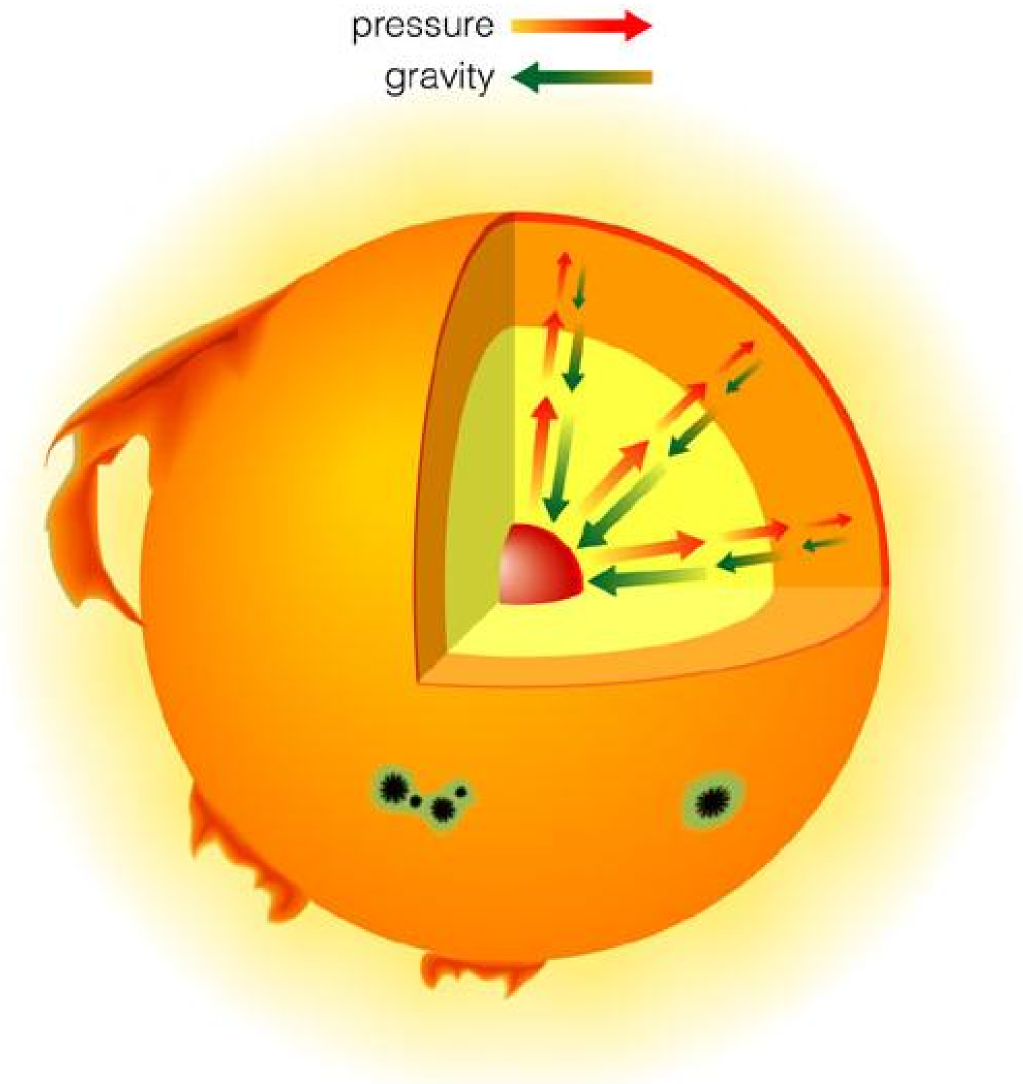
What is a Star?

Big ball of gas

Self-gravity	⇒	high pressure inside,
high pressure	⇒	high temperature inside,
high temperature	⇒	emit light.

Star's life: Protracted battle with gravity



ALWAYS

To support weight:

⇒ need high pressure

MOSTLY

⇒ need high temperature

⇒ will loose energy

⇒ need energy source:
- Gravitational contraction
- Nuclear fusion

Ultimately,
Can something else than thermal pressure balance gravity?

Physical Ingredients for Constructing a Star (or Planet)

1. Support against gravity
   - Locally: Pressure balance: hydrostatic equilibrium
     Requires equation of state
   - Globally: Energy balance: virial theorem

2. Source of energy
   - Contraction
   - Nuclear fusion

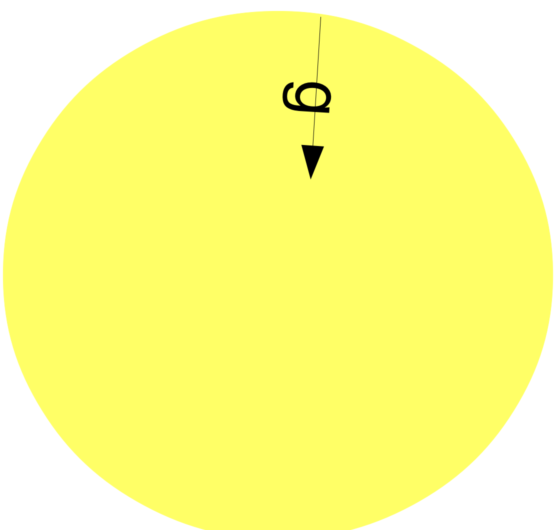
3. Energy transport
   - Photon propagation
   - Convection

What supports the star against its self-gravity?

I. What happens if there is no support?

II. What provides support?

III. What is sufficient support?



Gravitational acceleration $$g_r = \frac{GM_r}{r^2}$$

- Independent of density profile if spherically symmetric
- M_r is matter inside radius r
- Matter outside doesn't count

Example 2.2.1 of the textbook (p. 33)

I. No support: hydrodynamic collapse

Radial infall velocity: $v_r = \frac{dr}{dt}$,

Radial acceleration: $\frac{dv_r}{dt} = \frac{d^2 r}{dt^2} = -\frac{GM_r}{r^2}$

Accelerated fall, t_{ff} *is the time to reach the centre*

$$v_r \sim \frac{r_0}{t_{ff}}, \quad \frac{dv_r}{dt} \sim \frac{r_0}{t_{ff}^2} \sim \frac{GM_0}{r_0^2}$$

r_0: *initial radius,* M_0: *total mass*

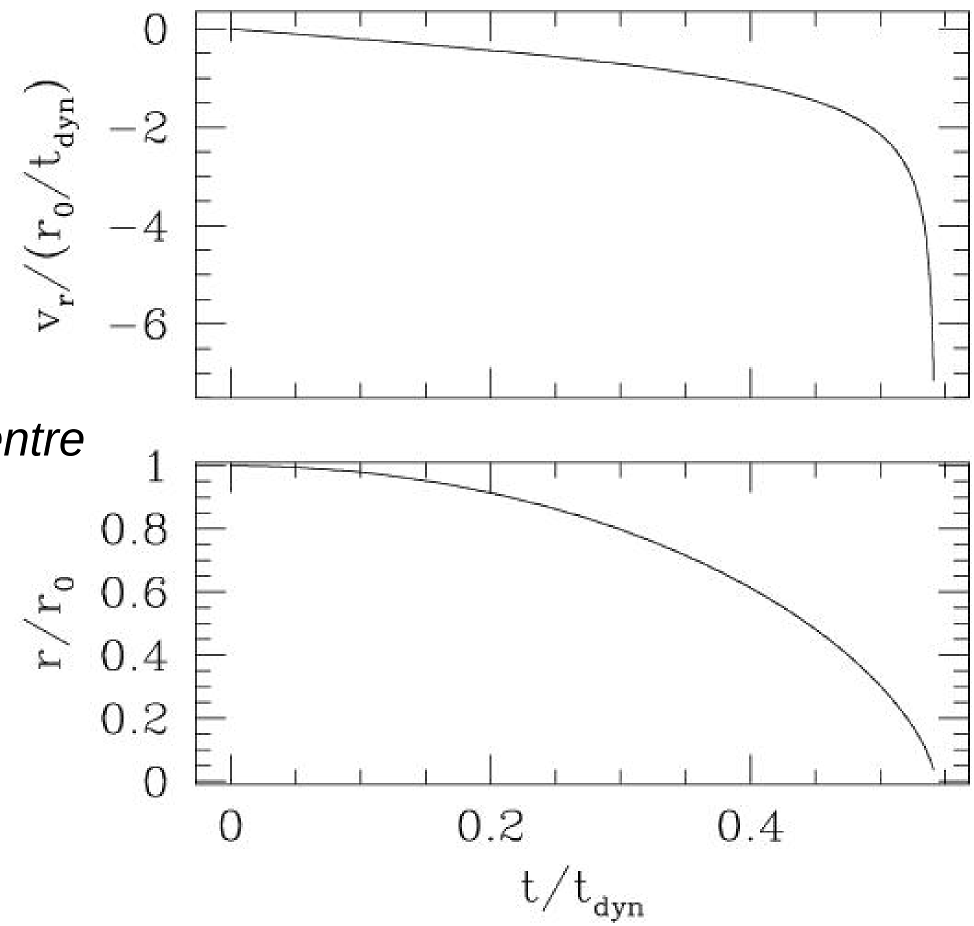
Hence, $t_{ff} \sim \sqrt{\frac{r_0^3}{GM_0}} \sim \sqrt{\frac{1}{G\bar{\rho}_0}}$

$\bar{\rho}_0$: *initial mean density*

Dynamical timescale t_{dyn}

Full answer: $t_{ff} = \sqrt{\frac{3\pi}{32}} \sqrt{\frac{1}{G\bar{\rho}_0}}$

Can also derive as 'half an orbit'



Examples: star formation, supernovae

Collapse timescale: only density matters, not size

$$t_{dyn} \sim \sqrt{\frac{1}{G\rho}}$$

	Size	*mass*	*mean density*	t_{dyn}
Earth	$\sim 10^{-2}\,R_\odot$	$\sim 10^{-6}\,M_\odot$	5.5 g/cm^3	
Jupiter	$\sim 10^{-1}\,R_\odot$	$\sim 10^{-3}M_\odot$	1.3 g/cm^3	~1 hr
Sun	$R_\odot$	$M_\odot$	1.4 g/cm^3	
White dwarf	$\sim 10^{-2}\,R_\odot$	$M_\odot$	1.4×10^6 g/cm^3	~3 s
Neutron star	10 km	1.4 $M_\odot$	7×10^{14} g/cm^3	~0.2 ms
giant molecular cloud	~100 pc	$\sim 10^6\,M_\odot$	$\sim 10^{-22}$ g/cm^3	$\sim 10^7$ yr
globular cluster of stars	~3 pc	$\sim 10^6\,M_\odot$	$\sim 3\times10^{-18}$ g/cm^3	$\sim 10^5$ yr
cluster of galaxies	~3 Mpc	$\sim 10^{16}\,M_\odot$	$\sim 10^{-25}$ g/cm^3	$\sim 10^8$ yr
the observable universe	~14Gly	$\sim 10^{22}\,M_\odot$	$\sim 3\times10^{-30}$g/cm^3	$\sim 10^{11}$ yr

Dynamical timescale:
if no support against self-gravity, object collapses in dynamical timescale.

Relevant for: solar oscillation, stellar pulsation, star formation in molecular clouds...

What prevents the Sun from collapse?

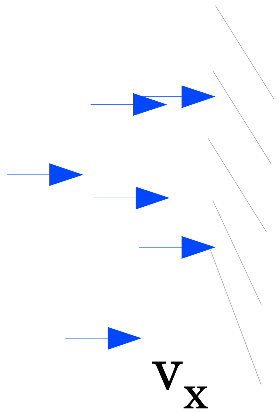
II. Stars are supported against self-gravity by gas pressure.

Gas exerts pressure on its surrounding; it resists being compressed.

Pressure arises from kinetic energy of the gas particles;
exert force (momentum exchange) when they are reflected

Pressure: force/unit area

For stars, the plasma is well described as as an Ideal Gas



Ideal gas: no correlation between particles.

All particles are energetically indistinguishable
mean kinetic energy per particle = 3/2 $k_B T$

= <average over time for one particle>
= <average over all particles at one time>

$P = n\, k_B\, T$ *(see textbook § 10.2)*

$n = N/V$, number density

Non-ideal gas: correlation between particles

particles are also waves, correlate if wavelengths overlap

1) Fermion gas $P = P(n)$ *electrons, protons, neutrons, quarks ("degenerate gas")*

2) Boson gas $P = P(T)$ *photons, gravitons, strong/weak gauge bosons*

Pressure of Ideal Gas:

$$P = n\, k_B\, T$$

n: number density, number of all particles per unit volume

Can be written in another form using mass density ρ (mass per unit volume)

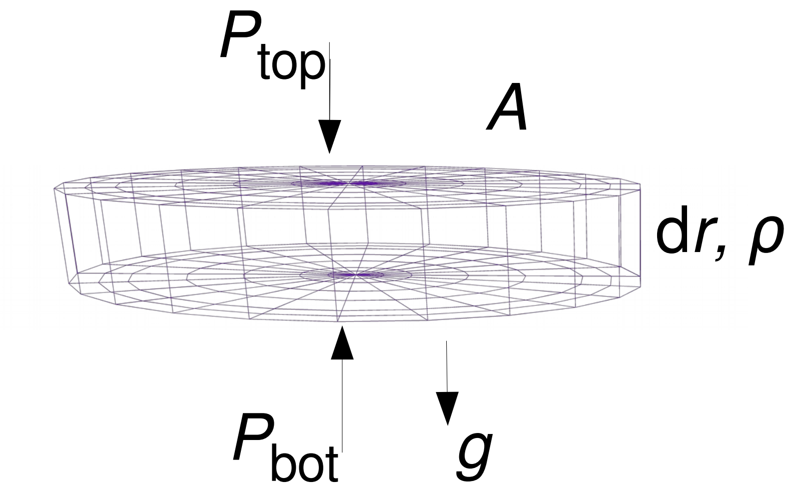
$$P = n\, k_B\, T = (\rho/\mu m_H)\, k_B\, T$$

Mean molecular weight (μ): $\rho/n = \mu m_H$

Average mass of molecules/atoms measured in the unit of hydrogen atomic mass

III. Pressure support and Hydrostatic Equilibrium

Actually need pressure gradient to support star against gravity.



$$m\,g_r = \rho\,dr\,A\,g_r = P_{bot}\,A - P_{top}\,A$$

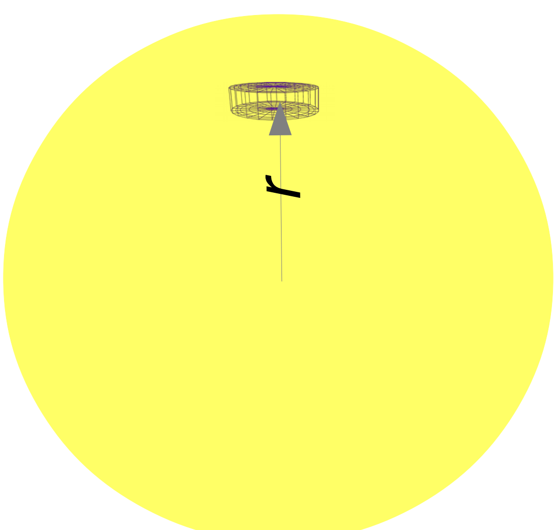
$$P_{top} = P_{bot} + \frac{dP}{dr}\,dr$$

$$\text{Hence, } \frac{dP}{dr} = -\rho\,g_r$$

equation of hydrostatic equilibrium

Applies to any 'fluid' (ocean, rock, air...)

Pressure has to increase inward



H.E. in the astronomer's toolkit

$$\frac{dP}{dr} = -\frac{G M_r \rho}{r^2}$$

$$P \approx \frac{GM^2}{R^4} \text{ or } P \propto M^2/R^4$$

$$\rho \approx \bar{\rho} \text{ or } \rho \propto M/R^3$$

With ideal gas law, $P = \frac{\rho}{\mu m_H} k_B T$,

$$kT \approx \frac{GM \mu m_H}{R} \text{ or } T \propto M/R$$