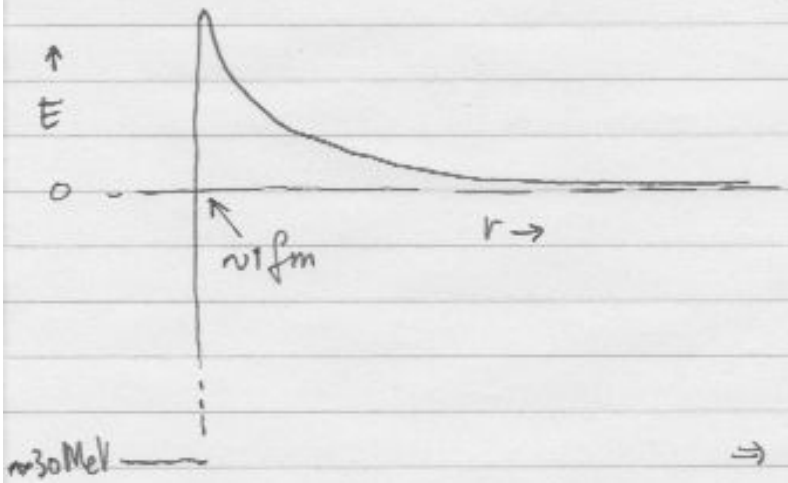
Fusion → Tunneling



To get to ~1 fm

$$\Rightarrow E_{Coul} = \frac{Z_1 Z_2 e^2}{r} \approx Z_1 Z_2 \text{ MeV}$$

Problem: $kT \approx 1 \text{ keV} \left(T/10^7 \text{K}\right)$

III

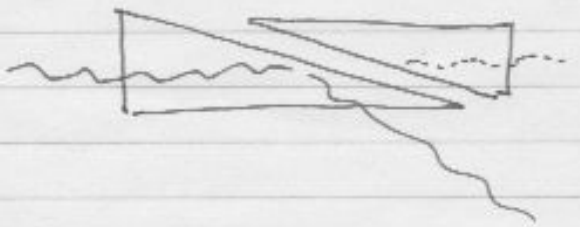
T_7

$\Rightarrow$ factor 1000 too small

only get to $r_{Coul} = \frac{Z_1 Z_2 e^2}{E} \approx Z_1 Z_2$ pm

Tunneling

Example: 2 prisms



Qualitative: compare r_{coul} w/ "size" of nucleus

(Uncertainty principle: given low velocity cannot know position precisely.)

$$\Delta x = \frac{\hbar}{\Delta p} \approx \frac{\hbar}{\sqrt{2mE}}$$

$$\Rightarrow P \propto e^{-r_{coul}/\Delta x} \sim e^{\frac{Z_1 Z_2 e^2}{E} / \hbar/\sqrt{2mE}} \sim e^{-Z_1 Z_2 e^2 / \sqrt{E/2m}\,\hbar}$$

Exponent: $\frac{Z_1 Z_2 e^2}{1 \text{ fm}} \sim Z_1 Z_2 \text{ MeV}$

OR

$$\frac{\sqrt{kT}}{\frac{\hbar}{1 \text{ fm}\sqrt{2m}}} \sim \sqrt{\sim 1 \text{keV}\, T_7}\ \sqrt{\sim 1650 \text{ MeV}}$$

≈ -1

e^{-1} ? TOO FAST ?

NOTE: generally only one stage at a time

Too fast ?

Reaction prob. needed is $\sim e^{-10}$, otherwise too fast

$\Rightarrow$ H fusion at $T_7 = 0.1^2 = 0.01 \Rightarrow T = 10^5 k$? No!

But: do have other fusion at lower temperatures!

Problem set: ${}^1_1H + {}^7_3Li \rightarrow 2\,{}^4_2He$ at $\sim 10^6 k$

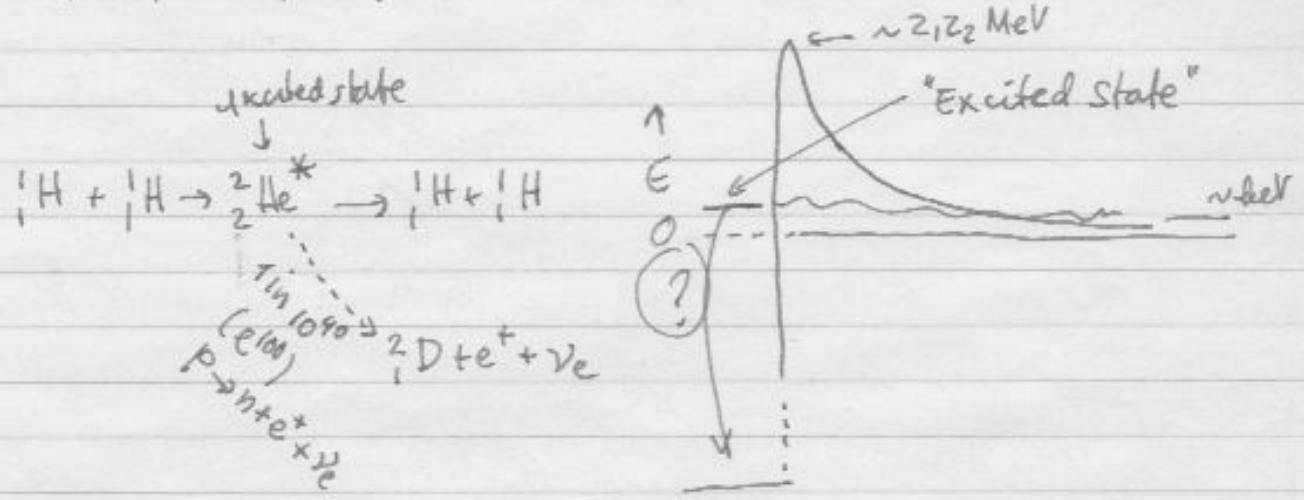
$$\underset{D}{\text{exponent}} - \frac{Z_1 Z_2}{T_7^{1/2}} \approx -10 \Rightarrow \text{ok}$$

Also, ${}^1_1H + {}^2_1H \rightarrow {}^3_2He + \gamma$ at $10^5 k$ $\rightarrow$ also ok

$\hookrightarrow$ distinction between planets & brown dwarfs

Indeed, at $10^7 k$ ${}^1_1H + {}^{12}_6C \rightarrow {}^{13}_7N + \gamma$ works!

Then, why not ${}^1_1H + {}^1_1H$? Need to tunnel AND to react!



For many reactions, resonances help to make success rate close to unity at specific energies

p-p chain

$^1_1H + {}^1_1H \rightarrow {}^2_1D + e^+ + (\nu_e)$ → what happens to the neutrino?

very hot for ^2_1D $\quad {}^2_1D + {}^1_1H \rightarrow {}^3_2He + \gamma$

next? $\quad {}^1_1H + {}^3_2He \rightarrow {}^4_3Li$? → no, immediately falls apart

He + He? $\quad -\frac{Z_1 Z_2}{T_+^{1/2}} \approx -4 \Rightarrow$ easily Ok

$^3_2He + {}^3_2He \rightarrow {}^4_2He + 2\,{}^1_1H$ pp 1

$^3_2He + {}^4_2He \rightarrow {}^7_4Be + \gamma$

$^7_4Be + e^- \rightarrow {}^7_3Li (+\nu)$

$^7_3Li + {}^1_1H \rightarrow {}^4_2He + {}^4_2He$

pp 2

$^7_4Be + {}^1_1H \rightarrow {}^8_5B + \gamma$

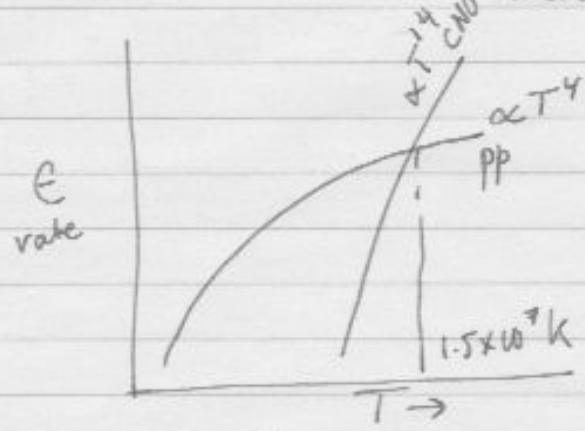
$^8_5B \rightarrow {}^8_4Be + e^+ + \nu$

$^8_4Be \rightarrow 2\,{}^4_2He$

^8_4Be ↑ unstable

${}^4_2He$ ↖ very stable 2p 2n in inner shell

Slowest step?



CNO cycle

$^{12}_{6}C + {}^{1}_{1}H \rightarrow {}^{13}_{7}N + \gamma$ stable? more p than n → no

$^{13}_{7}N \rightarrow {}^{13}_{6}C + e^{+} + \nu_e$

$^{13}_{6}C + {}^{1}_{1}H \rightarrow {}^{14}_{7}N + \gamma$

$^{14}_{7}N + {}^{1}_{1}H \rightarrow {}^{15}_{8}O + \gamma$

$^{15}_{8}O \rightarrow {}^{15}_{7}N + e^{+} + \nu_e$

$^{15}_{7}N + {}^{1}_{1}H \rightarrow {}^{12}_{6}C + {}^{4}_{2}He$

$\rightarrow {}^{16}_{8}O + \gamma$

$^{16}_{8}O + {}^{1}_{1}H \rightarrow {}^{17}_{8}F + \gamma$

$^{17}_{9}F \rightarrow {}^{17}_{8}O + e^{+} + \nu_e$

$^{17}_{8}O + {}^{1}_{1}H \rightarrow {}^{14}_{7}N + {}^{4}_{2}He$ → Oxygen also used in CNO cycle as catalyst

$\rightarrow {}^{18}_{9}F + \gamma$

Slowest step? Highest charge: N

largest reduced mass: ^{15}N

But small difference in change of success ⇒ ^{14}N

⇒ most of C, N, O turned into $^{14}_{7}N$