Quiz: cannibalism in close binary stars

star m_1 bloats up, part (d m) of its envelope becomes dominated by the gravity of m_2 and is transferred from m_1 to m_2

-- does the binary unbind or spiral-in?

How to estimate?

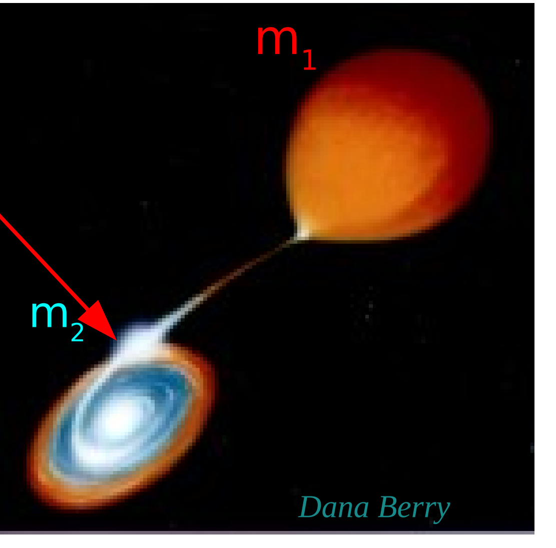
- Use mass conservation? Yes $\dot{m}_2 = -\dot{m}_1$
- Use energy conservation? No! Energy lost from binary
- Use angular momentum conservation?

For a circular orbit, $L = \frac{m_1 m_2}{M} \sqrt{GMa}$

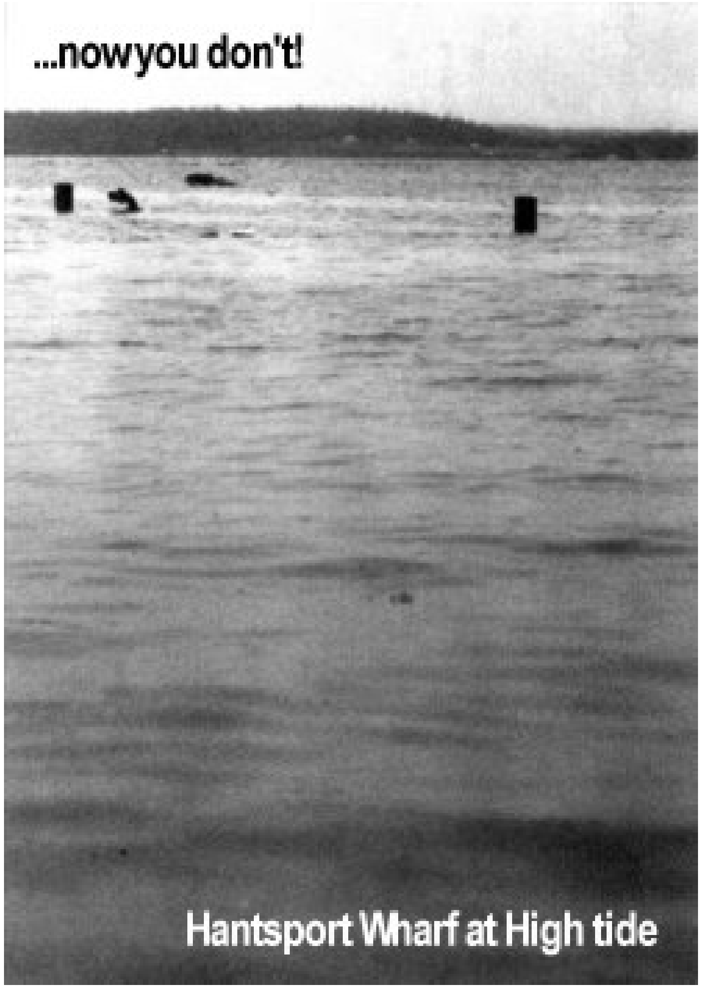
Hence, $\dot{L} = L\left(\frac{\dot{m}_1}{m_1} + \frac{\dot{m}_2}{m_2} + \frac{1}{2}\frac{\dot{a}}{a}\right) = 0$

With $\dot{m} = -\dot{m}_1 = \dot{m}_2$, *one finds* $\frac{\dot{a}}{a} = 2\frac{\dot{m}(m_2 - m_1)}{m_1 m_2}$

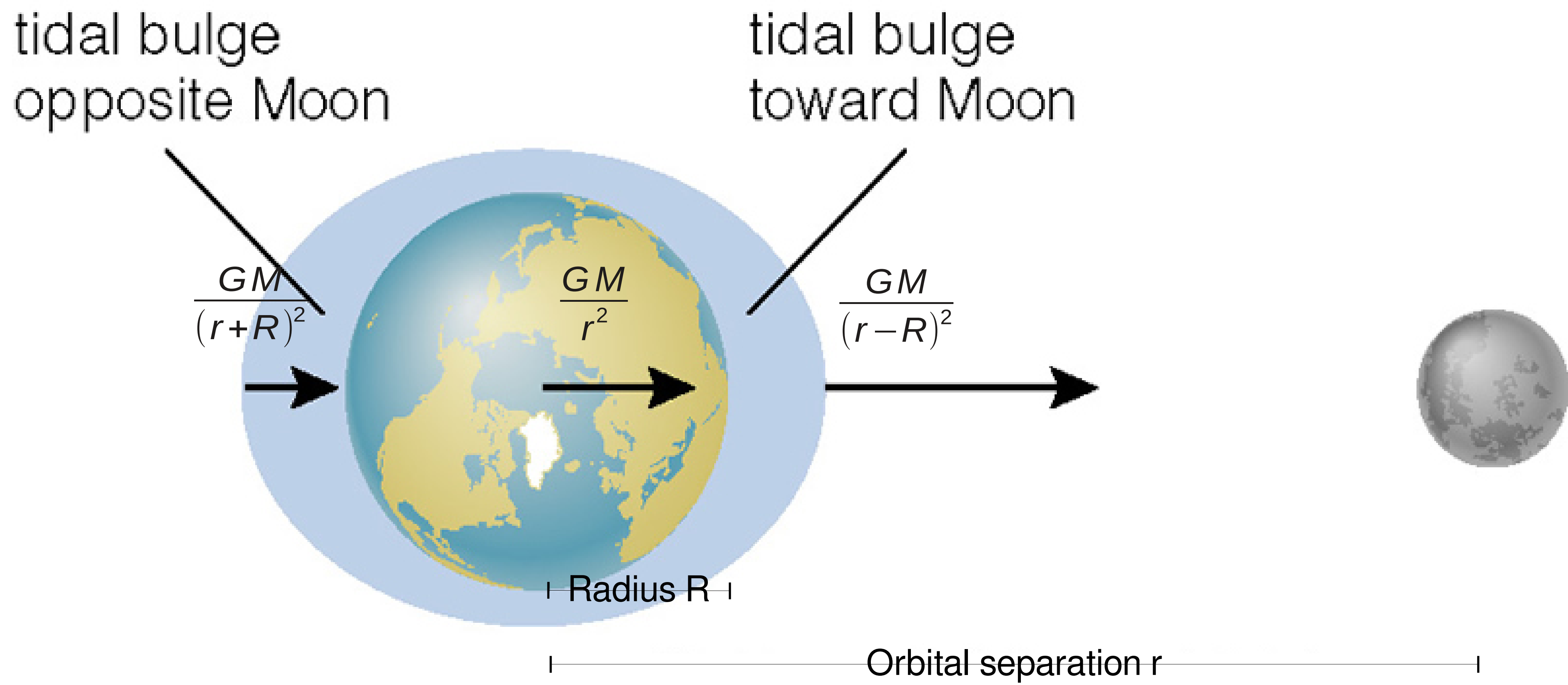
Cataclysmic Variable



Laws of Gravity III Tides

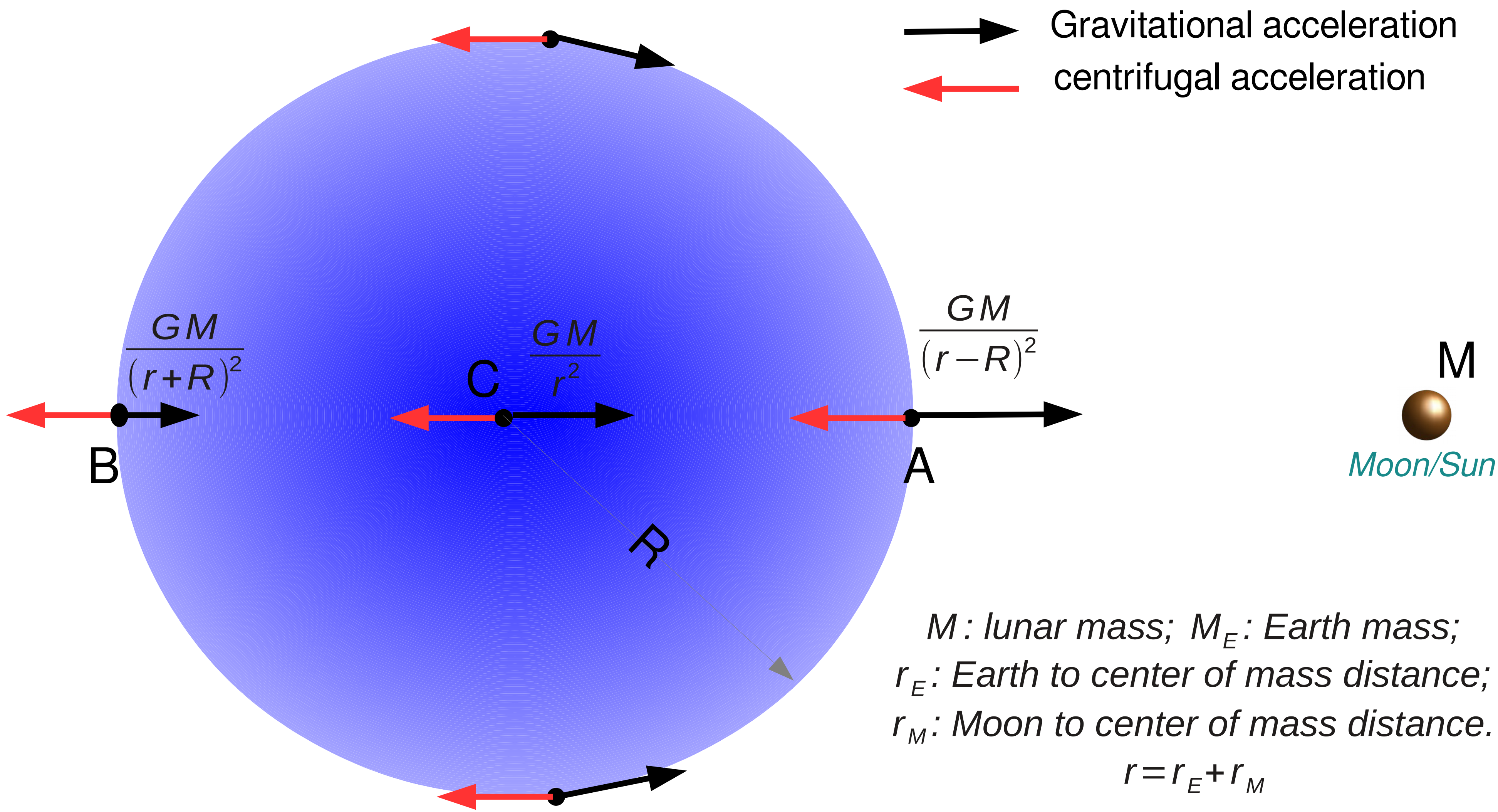


Tidal forces and the tidal bulge



The tidal acceleration is due to
differential
Lunar gravity across the Earth.

Force = Lunar gravity + centrifugal force*



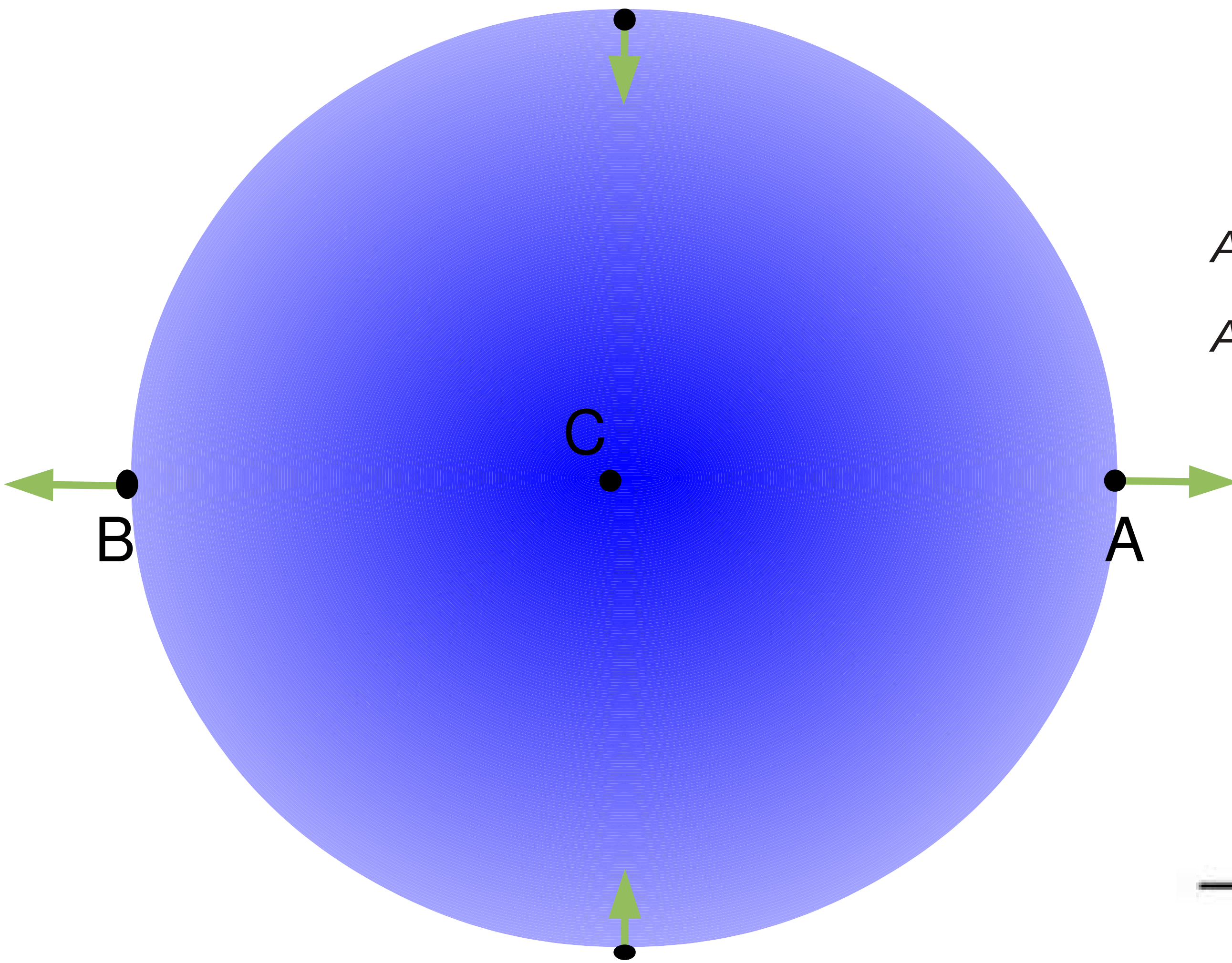
M : lunar mass; M_E: Earth mass;
r_E: Earth to center of mass distance;
r_M: Moon to center of mass distance.

$$r = r_E + r_M$$

Centrifugal accelation: $\Omega^2 r_E = \frac{GM}{r^2}$

*The centrifugal force is relevant because we are considering a rotating coordinate system fixed to Earth.

Net acceleration



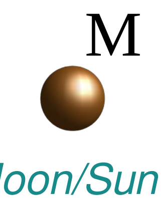
$$\text{At A:}\quad a_A = \frac{GM}{(r-R)^2} - \Omega^2 r_E$$

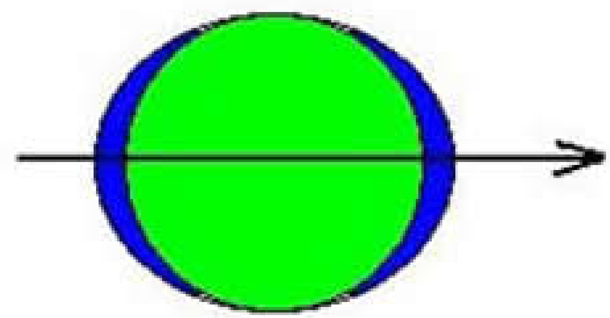
$$\approx \frac{GM}{r^2} + \frac{GM}{r^2}\frac{2R}{r} - \Omega^2 r_E$$

$$\approx \frac{GM}{r^2}\frac{2R}{r},$$

$$\text{At B:}\quad a_B \approx -\frac{GM}{r^2}\frac{2R}{r},$$

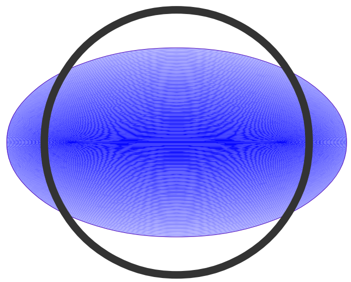
$$\text{At C:}\quad a_C = 0$$





Tidal bulges points at and away from the Moon

Tidal periods --- lunar & solar tides



Solar tides and lunar tides have comparable heights

Rotation of the Earth causes:

semi-diurnal tides	--	earth rotates every 24 hours tide rises and falls every ~ 12.4 hours dominates in atlantic coasts
diurnal tides	--	dominates in some pacific coasts

resonance of tidal forcing with ocean basin

Orbital phases of the Sun and the Moon:

'spring' tides and 'neap' tides

Orbit of the Earth causes:

fortnightly tides	--	moon's distance from us varies, e=0.055
semi-annual tides	--	earth's orbit around the Sun, e=0.017

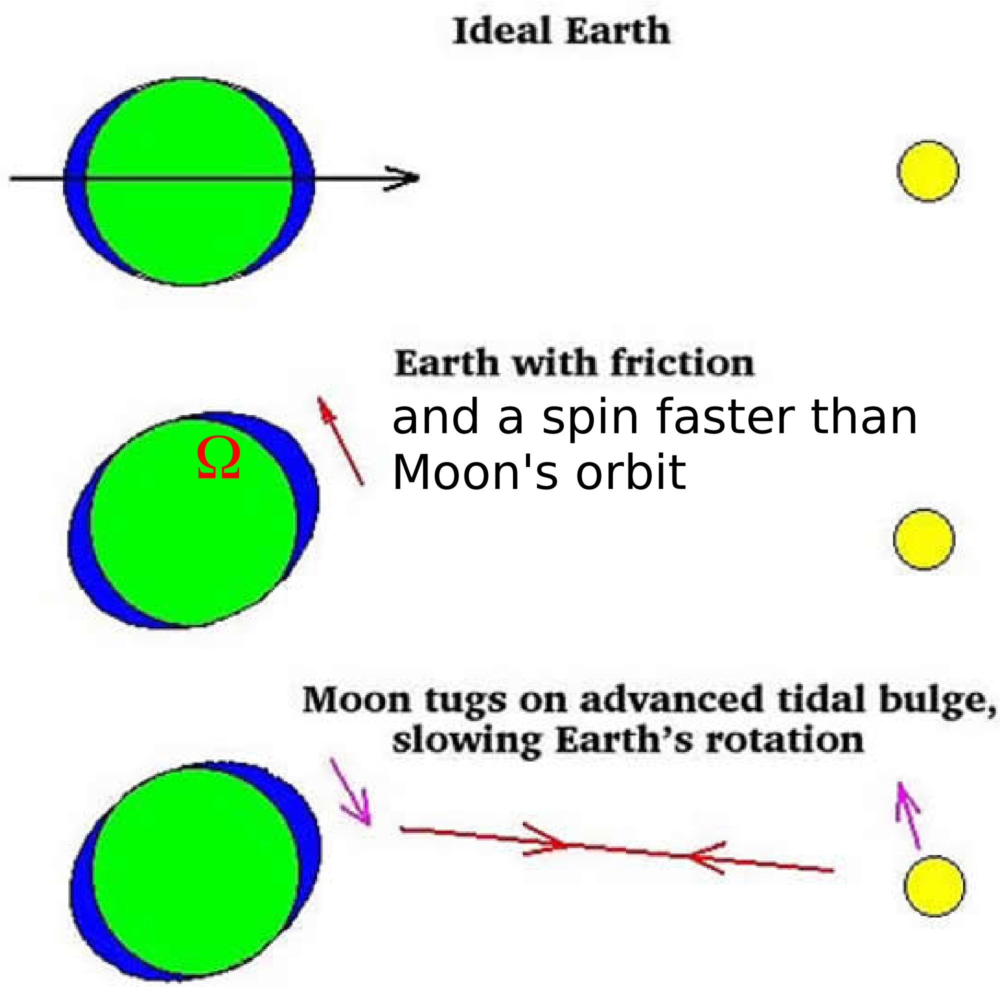
Tidal Height

Tides = ocean tide + atmosphere tide + solid tide

Over most of the world, ocean tide ~ 0.7 metres,
balancing enhanced self-gravity (due to tidal bulge) with
the tidal acceleration *(see extra note at end for an order-of-magnitude estimate)*

Bay of Fundy: tidal height ~ 9 m (highest in the world)
Nearby PEI: ~ 2.5 m

Tidal Evolution



Locally (observer on Earth)

tidal sloshing
energy dissipation (into heat)

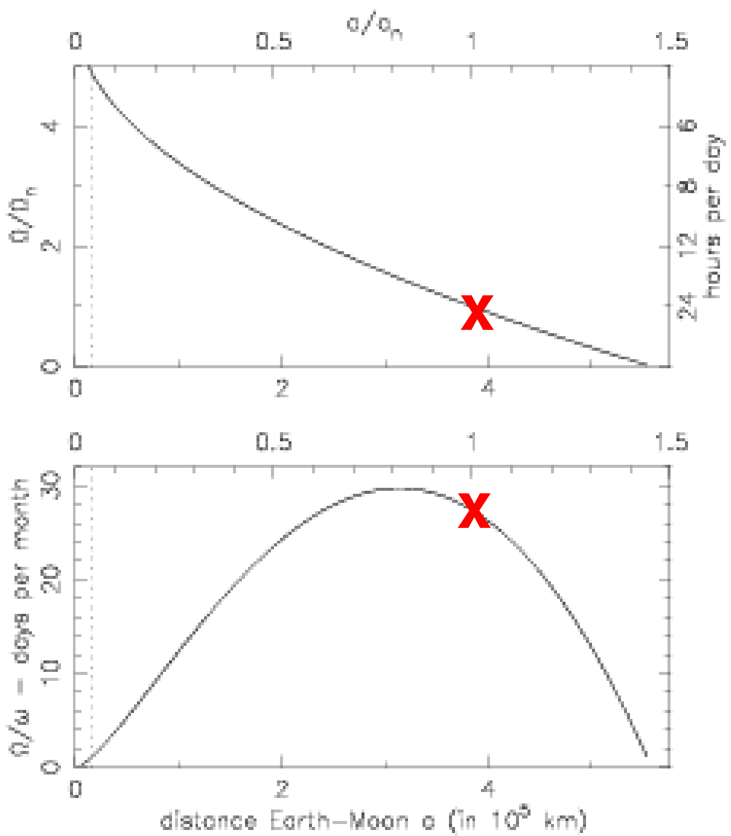
Globally

tidal bulges lead Earth-Moon line
lunar torque on bulges
- slows Earth's spin;
- pushes away Moon

Total energy (orbit + spin) steadily decreases over Gyr timescale, while the total angular momentum (orbit + spin) is conserved.

Final state: synchronised & circularised

Tidal Evolution Earth-Moon system



(F.Verbunt)

As a result of tidal dissipation:

Earth is spinning down

--angular momentum transfer--

and the Moon is receding

Observable Consequences:

lengths of day & month are increasing

number of days in a month is decreasing

Evidence from:

laser ranging,

historical eclipse records,

coral & nautilus fossil,

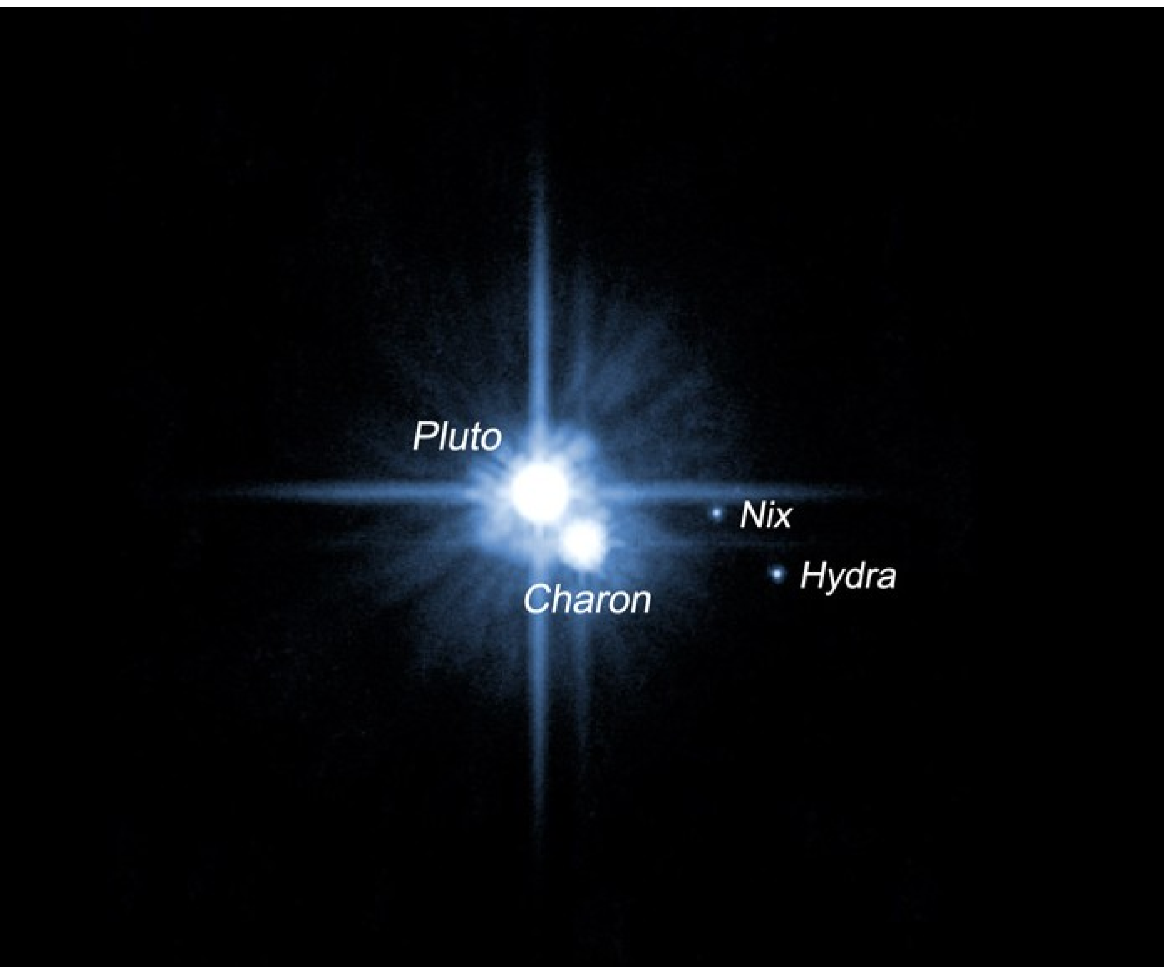
mud deposit

For an excellent review (by F. Verbunt), see www.astro.utoronto.ca/~mhvk/AST221/verbunt.pdf

Tidal Evolution final state

Pluto & Charon
Orbital period: 6.387 days
Pluto spin period: 6.387 days
Charon spin period: 6.387 days
e=0

Earth-Moon:
orbital period: 27.32 days
Earth spin period: 1 day
Moon spin period: 27.32 days
e = 0.05



Extra Note: Taylor expansion
-- used to derive tidal acceleration
See also http://en.wikipedia.org/wiki/Taylor_series

Generally, any function $f(x)$ *can be Taylor-expanded around some* x_0

$$f(x)=f(x_0)+(x-x_0)f'(x_0)+\frac{1}{2}(x-x_0)f''(x_0)...$$

$$\text{where } f'(x)\equiv\frac{d\,f(x)}{dx}\text{, etc.}$$

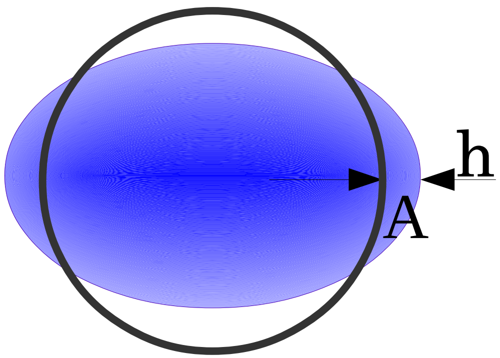
$$\text{E.g., expanding } f(x)=\frac{1}{1+x} \text{ around } x_0=0,$$

$$f(x)=1+x\left[\frac{-1}{(1+x)^2}\right]_{x=0}...\simeq 1-x \text{ to first order}$$

$$\text{Similarly, }\frac{GM}{(r-R)^2}=\frac{GM}{r^2(1-R/r)^2}=\frac{GM}{r^2}\frac{1}{1-2R/r+(R/r)^2}$$

$$\simeq\frac{GM}{r^2}(1+2R/r+(R/r)^2)\simeq\frac{GM}{r^2}(1+2R/r)$$

Extra Note: Height of the Tidal Bulges
-- an *order-of-magnitude* estimate



$$\textit{Tidal acceleration} \sim 2\frac{GM}{r^2}\left(\frac{R_E}{r}\right) \sim 2\frac{M}{M_E}\left(\frac{R_E}{r}\right)^3 g_E$$

Tidal bulge generates additional gravity
to balances the tidal acceleration

$$g' \sim \frac{G\left(M_E + \rho\, h\, R_E^2\right)}{R_E^2} - \frac{GM_E}{R_E^2}$$

$$\textit{with } \rho \sim \rho_E, \quad g' \sim \frac{GM_E}{R_E^2}\frac{h}{R_E} \sim \frac{h}{R_E} g_E$$

Equating the two, we obtain $\boxed{\frac{h}{R_E} \sim 2\frac{M}{M_E}\left(\frac{R_E}{r}\right)^3}$

factor of order unity correction
from using correct density, etc.

Moon raises tide on earth ~ 10^{-7} R_E	~ 60 cm
Sun on Earth	~ 25 cm
Earth on Moon	~ 1.7 m
Earth on Sun	~ (you do it!)

Extra Note: Tidal evolution

muscle flexing --> heat generated, tidal sloshing --> heat

Total energy (orbit + spin) is steadily decreasing over Gyr timescale,

$$E_{tot} = E_{orb} + E_{rot} = -\frac{G M M_E}{2a} + \frac{1}{2} I_E \Omega_E^2 + \frac{1}{2} I_M \Omega_M^2$$

$$\textit{Moment of inertia: } I_E = r_{g,E}^2 M_E R_E^2$$

$$\textit{Spin frequency: } \Omega_E \equiv \frac{2\pi}{P_E}$$

while the total angular momentum (orbit + spin) is conserved,

$$L_{tot} = \frac{M M_E}{M + M_E} \sqrt{G(M + M_E) a (1 - e^2)} + I_E \Omega_E + I_M \Omega_M$$

$$\textit{Final minimum energy state: } \frac{\partial E_{tot}}{\partial \Omega_M} = \frac{\partial E_{tot}}{\partial \Omega_E} = \frac{\partial E_{tot}}{\partial e} = 0$$

$$\textit{Hence, } \Omega_M = \Omega_E = \omega, \quad e = 0$$

synchronised & circularised

Currently: $\Omega_M = \omega, \quad e \approx 0, \quad \Omega_E \approx 27\,\omega$ *(free energy from Earth's spin)*