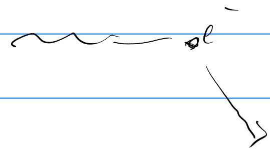
# Opacity sources

$e^-$ scattering

big $T$ } → everything ionized
low $\rho$ }



effect on energy transport ⇒ like a random walk / diffusion.

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 6.65\times10^{-29}\ m^2$$

indep of freq as long as $h\nu \ll m_e c^2$

$$\kappa_{es} = \sigma_T \frac{1+X}{2m_H} = $$

$$= 0.0200\,(1+X)\ m^2/kg$$

H: $1e^-$/nucleon → 1

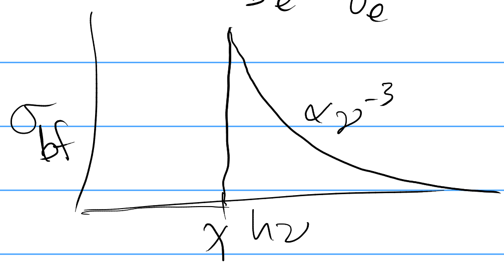
He: $2e^-$/4 nucleons
C, O
$^{56}_{26}Fe$: $26e^-$/56 nucleon
→ $\frac{1}{2}$

## Bound-free

$$h\nu = \chi + \tfrac{1}{2}m_e v^2$$

$$\sigma_{bf} \propto \frac{Z^4}{n^5\nu^3}$$

($n$ ← state)




From Saha eq: most material in higher ionization state

H: $\chi_{HI} = 13.6\,eV$, yet ionized already at $T \simeq 10000\,K$ $\approx 1\,eV$

at $T = 10^5\,K$, for pure Hydrogen

$$n_{H^0} \propto n_p n_e \left(\frac{h^2}{2\pi m_e kT}\right)^{3/2} e^{\chi/kT}$$

⇒ bf opacity scales w/ $\rho^2$

approximate total over all elements for interior

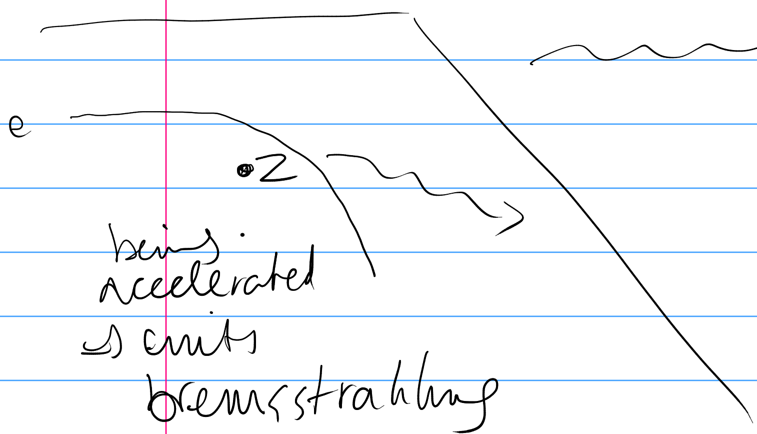
$$\propto Z\,(1+X)\,\rho\,T^{-7/2}$$

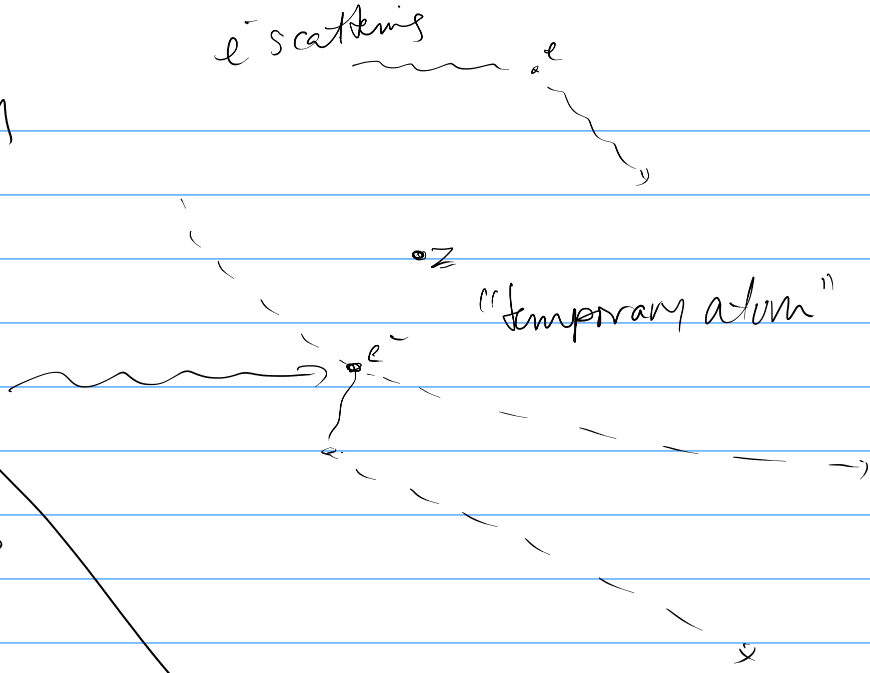
$Z$: because H & He are fully ionized

$(1+X)\rho$: $n_e$ density

$T^{-7/2}$: $\nu^3$, from Saha ion.

free-free opacity



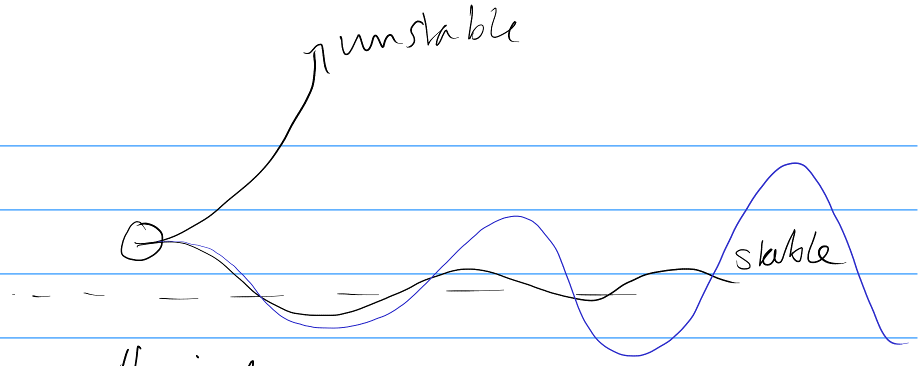




$$\sigma_{ff} \propto \frac{Z^2}{\nu^3} n_e$$

add all elements $\quad n_{ion} Z^2 = \sum_i \frac{\rho X_i}{m_H A_i} Z_i^2 = \frac{\rho}{m_H}\left(X \frac{1^2}{1} + Y\frac{2^2}{4} + \underbrace{\sum_i X_i \frac{Z_i^2}{A_i}}_{B}\right)$

$$\sigma_{ff} \propto \underbrace{\rho(1+X)}_{n_e} \overbrace{(X+Y+B)}^{n_{ion} Z^2/\rho} \underbrace{T^{-7/2}}_{\text{same as for b-f}}$$

# Convective flux




bottom line: c.t. is very efficient
$\Rightarrow$ net temperature gradient that star endres to is nearly adiabatic

Flux: $F_{conv} = \rho \bar{v}_{conv} \overbrace{\Delta q}^{\text{excess energy carried by blob}} = \rho \bar{v}_{conv} C_p \Delta T$

$\bar{v}_{conv}$: some average velocity

$\Delta T$ after traveling some relevant distance where the blob dissolves
Call it $\ell_{mix} = \alpha_{mix} H_p$

$\alpha_{mix}$: measure of our ignorance
$H_p$: pressure scale height

$$= \rho \bar{v}_{conv} C_p T \frac{\ell_{mix}}{2H_p} \underbrace{(\nabla - \nabla_{ad})}_{\text{what we want to know:}}$$

$\nabla$: true temperature gradient in the surrounding

## How to estimate the velocity

$$-\underset{\text{area}}{A} \rho v^2 = \vec{F}_{drag}$$

$$\vec{F}_{buoyance} = V g \Delta \rho = g \rho V \frac{\Delta T}{T}$$

define $\frac{V}{A} = \beta \ell_{mix}$

$$\Rightarrow v_{conv}^2 = \frac{\beta g}{H_p} \frac{\ell_{mix}^2}{2} (\nabla - \nabla_{ad})$$

$$\Rightarrow F_{conv} = \rho C_p T \alpha_{mix}^2 \sqrt{\frac{\beta g H_p}{8}} (\nabla - \nabla_{ad})^{3/2}$$

$$\frac{L_\odot}{4\pi R_\odot^2} = F_{conv} \Rightarrow \nabla - \nabla_{ad} \text{ very small}$$

<u>except</u> near photosphere