HE $\quad \frac{dP}{dr} = -\frac{GM_r}{r^2}\rho \Rightarrow \frac{P_c}{R} \approx \frac{GM}{R^2}\rho \approx \frac{GM^2}{R^5}$

$$P_c \approx C_{const} \frac{GM^2}{R^4}$$

disturb it $\Rightarrow$ R a little smaller $\rightarrow$ homologously

to be stable: real $P \propto R^{-4}$ or steeper

what we know: $\rho \propto R^{-3}$

ideal gas $P \propto \rho T$ isothermal $P \propto \rho \propto R^{-3}$

not steep enough $\Rightarrow$ unstable

adiabatic $\quad P \propto \rho^{\gamma} \propto R^{-3\gamma}$

$\gamma \leftarrow$ adiabatic index

stability requires $\boxed{\gamma > \frac{4}{3}}$

ideal gas $\gamma = \frac{5}{3} \Rightarrow$ stable

dominated by radiation pressure

$$P = \frac{1}{3}aT^4$$

compress it $\quad n_\gamma \propto R^{-3}$

mean energy of photons $e_\gamma \propto R^{-1}$

$\Big\}\ P \propto R^{-4}$ marginally stable

$$P_{true} = P_{rad} + P_{ideal}$$

$\Rightarrow$ need to have some suitable average of γ (can have small isothermal core)

$\Rightarrow$ how to get $\gamma < \frac{4}{3}$?

Ionisation zone $\rightarrow$ mix of H^0, p^+, e^-

compress a little $\rightarrow T\uparrow \Rightarrow$ ionize more H^0

$\rightarrow$ costs energy $\Rightarrow T\downarrow$ a little $\Rightarrow$ T not up as fast

$\Uparrow$ γ lower

Timescales

Dynamical timescale

$g = \frac{GM}{R^2}$

not in HE: $\rho \frac{\partial v}{\partial t} = \rho \frac{\partial^2 r}{\partial t^2} = -\nabla P + \rho \nabla \phi = -\frac{\partial P}{\partial r} - \rho g$

ignore P: $\rho \frac{\partial^2 r}{\partial t^2} = -\rho g \Rightarrow \frac{R}{t_{ff}^2} \approx -g$

$\Rightarrow t_{ff} \approx \sqrt{\frac{R}{g}} \approx \frac{1}{\sqrt{G\rho}}$ $\qquad \rho_\odot \approx 1\,g/cm^3 \Downarrow t_{ff} \approx 1\,hr$

ignore gravity $\rho \frac{\partial^2 r}{\partial t^2} = -\frac{\partial P}{\partial r}$

$\rho \frac{R}{t_{expl}^2} \approx \frac{P}{R} \Rightarrow t_{expl} \approx R\sqrt{\frac{\rho}{P}} \approx R/c_s$

$P \approx \frac{GM}{R}\rho \Rightarrow t_{expl} \approx \frac{1}{\sqrt{G\rho}}$

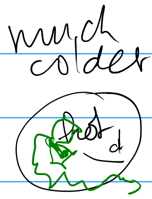
HE: fast $\Rightarrow$ safe to assume stars are in HE

Thermal equilibrium

$NkT \approx \frac{M}{\mu m_H} kT$

$t_{th} = E_{th}/L \approx E_{pot}/L \approx \frac{GM^2}{RL}$ $\qquad \odot$: few 10^7 yr

no TE in a small spot, over some distance d



$F = -\frac{1}{3} v l_{mfp} \nabla U \approx \frac{1}{3}\frac{c}{\kappa\rho}\frac{U}{d}$

$\tau_{adj} = \frac{U d^3}{F d^2} = \frac{E_{blob}}{L_{out\,of\,blob}}$

$= \frac{3 U d^{\cancel{3}\,2}}{v l_{mfp} \cancel{U d}} \approx \frac{3d^2}{v l_{mfp}} \approx \underbrace{\left(\frac{l_{mfp}}{v}\right)}_{t_{step}\text{ in randomwalk}}\underbrace{\left(\frac{d}{l_{mfp}}\right)^2}_{N_{steps}}$

Nuclear timescale

$\eta \approx \frac{1\,MeV/nucleon}{m_{nucleon}c^2} \approx 0.1\%$ $\approx GeV$ $\qquad t_{nuc} = \frac{E_{nuc}}{L} = \frac{\eta M_{fuel} c^2}{L}$

Polytropes $\qquad P \propto \rho^{\gamma}$ adiabatic expansion
contraction

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$P = K\rho^{\gamma}$$

polytropic equation
of state

$$\frac{dP}{dr} = -\frac{GM_r}{r^2}\rho$$

normally $P = P(\rho, T, \text{abundances})$

$$P = K\rho^{1+\frac{1}{n}}$$ ← polytropic index

$\gamma = \frac{5}{3} \Rightarrow n = 1.5$

$\gamma = \frac{4}{3} \Rightarrow n = 3$

$$P_c = \text{Const}_1 \frac{GM^2}{R^4}$$

$$\rho_c = \text{Const}_2 \, M / \tfrac{4}{3}\pi R^3$$