Transit Light Curves
Exoplanet Transits

- Detection is dependent on the geometry of the system with respect to our viewing orientation from Earth
  - We need the orbit of the exoplanet to be near edge-on to eclipse the disc of the star
What do we know?

• Transit Depth
• Transit Duration
• Transit Period
• Ingress and Egress Times
Sky-projected distance between the center of the star and planet. The total transit duration is heavily dependent on the impact parameter.

\[ b = a \cos i \]

To observe the transit the inclination must be,

\[ a \cos i \leq R_\ast + R_p \]
Transit Duration

- An observing program is designed around the transit duration time and frequency of transit (orbital period of planet)
- From Kepler’s third law the orbital period is,
\[ P = \sqrt{\frac{4\pi^2 a^3}{GM_\star}} \]
- The transit duration is also dependent on the crossing path across the stellar disk, i.e. the impact parameter (b)
Transit Duration

- The length of the transit is,
  $$l = \sqrt{(R_\star + R_p)^2 - b^2}$$

- The angle ($\alpha$) crossed by the planet during transit is,
  $$\left| \sin\left(\frac{\alpha}{2}\right) \right| = \frac{l}{a}$$
Transit Duration

- The time for the planet to traverse A to B,

\[ T_{duration} = P \frac{\alpha}{2\pi} \]

\[ T_{duration} = \frac{P}{\pi} \sin^{-1} \frac{l}{a} = \frac{P}{\pi} \sin^{-1} \sqrt{\frac{(R_* + R_p)^2 - b^2}{a}} \]
Transit
Depth
Limb
Darkening
Ingress
Egress
Limb darkening

- Flux from a stellar disk is non-uniform since optical depths ($\tau$) for a given viewing angle probes a different depth interior to the star
- Effectively, for a given viewing angle you observe a different effective temperature and density interior to the star
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Limb darkening effects

- Light curves with and without limb darkening with differing impact parameters (b)
Fitting for limb darkening

- Modeling limb darkening in our transit curves is non-trivial and there are several fitting methods used.
- Intensity variation are determined from stellar atmosphere models (e.g. Phoenix) where its dependent on viewing angle.
  - This is modeled then for a given filter bandpass and fitted with limb darkening laws.
    - Linear
      \[ I_\lambda(\mu) = I_\lambda(1) [1 - u(1 - \mu)] \]
    - Quadratic
      \[ I(\mu) = I(1) [1 - u_1(1 - \mu) - u_2(1 - \mu)^2] \]
    - Non-linear laws
Ingress and egress of transit

- Ingress is the initial slope downwards as the planet partially eclipses.
- Egress is the slope upwards as the planet partially eclipses.
- The shape of ingress and egress are effected by limb darkening.

\[ t_F = \text{Full depth eclipse duration} \]
\[ t_T = \text{Transit duration between start of ingress and end of egress} \]
Approximating inclination from light curve

• You can approximate the impact parameter with the observed transits times ($t_F$ and $t_T$) and flux ratio ($\delta$)
  
  – Assuming $a \gg R_*$ and $\pi t_T \ll P$,

  \[
  b = \sqrt{\frac{(1-\sqrt{\delta})^2 - (t_F / t_T)^2 (1+\sqrt{\delta})^2}{1-(t_F / t_T)^2}} \quad ; \quad b, \text{ impact parameter}
  \]

  – Now you can solve for inclination assuming circular orbit,

  \[
  i = \cos^{-1}\left(\frac{bR_*}{a}\right)
  \]
Approximating the orbital period from light curve

- Under the approximation that $M_p << M^*$ and using Kepler’s 3rd law you can determine the orbital period ($P$) with transit times, flux ratio, stellar mass and radius,

$$P = \frac{M_* G \pi (t_T^2 - t_F^2)^{3/2}}{32 R_*^3 \delta^{3/4}}$$