AST 222 Winter 2011

Assignment #3

Due 12pm, Mon March 14

This assignment is due before class on Mar 8; that is, at 12:10 **sharp**. Assignments handed in late — including at the end of class — will lose 20% credit. Answers should be posted online by about Wednesday; once answers are posted, of course, no further assignments will be accepted.

Questions can be asked by email to `ast222@astro.utoronto.ca`, or to the instructor or TAs at their office hours.

Show your work, and good luck!

Question 1 - Smashing Some Galaxies Together [55 pts]

`GalCrash` is a nice Java Applet at Case Western that we demonstrated in a previous class that lets you set some parameters and run a real, if simplified, galaxy interaction simulation on your computer through the web browser. It is at:

http://burro.cwru.edu/JavaLab/GalCrashWeb/main.html

We're going to use this as a mini-laboratory examining some issues we talked about in class. Click on 'Applet', and run some galaxy interactions to familiarize yourself with the program. The Green galaxy is the primary galaxy, with mass 1.0; the red galaxy is the companion galaxy. you can change the inclinations of the galaxies with the dials. 'Green Centred' means that the display is in the Green galaxies frame of reference; you can turn on or off dynamical friction and make the dark matter halos normal-sized or large. 'Peri' here means periapse, the distance of closest approach of the orbits of the two galaxies if there were no interactions between them other than as point masses. You can 'drag' the results of the simulations to see it from different angles, and shift-drag to zoom in or out.

We're going to do all the simulations for this question with 2000 stars and with zero inclinations. I find it's easier to see what's going on with 'Green Centered' on, but you can use either setting. We're going to do questions based on the suggested lab exercises. For each simulation, keep track of the time it takes for a merger to take place; for our purposes, the merger happens when the green and red circles stay touched together, and the merger time is the time from the original periapse (that is, *not* from time 0) to the merger. We'll say that anything that doesn't merge within $2, \mathrm{Gyr}$ doesn't merge at all.

1 (a)

(5 pts) Run an interaction simulation with a periapse of 4.0 kpc, a red galaxy mass of 0.8, and no friction. What happens? Now turn on dynamical friction. What is the difference? Does changing the inclination of either galaxy change things?

1 (b)

(10 pts) For a series (at least 5) of companion masses including the range 0.1 to 0.5, re-run the simulation with periapse of $1.0\,\mathrm{kpc}$ and tabulate time from peripse until merger as a function of mass. Measure the times from first closest approach until you can no longer see a distance between the two galaxies (the red line stays at zero). What is the uncertainty in your time - what is the largest and smallest time you could plausibly claim for this time? We'll take the average of those as the merger time, and half the difference to be the uncertainty.

1 (c)

(10 pts) In class, we said that the force due to dynamical friction was

$$f_d = C\frac{(Gm)^2\rho}{v_m^2}$$

where m was the mass of the inteloping galaxy, and v_m it's speed. We'll consider that the green galaxy is like the Milky way; it's mass is $M = 7 \times 10^{11}\, M_\odot$ (so that the red mass m is a fraction m/M of that; it's that fraction which one sets in the applet). We'll say the Green galaxy's density can be approximated as

$$\rho = \frac{v_c^2}{4\pi G r^2}$$

to give a flat rotation curve. We're told that the initial velocity at closest approach was $v = 2v_e = 4v_c$. By approximating the time for decay of the orbit as

$$t = \frac{L_0}{\tau_0} = \frac{m v_m r}{f_d r},$$

show that we can write the decay time as

$$t = \frac{32\pi}{C}\sqrt{\frac{r^3}{Gm}} = \frac{56.6\,\mathrm{Myr}}{C}\sqrt{\left(\frac{r}{1\,\mathrm{kpc}}\right)^3\left(\frac{m}{M}\right)^{-1}}.$$

Correction:

$$t = \frac{256\pi(1\,\mathrm{kpc})^{3/2}}{\sqrt{GM}}\frac{1}{C}\left(\frac{m}{M}\right)^{-1}\left(\frac{r}{1\,\mathrm{kpc}}\right)^{3/2} = 453\,\mathrm{Myr}\frac{1}{C}\left(\frac{m}{M}\right)^{-1}\left(\frac{r}{1\,\mathrm{kpc}}\right)^{3/2}.$$

1 (d)

(10 pts) Plot your data, with errorbars, and either fit or choose by eye a value for C consistent with the data using the formula above. Print and submit the best-fit plot through your data with errorbars. (Instructions for plotting with errorbars, and fitting, will be on the course webpage under "Problem Sets" shortly.) Once you've gotten a fit either way, adjust C; what is the largest and smallest C you can plausibly claim fit your data – at what point is it clearly to once side or the other of the data?

1 (e)

(20 pts) Another way to tackle the same question is to focus in on one orbit. At the beginning, the red galaxy has some distance (and thus potential energy) and velocity (and thus, kinetic energy), so the total energy is

$$E_0 = \frac{1}{2}mv_0^2 + \frac{GMm}{R_0}.$$

at the maximum distance after the first pass, the energy is

$$E_1 = \frac{GMm}{R_1}$$

so that the energy "lost" – *eg*, done on the galaxy by dynamical friction – in that first half-orbit must be

$$f_d d \approx \frac{C}{64\pi}\frac{Gm^2}{r^2}\pi r = E_1 - E_0.$$

For at least three masses in the same mass range, calculate the work done on the galaxy, and plot that versus mass as above but without error bars, and again try to fit it using a version of the expression above; submit the plot with your assignment. What is your best-fit value for C? Is it consistent with the expression found earlier? What might account for any difference?

Question 2 - Stars in Merging Galaxies (15 pts)

Galaxies have lots of stars, but they're really big, too. Consider two Milky-Way-type spiral galaxies merging face on — that is, one falling right on top of another — and passing through (for instance, the first pass before a merger). Let's try to estimate how many disk stars actually collide.

2 (a)

(5 pts) The local stellar density is about $n \approx 0.15\,\mathrm{pc}^{-3}$. Consider a cylinder centred on the sun, of the Sun's radius, extending up and down through the whole thick disk (which extends about $1\,\mathrm{kpc}$ either side of the midplane). If that density is constant through the thick disk (an overestimate), how many other stars on average are in that cylinder? The sun has a radius of about $6.95 \times 10^5\,\mathrm{km}$.

2 (b)

(3 pts) That's an awfully small number, does it make sense? If the local stellar density is as above, eg a cube of size $1\,\mathrm{pc}$ has 0.15 stars in it on average, what is the fraction of space in the local area that is *not* stars? Use the Sun's radius as typical.
When one galaxy falls through the other, that cylinder of stars will fall through the Sun. If the two galaxies are similar, the stellar densities will be similar. So the odds of the Sun being hit are the number of stars in that cylinder. (Of course, the galaxies are rotating, but if they're similar, they're rotating at similar speeds at similar positions...)

2 (c)

(2 pts) What are the odds of *any* stars in the first galaxy being hit? There are about 10^{11} stars in the galaxy; we'll say all stars are in the disk, which isn't a bad approximation.

2 (d)

(5 points) Of course, the stellar densities are higher further in. If the average disk stellar density were 100 times that in the local disk, what number of stars on average would collide then?

Question 3 - Cluster Gas and Galaxies (15 pts)

We've mentioned that a typical velocity dispersion of a galaxy cluster is $\sigma \approx 1000\,\mathrm{km\,s^{-1}}$, and that they typically emit in X-rays implying a temperature of about 10^7–$10^8\,\mathrm{K}$. Are these two numbers consistent? That is, are the random velocities of both the galaxies and the particles that make up the gas plausibly both the same thing – the mass of the cluster? Recall that the kinetic energy of a particle of monatomic gas is

$$\frac{3}{2}kT$$

and we'll consider that the gas is all hydrogen. (In fact, it's ionized hydrogen, which makes no real difference here.) What temperature is implied by a velocity dispersion of $1000\,\mathrm{km\,s^{-1}}$? Is it consistent with the quoted Xray temperatures?

Question 4 - Fast Galaxy Interactions: Impulse Approximation [15pts]

In our discussion of galaxy interactions, we talked about the 'Impulse Approximation' for fast interactions; if the galaxies just whip past each other, then there is no time for the large scale structure of the galaxy, and thus its gravitational potential, to change (recall that a typical orbital time for a galaxy is of the order of hundreds of Myr, and it typically takes times of that order to make global-scale changes to the system). However, individual stars will be given kicks in various directions by the interaction. Thus, the gravitational potential remains the same, $U_i = U_0$, but the kinetic energy of the motions within the galaxy are increased by some amount, $K_i = K_0 + \Delta K$. (The extra energy comes from the overall motions of the galaxies interacting). But this is no longer in virial equilibrium; the argument was that once the system re-virialized (conserving energy in this phase), the kinetic energy would actually *drop* to $K_f = K_0 - \Delta K$, and the gravitational potential energy would go up (the system would be less bound) to $U_f = U_0 + 2\Delta K$.

Using the equation of virial equillibrium, $2K = -U$, and energy conservation ($K + U = \mathrm{const}$), show that this is true.