Intro to Photometry and Astrometry
Goals of Lab #3: Astrometry

• Learn more about imaging properties of a CCD and coordinate systems
• Further applications of least-square fitting
• Query astronomical databases and catalogs
• Data reduction techniques for imaging data
• Learn about proper motions and astrometry
The Needs of Astronomers

Astronomical Object

Detection

Do we see it?
Initial discovery of astronomical objects

Astrometry

Does it move?
Measure gravitational interactions between objects

Photometry

How bright is it?
Measure rudimentary properties of astronomical objects

Spectroscopy

What kinds of light does it emit?
Look at astrophysical processes that govern astronomical objects

What are the physical laws that govern these objects?
I. Stellar Photometry
What are the essential factors that define what an “image” looks like?

• Diffraction-limit:
  – telescope and/or imaging system (e.g., your eye’s pupil), \( f(\lambda) \)

• Sensitivity:
  – total collecting area of the telescope, \( f(\lambda) \)
  – throughput of optics and atmosphere, \( f(\lambda) \)
  – QE of detector, \( f(\lambda) \)
  – Frame rate of detector
  – Detector spatial sampling (pixel size and plate scale)

• Seeing:
  – Atmospheric turbulence, \( f(\lambda) \)
Point Spread Function of DI Telescope Star Field
Why do bright stars “look” bigger than faint stars?

• The FWHM of the stars are the same, but the intensity of the stars are different with respect to the background (or image stretch).
Principles of photometry

• The light from a star is spread over several pixels non-uniformly

• How do we sum the light to get a measure of the total flux from the star?
  – Identify the location of the star
  – Select the associated pixels that contain the stellar flux by generating a masking region
  – Sum up the light
    • Ensure that the noise and background is not included
Determining the center

• For each star we can find its centroid by determining the first moments along a 2d array,

\[
\langle x \rangle = \frac{\sum x_i I_i}{\sum I_i} \quad \quad \langle y \rangle = \frac{\sum y_j I_j}{\sum I_j}
\]

where \( I \) is the intensity at each pixel location (\( x, y \))
Aperture photometry

- How would you define the aperture mask on the star?
  - What radius of an aperture?
  - Where would you define the sky region?
Photometric Model

- **Star**
  - Brightness
  - Center \((x_0, y_0)\)
  - Width \((\sigma)\)

- **Sky background in annulus**
  - \(B\)

- **Detector**
  - \(QE\), readnoise, dark current

- **Aperture sizes**
  - \(r_1\), \(r_2\), \(r_3\)
Photometric Model

- Write down an expression for the signal, $S_i$, in units of photoelectrons
  - In an individual pixel
    \[ S_i = F_i + B_i + Q_i + E_i \]
  - $F_i$ is the stellar signal = $f_i \cdot t$ at pixel $i$ [e⁻]
    - Different for every pixel
  - $Q_i$ is the dark charge = $i_i \cdot t$ [e⁻] in a given pixel
    - The dark current $i_i$ varies from pixel to pixel
    - For SNR model assume constant
  - $B_i$ is the sky background = $b_i \cdot t$ assumed uniform [e⁻]
    - Varies from pixel to pixel, for SNR model assume constant
  - $E_i$ is the readout electronic offset or bias [e⁻]
    - Varies from pixel to pixel, for SNR model assume constant
The Stellar Signal

- The stellar signal is found by subtracting the background from $S_i$ and summing over the $N$ pixels that contain the star

$$F_i = S_i - \left( B_i + Q_i + E_i \right) \quad ; \text{Flux per pixel}$$

$$F_N = \sum_{i=1}^{N} F_i = \sum_{i=1}^{N} S_i - \left( B_i + Q_i + E_i \right) \quad ; \text{Total flux of star in circular aperture}$$

$$N_1 = \pi r_1^2$$

- Error in $F_N$ is due to noise in the signal itself, $F_N$
- Noise due to dark charge per pixel, $Q_i$
- Noise from the background per pixel (e.g., from atmosphere), $B_i$
- The read out noise $\sigma_{RO}^2$ can define the Error, $E_i$
Noise in the stellar aperture

- The total noise is a function of the Poisson noise of the stellar signal and noise (background, dark current, readnoise) in the stellar aperture ($N_1$)

$$F_N = \sum_{i=1}^{N_1} F_i = \sum_{i=1}^{N_1} \left[ S_i - (B_i + Q_i + E_i) \right]$$

$$\sigma_F^2 = \frac{F_N}{\text{Poisson signalnoise}} + \frac{N_1 \left( \langle B \rangle + \langle Q \rangle + \sigma_{RO}^2 \right)}{\text{Poisson noise within } r_1}$$

- Where $\langle B \rangle$ and $\langle Q \rangle$ is the mean background and dark current per pixel and $N_1 = \pi r_1^2$
Including all noise sources

- You must also include the noise from the background annulus \((N_{23})\),

\[
\sigma_{Sky}^2 = \left( \langle B \rangle + \langle Q \rangle + \sigma_{RO}^2 \right) / N_{23}
\]

\[
N_1 = \pi r_1^2, \quad N_{23} = \pi r_3^2 - \pi r_2^2
\]

- Then you get the total noise source from stellar aperture and sky annulus,

\[
\sigma_F^2 = \underbrace{F_N}_{\text{Poisson signal/noise}} + \underbrace{N_1 \left( \langle B \rangle + \langle Q_d \rangle + \sigma_{RO}^2 \right)}_{\text{Poisson noise within } r_1} + \underbrace{N_1 \left( \langle B \rangle + \langle Q_d \rangle + \sigma_{RO}^2 \right)}_{\text{Poisson noise within } r_2 < r < r_3} / N_{23}
\]
Signal-to-noise ratio for aperture photometry

\[
\text{SNR} = \sqrt{\frac{F_N^{\text{Signal}}}{F_N^{\text{Noise}}} + N_1(\langle B \rangle + \langle Q \rangle + \sigma_{RO}^2)} + \frac{N_1(\langle B \rangle + \langle Q \rangle + \sigma_{RO}^2)}{N_{23}}
\]

- How do we choose \( r_1, r_2, r_3 \)?
  - Signal increases with \( N_1 \)
  - Noise increases with \( N_1 \) and decreases with \( N_{23} \)

For more generalized with N number of frames see Mclean, “Electronic Imaging in Astronomy”
How does flux change with aperture radius?

• Suppose the stellar signal has a 2-d Gaussian shape

\[
F_i = \frac{F_0}{2\pi\sigma^2} \exp \left[ -\frac{1}{2} \left( \frac{r_i}{\sigma} \right)^2 \right], \quad r_i^2 = (x - x_0)^2 + (y - y_0)^2
\]

\[
F_N = \int_0^r 2\pi r F_i dr
\]

– This tells us how \( F_N \) changes with aperture radius
Stellar profile and integral

- You can determine what percentage of light is found per aperture radius
  - Useful for determining aperture radii
  - Useful when you need to use “aperture correction”
    - When the total flux is outside the aperture, like for crowded field photometry
    - Useful for defining SNR per radius
II. Astrometry
Astrometry

- Astrometry is the measurement of position and motion of celestial objects
- Oldest branch of astronomy
  - In 129 BC Hipparcos generated the 1st catalog of stars with apparent brightness and positional accuracy to 1°
  - Revolutionized by Tycho Brahe in 16th century and measured positions of stars to 1 arcminute
  - Positions were measured to 1 arcsecond precision by the 18th century with new instruments and the telescope
  - In 1989 Hipparcos satellite launched and had an astrometric precision of 0.001"

Courtesy ESA
Importance of astrometry

- Astrometry is used to determine:
  - Distances to nearby stars using parallax
  - Proper motions of stars
    - Kinematic structure of the galaxy
      - Distance scale of the Milky Way and beyond
    - Stellar formation and evolution
  - Orbits of solar system objects, binary stars, exoplanets, stellar populations
    - Direct way of determining the mass of celestial objects
    - Determine the mass of stellar black holes and the supermassive black hole at the Galactic Center
    - Determine the orbit of asteroids and comets
      - Probability of Earth getting slammed!
      - Solar System formation
  - Astrometric microlensing
    - Determine the mass of the lens source accurately
    - Number density compact objects
Proper Motion (from Lecture #7)

- Objects in our solar system, galaxy, and extragalactic sources have their own velocities
  - Closer the source and higher the velocities the greater motion on the projection of the sky, which is known as “proper motion”
    - Typical proper motion of nearby stars is 0.1” per year
    - Typical proper motion of main belt asteroids is ~0.5-1’ per hour
    - NEO’s can move have as fast as 2.5° per day
Astrometry discovered the dwarf planet Eris (the “Pluto killer”!):

- Eris moves at 1.75” per hour and was discovered by 1.2m telescope (Brown et al. 2005, ApJL)
  - 27% more massive than Pluto

Hubble

Palomar (1.2m telescope) Discovery Image!
Astrometry determined the mass of the MW’s supermassive black hole

- Using adaptive optics and 8-10m ground-based telescope the relative astrometric precision is 100 μas (0.0001")
- Monitor the stellar orbits over 15+ years
Reference frames

• The concept of position in space is relative
  – You need to define a coordinate system (e.g., Celestial coordinates, standard coordinates)
  – You need to define position relative to other celestial objects for given Equinox
    • Everything in the Universe is moving!

• A “reference frame” is a catalog of known positions and proper motions of celestial objects
  – For astrometry you use reference frames to define the position and motion of the object of study
Reference star catalogs

• All-sky surveys have generated catalogs of stars with positions per Equinox and proper motions
  – **Hipparcos** contains positions, proper motion, parallaxes of 118,218 stars. Complete to V=7.5 mag.
  – **Tycho-2** contains positions and proper motion of over 2 million stars. Complete to V=11.0 mag.
  – **USNO-B1** contains positions and proper motion of over 1 billion objects. All-sky coverage to V=21 mag.
  – **Sloan Digital Sky Survey** mapped ¼ of the sky in 5 optical filters and contains 287 million objects

We will be using the USNO-B1 catalog in Lab #3
International Celestial Reference System and Frame (ICRS)

• Fundamental reference system adopted by International Astronomical Union (IAU)
  – Reference system of celestial sphere defined by the barycenter of the solar system
  – Reference frame is with respect to distant extragalactic objects that are considered fixed
    • Based on radio positions from the VLBI of Quasars
    • Positional accuracy is 0.5 mas (0.0005")
Coordinate transformations to the image plane

• Celestial coordinates are defined on spherical coordinate system, but a CCD image is a Cartesian coordinate system on the plane of the sky

The projected coordinates \((X, Y)\) of a position on the celestial sphere \((\alpha, \delta)\) in the vicinity of a field centered at \((\alpha_0, \delta_0)\) are,

\[
X = - \frac{\cos \delta \sin(\alpha - \alpha_0)}{\cos \delta_0 \cos \delta \cos(\alpha - \alpha_0) + \sin \delta \sin \delta_0}
\]

\[
Y = - \frac{\sin \delta_0 \cos \delta \cos(\alpha - \alpha_0) - \cos \delta_0 \sin \delta}{\cos \delta_0 \cos \delta \cos(\alpha - \alpha_0) + \sin \delta \sin \delta_0}
\]

Derivation in the Lab #3 write-up
Converting to pixel coordinates \((x,y)\) of the CCD

- The CCD has plate scale (pixel per arcseconds) that is defined by the focal length \((f)\) of the camera and the size of the CCD pixels \((p)\).
- For an “ideal” camera and detector the \(x, y\) position on the detector can be found by,

\[
x = f \left( \frac{X}{p} \right) + x_0
\]
\[
y = f \left( \frac{Y}{p} \right) + y_0
\]

- Of course a number of factors make this non-ideal with optical distortion, anamorphic magnification and the CCD rotation on sky.
Let's think about Affine transformation.

\[ x = \frac{f}{p} (X \cos \theta - Y \sin \theta) + x_0 \]
\[ y = \frac{f}{p} (X \sin \theta + Y \cos \theta) + y_0 \]

\[ x = TX \]
\[ X = (X, Y, 1), \ x = (x, y, 1) \]

\[ T = \begin{pmatrix}
(f/p) a_{11} & (f/p) a_{12} & x_0 \\
(f/p) a_{21} & (f/p) a_{22} & y_0 \\
0 & 0 & 1
\end{pmatrix} \]

The constants \( a_{ii} \) refer to the scale, shear and orientation of the image, and \( x_0 \) and \( y_0 \) are pointing offsets in pixels.
Affine Transformation (from MathWorld): An affine transformation is any transformation that preserves collinearity (i.e., all points lying on a line initially still lie on a line after transformation) and ratios of distances (e.g., the midpoint of a line segment remains the midpoint after transformation). In this sense, affine indicates a special class of projective transformations that do not move any objects from the affine space to the plane at infinity or conversely. Geometric contraction, expansion, dilation, reflection, rotation, shear, similarity transformations, spiral similarities, and translation are all affine transformations, as are their combinations. In general, an affine transformation is a composition of rotations, translations, dilations, and shears.

Examples of Affine Transformation in matrix form:

Scaling (factors $a, b \neq 0$): \[
\begin{pmatrix}
a & 0 \\
0 & b
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} =
\begin{pmatrix}
ax \\
by
\end{pmatrix}
\]

Reflection (in line $y = x$): \[
\begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} =
\begin{pmatrix}
y \\
x
\end{pmatrix}
\]

Shearing (x-dir., factor $s$): \[
\begin{pmatrix}
1 & s \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} =
\begin{pmatrix}
x + sy \\
y
\end{pmatrix}
\]

Rotation (countercl., $45^\circ$): \[
\frac{\sqrt{2}}{2}
\begin{pmatrix}
1 & -1 \\
1 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} =
\frac{\sqrt{2}}{2}
\begin{pmatrix}
x - y \\
x + y
\end{pmatrix}
\]
Examples of Affine Transformation in matrix form:

- **Translation** by vector $\vec{t}$: $\vec{p}_1 = \vec{p}_0 + \vec{t}$.

- **Rotation** counterclockwise by $\theta$: $\vec{p}_1 = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \vec{p}_0$.

- **Nonuniform scaling** by $a$ and $b$: $\vec{p}_1 = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \vec{p}_0$.

- **Shear** by scalar $h$: $\vec{p}_1 = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \vec{p}_0$.

- **Uniform scaling** by scalar $a$: $\vec{p}_1 = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \vec{p}_0$.

- **Reflection** about the $y$-axis: $\vec{p}_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \vec{p}_0$. 
Telescope Plate Scale

F determines the pixel scale at the focal plane (the ratio between the angle on the sky and the physical dimension on the detector)

\[ \delta = \theta \times F \]

Suppose at 8.4-meter with an f/15 focal ratio, what is F?

The focal ratio gives the size of the diffraction limit on the focal plane/detector.

\[ x = F/D \times \lambda \]

Plate scale is defined as the angle per unit displacement.

What is the plate scale of a 5-meter f/16 telescope?

\[ f = 5 \times 16 = 80 \text{ m} = 80,000 \text{ mm} \]

\[ \text{Plate scale} = \left( \frac{1}{3600} \right) \times \left( \frac{\pi}{180} \right) \times 80,000 \text{ mm} \Rightarrow 1" = 0.387 \text{ mm} \text{ or } 2.58"/\text{mm} \]