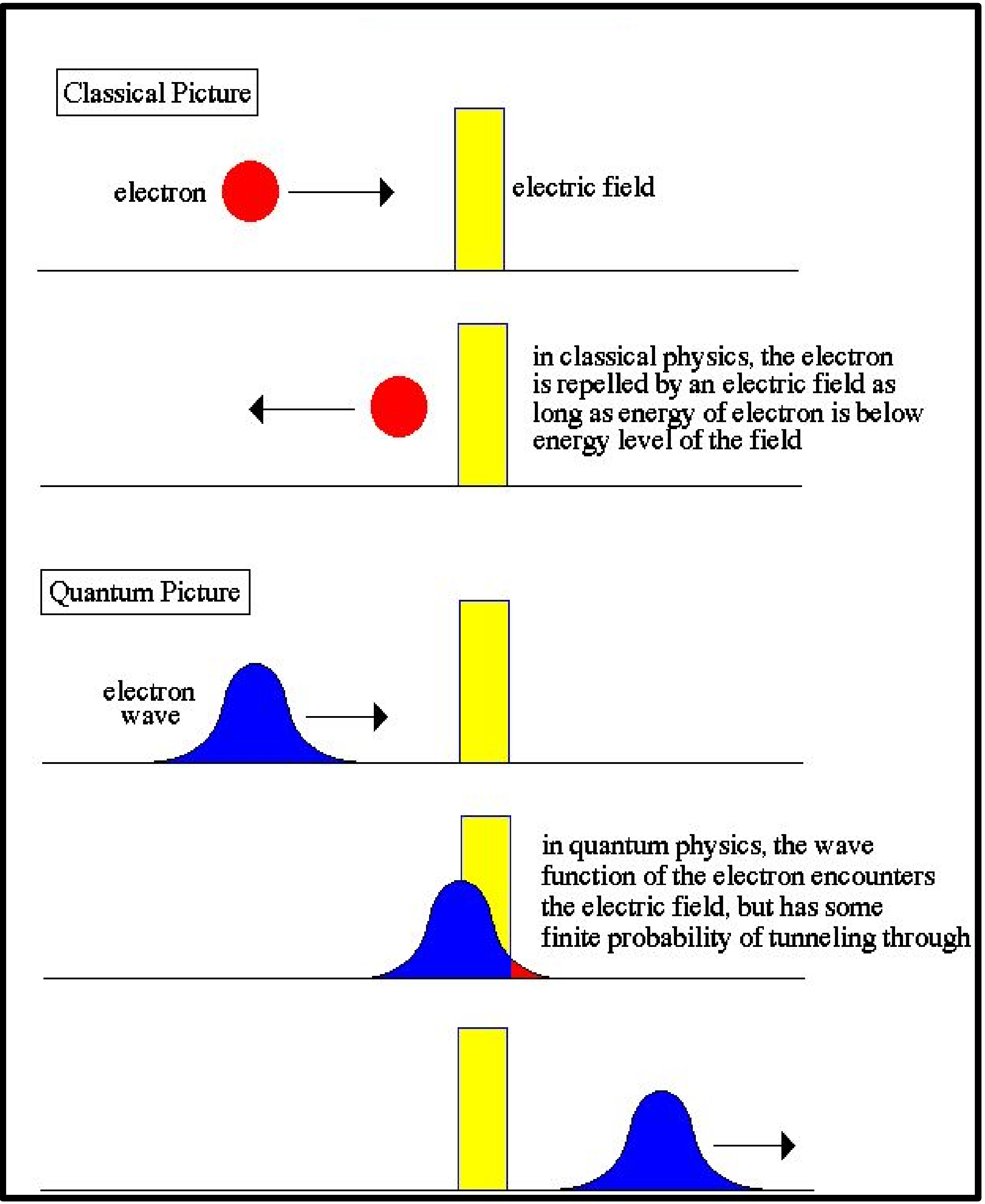
$$\frac{3}{2}kT=\frac{1}{2}m_p v_\infty^2=\frac{1}{2m_p}\left(\frac{h}{\lambda}\right)^2$$

$$T\ge T_{quantum}\sim\frac{m_p e^4}{12\pi^2\epsilon_0^2 k h^2}\sim10^7K$$

Due to Quantum Tunneling, hydrogen fusion is possible already at 10^7 K

(stars < $0.08M_\odot$ cannot reach this temperature at the center; “failed stars” or brown dwarfs)

Quantum tunneling and the ignition temperature



Quantum Tunneling:
a physical system spends some time in an energetically forbidden region:

there always exists
a certain probability
that two protons 'are' closer than ~1fm

Tunneling probability

$\exp(-2\pi^2\, U_c(\lambda(E))/E)$

~ 4×10^{-11} (E/k ~ T ~ 10^7 K)
~ 0.4 (E/k ~ T ~ 10^{10} K)

Nuclear reaction rates rise extremely steeply with T.
Hence, **ignition temperature** (~10^7 K for Hydrogen fusion)

How to set the Nuclear Fire? (cont'd)

Problem 2: How often do protons see each other?

1) space is empty for nuclei even at the Sun's center (ρ ~ 100 g/cm^3)
   mean separation ~ 10^{-11} m $\gg$ λ ~ 10^{-13} m

2) 4 $^1_1H \rightarrow {}^4_2He$ + energy release of ~7 MeV/nucleon
   All four in the same place at the same time? --- difficult
   Fusion proceeds through *a chain of 2-body reactions,*
   each satisfying conservation of energy, momentum, charge,
   lepton number (e^-, e^+, ν, anti-ν), and baryon number (A)

   a) At low T: **p-p chain** (involving only protons and products);
   b) At higher T: **CNO chain (**C,N,O as catalysts).

3) The more highly charged the nucleus, the higher the Coulomb barrier, and thus the higher the required temperature;
   Helium fusion requires $T > 10^8$ K; others $T > 10^9$ K
   (relevant after H exhaustion in the stellar core)