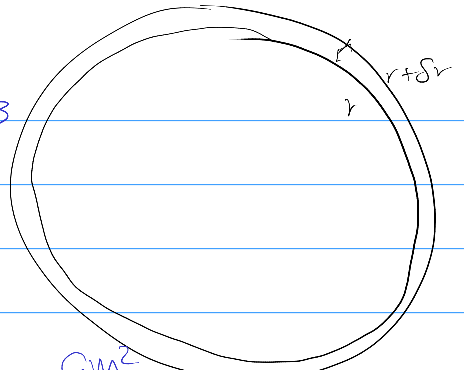
MC $\quad \frac{\partial m}{\partial r} = 4\pi r^2 \rho \Rightarrow \frac{M}{R} \approx R^2 \rho \Rightarrow \rho \propto M/R^3$



HE $\quad \frac{\partial p}{\partial r} = -\frac{G m \rho}{r^2} = -g_r \rho$

$$\Rightarrow \frac{P}{R} \approx \frac{G M \rho}{R^2} \Rightarrow p \approx \frac{GM^2}{R^4} \propto M^2/R^4$$

$$P = \frac{\rho}{\mu m_H} k T \Rightarrow T \propto \frac{P}{\rho} \propto \frac{GM^2/R^4}{M/R^3} \propto \frac{GM}{R}$$

$$\Rightarrow \boxed{T \propto \frac{M}{R}}$$

opacity: m²/kg ; luminosity at radius r

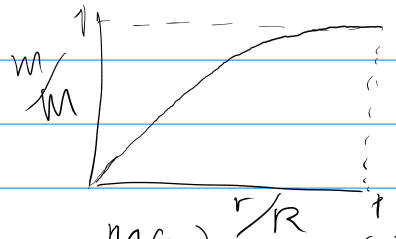
TE $\quad \frac{\partial T}{\partial r} = -\frac{3}{4ac} \frac{k\rho}{T^3} \frac{l}{4\pi r^2}$

$$\Rightarrow \frac{T}{R} \approx \frac{\langle k \rangle \rho}{ac T^3} \frac{L}{R^2} \Rightarrow T^4 \propto \frac{k \rho L}{R}$$

$$\boxed{T^4 \propto \frac{k M L}{R^4}}$$

$$\frac{M^4}{\cancel{R^4}} \propto \frac{\langle k \rangle M L}{\cancel{R^4}} \Rightarrow \boxed{L \propto \frac{M^3}{\langle k \rangle}}$$

$k(\rho, T, \text{abundances})$



Homology $\quad P_c \propto M^2/R^4$

↳ 2 stars have the same structure $\quad \frac{m_1(r)}{M_1} = \frac{m_2(r)}{M_2}$

$$\Rightarrow \frac{P_1(r/R)}{P_2(r/R)} = \frac{(M_1/M_2)^2}{(R_1/R_2)^4}$$

$$\frac{\rho_1(r/R)}{\rho_2(r/R)} = \frac{M_1/M_2}{(R_1/R_2)^3}$$

$T \propto M/R$ $\quad$ $T \approx$ const $\Rightarrow R \propto M$

$P \propto M^2/R^4 \propto M^{-2}$, $\rho \propto M/R^3 \propto M^{-2}$

$T_{c,\odot} = 1.57 \times 10^7$ K

$P_{c,\odot} = 2.34 \times 10^{16}$ N/m^2

$\rho_{c,\odot} = 1.53 \times 10^5$ kg/m^3

$P_{rad} = \frac{1}{3} a T^4 = 1.6 \times 10^{13}$ N/m^2

$\frac{P_{rad}}{P_{c,\odot}} \simeq 10^{-3}$

equal at $M \simeq \sqrt{1000} = 30 M_\odot$

$\Delta x \cdot \Delta p = \hbar$ $\quad$ $n_e \approx \frac{\rho}{m_H} \simeq 10^{32}$ m^{-3}

$\Rightarrow \Delta x \approx n_e^{-1/3} \simeq 2 \times 10^{-11}$ m

$\Delta p \simeq \frac{\hbar}{\Delta x} \approx 5 \times 10^{-24}$ kg m/s

ideal gas
$e = kT = 2 \times 10^{-16}$ J $\quad$ $e \approx \frac{p^2}{2m_e} \simeq 10^{-17}$ J $= \frac{1}{20} kT$

$n_e \propto \rho \propto M^{-2} \Rightarrow \Delta x \propto M^{2/3} \Rightarrow p \propto M^{-2/3}$

$\Rightarrow e \propto M^{4/3}$

$\Rightarrow e_{deg} = e_{th}$ at $\frac{M}{M_\odot} = 20^{-4/3} \simeq 0.02 M_\odot$

$L \propto \frac{M^3}{\langle \kappa \rangle}$

high masses: add P_{rad}
$\Rightarrow$ lower than expected T is enough to support star
$\Rightarrow$ L, M less steep

very high mass $L \propto M$

lower mass: add deg
$\Rightarrow$ lower than expected temp is enough to support star
$\Rightarrow$ L, M steeper

Suppose my star is pure He?

fusion temp for He: $\approx 10^8$ K

$T \propto M/R$

H → He $\quad$ 10^7 K → 10^8

R decreases by about factor 10

e_{th} factor 10 larger

1000 times denser ⇒ e_{deg} factor 100 larger ⇒ deg at higher mass $\sim 0.5 M_\odot$

Reality? nuclear fusion steep function of T

$R \propto M$ roughly ok

↳ massive stars are slightly hotter

$\Rightarrow$ $\boxed{R \propto M^{0.8}}$ (reality)

opacity not constant w/ ρ, T, but goes down with $T\uparrow$

up w/ lower T

but compensated by convection

$\boxed{L \propto M^4}$

$L \propto \frac{M^3}{\langle\kappa\rangle}$

main sequence stars $\Rightarrow$ balls of gas $\longrightarrow$ everything dominated by the total mass M

giants : denser cores $\Rightarrow$ shell sources of fusion

extreme case : every property of the giant dominated by M_{core}