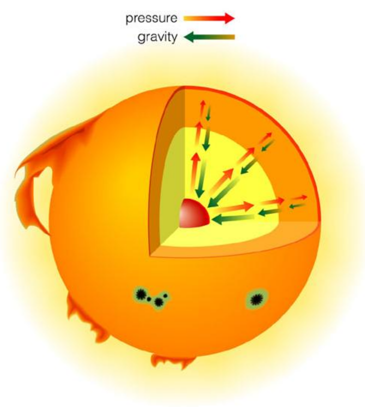
## What is a Star?

*Big ball of gas*

| | |
|---|---|
| Self-gravity | ⇒ high pressure inside, |
| high pressure | ⇒ high temperature inside, |
| high temperature | ⇒ emit light. |

## Star's life: Protracted battle with gravity




ALWAYS

To support weight:
⇒ need high pressure

MOSTLY

⇒ need high temperature
⇒ will loose energy
⇒ need energy source:
- Gravitational contraction
- Nuclear fusion

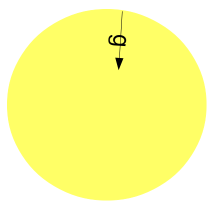
Ultimately,
***Can something else than thermal pressure balance gravity?***

## Physical Ingredients for Constructing a Star (or Planet)

1. Support against gravity
   - Locally: Pressure balance: hydrostatic equilibrium
     Requires equation of state
   - Globally: Energy balance: virial theorem
2. Source of energy
   - Contraction
   - Nuclear fusion
3. Energy transport
   - Photon propagation
   - Convection

### What supports the star against its self-gravity?

**I.** What happens if there is no support?
**II.** What provides support?
**III.** What is sufficient support?




Gravitational acceleration $g_r = \frac{GM_r}{r^2}$

- Independent of density profile if spherically symmetric
- $M_r$ is matter inside radius r
- Matter outside doesn't count

*Example 2.2.1 of the textbook (p. 33)*

## I. No support: hydrodynamic collapse

*Radial infall velocity:* $v_r = \frac{dr}{dt}$,

*Radial acceleration:* $\frac{dv_r}{dt} = \frac{d^2 r}{dt^2} = -\frac{GM_r}{r^2}$

*Accelerated fall, $t_{ff}$ is the time to reach the centre*

$$v_r \sim \frac{r_0}{t_{ff}}, \quad \frac{dv_r}{dt} \sim \frac{r_0}{t_{ff}^2} \sim \frac{GM_0}{r_0^2}$$

*$r_0$: initial radius, $M_0$: total mass*

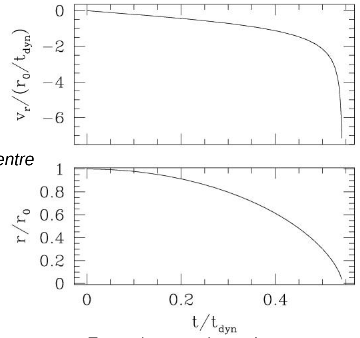
*Hence,* $t_{ff} \sim \sqrt{\frac{r_0^3}{GM_0}} \sim \sqrt{\frac{1}{G\bar{\rho}_0}}$

*$\bar{\rho}_0$: initial mean density*

**Dynamical timescale $t_{dyn}$**

*Full answer:* $t_{ff} = \sqrt{\frac{3\pi}{32}} \sqrt{\frac{1}{G\bar{\rho}_0}}$

*Can also derive as 'half an orbit'*




*Examples: star formation, supernovae*

Collapse timescale: only density matters, not size $\quad t_{dyn} \sim \sqrt{\frac{1}{G\rho}}$

| | Size | *mass* | *mean density* | $t_{dyn}$ |
|---|---|---|---|---|
| Earth | ~$10^{-2}$ $R_\odot$ | ~$10^{-6}$ $M_\odot$ | 5.5 g/cm$^3$ | |
| Jupiter | ~$10^{-1}$ $R_\odot$ | ~$10^{-3}$$M_\odot$ | 1.3 g/cm$^3$ | ~1 hr |
| Sun | $R_\odot$ | $M_\odot$ | 1.4 g/cm$^3$ | |
| White dwarf | ~$10^{-2}$ $R_\odot$ | $M_\odot$ | 1.4x$10^6$ g/cm$^3$ | ~3 s |
| Neutron star | 10 km | 1.4 $M_\odot$ | 7x$10^{14}$ g/cm$^3$ | ~0.2 ms |
| giant molecular cloud | ~100 pc | ~$10^6$ $M_\odot$ | ~$10^{-22}$ g/cm$^3$ | ~$10^7$ yr |
| globular cluster of stars | ~3 pc | ~$10^6$ $M_\odot$ | ~3x$10^{-18}$ g/cm$^3$ | ~$10^5$ yr |
| cluster of galaxies | ~3 Mpc | ~$10^{16}$ $M_\odot$ | ~$10^{-25}$ g/cm$^3$ | ~$10^8$ yr |
| the observable universe | ~14Gly | ~$10^{22}$ $M_\odot$ | ~3x$10^{-30}$g/cm$^3$ | ~$10^{11}$ yr |

Dynamical timescale:
if no support against self-gravity, object collapses in dynamical timescale.

Relevant for: solar oscillation, stellar pulsation, star formation in molecular clouds...

What prevents the Sun from collapse?

## II. Stars are supported against self-gravity by gas pressure.

Gas exerts pressure on its surrounding; it resists being compressed.

Pressure arises from kinetic energy of the gas particles;
exert force (momentum exchange) when they are reflected

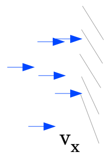
**Pressure: force/unit area**

For stars, the plasma is well described as as an Ideal Gas

**Ideal gas: no correlation between particles.**

All particles are energetically indistinguishable
mean kinetic energy per particle = 3/2 $k_B T$

*= <average over time for one particle>*
*= <average over all particles at one time>*




$P = n\, k_B\, T$ *(see textbook § 10.2)*
$n = N/V$, number density

**Non-ideal gas: correlation between particles**

particles are also waves, correlate if wavelengths overlap

1) Fermion gas $\quad P = P(n) \quad$ *electrons, protons, neutrons, quarks ("degenerate gas")*
2) Boson gas $\quad P = P(T) \quad$ *photons, gravitons, strong/weak gauge bosons*

Pressure of Ideal Gas:

$P = n\, k_B\, T$

n: number density, number of all particles per unit volume

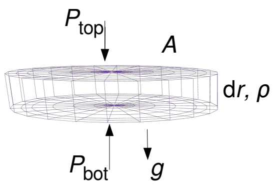
Can be written in another form using mass density ρ
(mass per unit volume)

$P = n\, k_B\, T = (\rho/\mu m_H)\, k_B\, T$

Mean molecular weight ($\mu$): $\quad \rho/n = \mu m_H$
Average mass of molecules/atoms measured
in the unit of hydrogen atomic mass

## III. Pressure support and Hydrostatic Equilibrium

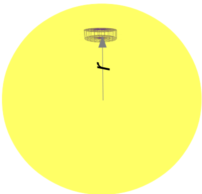
Actually need pressure gradient to support star against gravity.




$$m\, g_r = \rho\, dr\, A\, g_r = P_{bot} A - P_{top} A$$

$$P_{top} = P_{bot} + \frac{dP}{dr} dr$$

$$\text{Hence, } \boxed{\frac{dP}{dr} = -\rho\, g_r}$$




equation of hydrostatic equilibrium

*Applies to any 'fluid' (ocean, rock, air...)*

***Pressure has to increase inward***

## H.E. in the astronomer's toolkit

$$\frac{dP}{dr} = -\frac{G M_r \rho}{r^2}$$

$$P \approx \frac{GM^2}{R^4} \text{ or } P \propto M^2/R^4$$

$$\rho \approx \bar{\rho} \text{ or } \rho \propto M/R^3$$

$$\text{With ideal gas law, } P = \frac{\rho}{\mu m_H} k_B T,$$

$$kT \approx \frac{GM \mu m_H}{R} \text{ or } T \propto M/R$$