Dating stars using Lithium (due Feb 9)

Marten van Kerkwijk

January 26, 2022

1 The Lithium Depletion Boundary

As pre-main-sequence stars contract, at some point their internal temperature becomes sufficiently hot that Lithium is destroyed. This will occur earlier for more massive stars, and hence one can estimate an age for a group of stars by finding the highest-mass star that still has Lithium.

In this problem set, we will derive this so-called *Lithium Depletion Boundary* in a simple, nearly analytic way. We will make four assumptions: (i) that the stars are completely convective and can be described by the appropriate, $n = 1.5$ polytrope; (ii) that convective mixing ensures lithium is depleted throughout the star; (iii) that the equation of state is that of an ideal gas; and (iv) that the surface temperature remains constant during contraction.

Note: This problem set follows Bildsten et al., `http://adsabs.harvard.edu/abs/1997ApJ...482..442B`, which in turn is based on a problem set he posed his class [you do not need the reference, but feel free to look it up if you want to]. The first three parts below follow AST 320 mini problem sets VIII, IX, and X, but given at a higher level, e.g., asking simply to derive rather than to show a derivation gives a certain result. I think you'll learn most if you try up front, but if you get stuck, feel free to look at the AST 320 sets to see if that helps to keep you going.

2 Contraction

Use the above assumptions to derive $R(t)$ and $T_c(t)$, the way the stellar radius and central temperature change with time. Specifically, calculate E for a completely convective star, use that $L = dE/dt$ to derive dR/dt, and integrate. Scale to useful numbers: $R/R_\odot$, $M/0.1M_\odot$, $T_{\rm eff}/3000\,{\rm K}$, and $T_c/10^6\,{\rm K}$.

3 Scalings

Lithium is destroyed by fusion with a proton: ${}^7\mathrm{Li} + \mathrm{p} \to 2\,{}^4\mathrm{He}$. The velocity-averaged interaction cross-section $\langle\sigma v\rangle$ is an extremely steep function of temperature, and we approximate it with a power law of the form $\langle\sigma v\rangle_r(T/T_r)^\nu$ around a reference temperature T_r.

- The total number of Lithium atoms will decline as

$$\frac{dN_{\rm Li}}{dt} = -\int_r n_{\rm Li} n_{\rm H} \langle\sigma v\rangle 4\pi r^2 dr.$$

  Scale the integrand relative to the center and the stellar radius, and write the remaining non-dimensional integral in terms of polytropic variables ξ and θ (w, z in KWW). Show that

$$\frac{d\ln X_{\rm Li}}{dt} = -\frac{X\rho_{\rm c}}{m_{\rm H}}\langle\sigma v\rangle_r \left(\frac{T_{\rm c}}{T_{\rm r}}\right)^\nu \times \frac{3(\rho_{\rm c}/\overline{\rho})}{\xi_1^3}\int_\xi \xi^2\theta^{2n+\nu}d\xi.$$

- Consider two stars that differ in mass by a factor two. How much sooner would the more massive star start depleting Lithium for the case that $\nu \to \infty$? And what would it be for $\nu \to 0$? (Ensure you understand this physically!)

4 Dating stars

To derive actual depletion times, we need to insert actual numbers for fusion rate and to evaluate the integral.

- Bildsten et al. list a reaction rate $N_{\rm A}\langle\sigma v\rangle = S_0 f_{\rm scr} T_6^{-2/3}\exp(-aT_6^{-1/3})$, where $N_{\rm A} \equiv 1/m_{\rm H}$, $T_6 \equiv T/10^6\,{\rm K}$ and $S_0 = 7.2\times10^{10}$, $f_{\rm scr} \simeq 1$, and $a = 84.72$ (check you can reproduce a). Find $\langle\sigma v\rangle_r$ and ν for a reference temperature $T_r = 3\times10^6\,{\rm K}$ (check for yourself that the power-law approximation is reasonable at, e.g., 2.5 and $3.5\times10^6\,{\rm K}$).

- Do the integral over ξ two ways, (i) using your polytrope integrator, and (ii) using the second-order approximation $\theta \approx 1 - \xi^2/6$ near the center (why is this reasonable?). For the latter, it will help to know that $\int_x x^2(1-x^2)^{b-1}dx = \frac{1}{2}{\rm B}(3/2, b) \approx \frac{1}{2}\Gamma(3/2)b^{-3/2} = \frac{1}{2}(\sqrt{\pi}/2)b^{-3/2}$ (where the approximation holds for large b).

- Now determine the time required to reduce $X_{\rm Li}$ by a factor 2. (Would it matter much to take the time required to deplete by a factor 10 instead of a factor 2?)

Sanity checks

- Compare with the numerical result of Bildsten et al.,

$$t_{\rm depl} = 50.7\,{\rm Myr}(M/0.1\,M_\odot)^{-0.663}(T_{\rm eff}/3000\,{\rm K})^{-3.50}(\mu/0.6)^{-2.09}.$$

- Jeffries and Oliviera `http://adsabs.harvard.edu/abs/2005MNRAS.358...13J` find that in NGC 2457, stars with mass less than $0.17M_\odot$ still have Lithium, and they infer an age of about 34 Myr. Does this match what you would infer?

5 Validity limits

Stars with masses below $0.06\,M_\odot$ do not destroy Lithium, while stars with masses above about $0.5\,M_\odot$ destroy Lithium only on the main-sequence. Think why this would be (or, more specifically, which of the four assumptions breaks down). Then, estimate the limits yourself.

6 Applications

Not for the PS, but more for fun

- Replacing the above integral with one over ϵ, you can derive for yourself the brown dwarf limit, i.e., above what mass stars get hot enough for nuclear fusion to match their luminosity.

- You can also reproduce the luminosity of the Sun, though this needs some care: its structure is that of a $n = 3$ polytrope, i.e., $\theta = (\rho/\rho_c)^{1/3}$, implying $P \sim \rho^{\gamma_1}$ with $\gamma_1 = 4/3$, but the temperature depends on density as $T/T_c = (\rho/\rho_c)^{\gamma_3 - 1}$, with $\gamma_3 \approx 5/3$; thus, for the energy generation rate one gets $\epsilon/\epsilon_c = (\rho/\rho_c)^\lambda (T/T_c)^\nu = \theta^{n(\lambda+\nu(\gamma_3-1))}$.

- I used it myself to derive a carbon-burning luminosity in a white dwarf (following Woosley `http://adsabs.harvard.edu/abs/2004ApJ...607..921W`).