AST 222 Winter 2011

Assignment #1

Due 12pm, Wed Feb 2

This assignment will be marked out of 100, and makes up the first of five assignments this term. Your term mark will be made up 50% from your assignments, taken from the top four of your five assignment grades.

This assignment is due before class on Feb 2; that is, at 12:10 **sharp**. Assignments handed in late — including at the end of class — will lose 50% credit. Answers should be posted online soon afterwards; once answers are posted, of course, no further assignments will be accepted. We'll have them returned promptly.

Assignments may be handed in in person at class or submitted through the dropbox on blackboard.

The instructor will have office hours after class on Mon and Wed; the TAs have office hours 3-4pm Tues and 4-5 on Thursday. Questions can also be asked by email to `ast222@astro.utoronto.ca`, or posted on the blackboard forum (this is especially good for questions you think others will have.) An online office hour will be announced for 6pm Feb 1, the day before the assignment.

Show your work, use a sensible number of significant digits, and good luck!

Question 1 - Short Answer [15pt]

1 (a)

(6 pts) Milky Way Scavenger Hunt: Where (of the stellar halo, dark matter halo, disk, or bulge) would one be most likely to find the following?

- Young stars?
- Old stars?
- A huge black hole?
- Magnetic fields?
- Gas and Dust?
- Globular Clusters?

1 (b)

(2 pts) The Milky Way's supermassive black hole is thought to have a mass of approximately $3.5 \times 10^6\,M_\odot$. Give two independant reasons why we believe that that mass is enclosed in a small area (and thus, why we think it may be a black hole).

1 (c)

(2 pts) The black hole is understood to be contained in a radius of less than $200\,\mathrm{AU}$. Compare this to a large dense stellar system, the Omega Centauri globular cluster, with a similar mass but a radius of approximately 5×10^6 AU. Assuming the object is spherical, how does its density (in $M_\odot/\mathrm{AU}^3$) compare to Omega Centauri? The local disk stellar density (about $0.1\,M_\odot\,\mathrm{pc}^{-3}$)?

1 (d)

(2 pts) Why, briefly, did star count-based approaches to examining the local Universe place the Sun too close to its centre?

1 (e)

(2 pts) The radial scale height of the disk is observed to be approximately $2.25\,\mathrm{kpc}$. If at our location ($R_0 = 8.\,\mathrm{kpc}$ out) the local stellar density is, say, $0.1\,\mathrm{pc}^{-1}$, what would we expect it to be at at $R = 10.75\,\mathrm{kpc}$?

1 (f)

(1 pt) When looking in the general direction of the galactic centre, where would you see a higher density of stars in the sky - at high galactic lattitude ($b \approx 90$) or at low galactic lattitude ($b \approx 0$)?

Question 2 - Oort Constants [35 pts]

Seeking fame and fortune, the AST222 students quit school and head out to mine the asteroid belt for gold (as seen in the Discovery Network reality TV show, "Gold Rush Asteroid". Their instructor sets up a home base on Ceres, a large body approximtely $2.7663\,\mathrm{AU}$ from the sun, while the students begin exploring neighbouring smaller bodies, each remaining about $1.5 \times 10^5\,\mathrm{km}$ ($0.001\,\mathrm{AU}$) from the Ceres base. We assume circular orbits.

2 (a)

(2 pts) What is Ceres' orbital speed, in $\mathrm{km\,s^{-1}}$ (assuming, correctly, that this is dominated by the Sun's mass?)

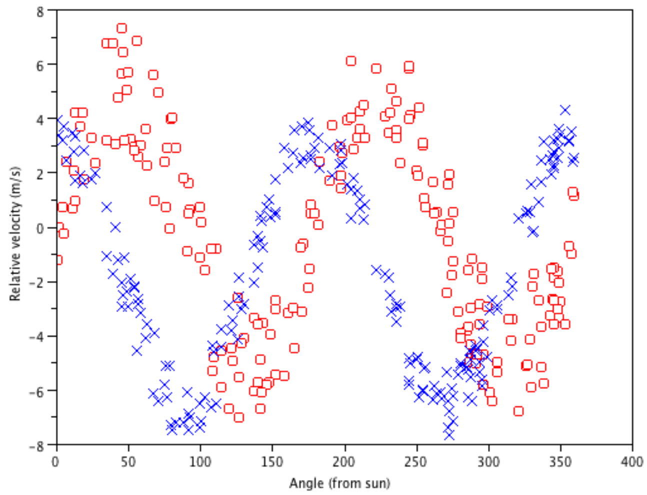
2 (b)

(3 pts) At Ceres' position, what is the slope of the rotation curve (dV/dr) in $\mathrm{km\,s^{-1}AU^{-1}}$?

2 (c)

(3 pts) In the same units, what are the expected values for the Oort Constants A and B around Ceres?

Using the surveying equipment brought for the mining expedition, the instructor observes the relative radial and tangential velocities of the asteroids being explored by the students, and records the data below. Sadly the instructor trips over a rocket pack and is flung towards Jupiter before labelling the graph as to which line is which. (Moral of the story: Always label your graphs right at the beginning! At least he had time to put units on his axes).



2 (d)

(2 pts) Which line is which, and why?

2 (e)

(4 pts) By eye, estimate A and B and quote uncertainties (eg, what are the highest and lowest numbers you could plausibly assign). Are they consistent with the expected values?

The asteroid belt, of course, has mass (approximately 3.5×10^{21} kg) and so might affect the local rotation curve. Consider the asteroid belt to be a uniform cylindrical annulus of density ρ and height $h = 0.1$ AU between the $R_i = 2.06$ AU and $R_0 = 3.27$ AU, so that the mass interior to an orbit at any point within the annulus is $M(< R) = 1\,M_\odot + \pi\rho[R^2 - R_i^2]h$.

2 (f)

(3 pts) Write an expression for $dM(< R)/dR$ in terms of the local density ρ.

2 (g)

(5 pts) Using the equation of circular orbital velocities,

$$V(R)^2 = \frac{GM(< R)}{R}$$

show that, to get a flat rotation curve, the local density ρ must be comparable to the average density in the enclosed volume $\approx M/R^3$.

2 (h)

(4 pts) Calculate an average density for the asteroid belt. Is it enough to significantly alter the rotation curve within the belt (is ρ comparable to $M_\odot/R^3$?)

It's worth exploring what the Oort constants would be if the asteroid belt changed the rotation curve significantly from Keplarian.

2 (i)

(6 pts) Consider rotation curves at Ceres' current position (R_c) and rotational velocity (V_c), in the form of a power law

$$V(R) = V_c \left(\frac{R}{R_c} \right)^p ,$$

and find dV/dR, and then A and B, as a function of p. What would Ceres' Oort contants A and B be if the rotation curve was locally flat ($p = 0$)? Solid body ($p = 1$)?

2 (j)

(3 pts) Are the Oort constants sensitive enough measures to distinguish between these cases – if with more measurements, the students' measured values of the Oort constant at Ceres were uncertain by 10%, would they still be able to rule out flat and solid body rotation curves? How about with the data as plotted above?

Question 3 - Tangent Point Method [25 pt]

In class, we discussed the Tangent Point Method, and stated (with a qualitative geometric argument) that when observing at a line of sight at Galactic longitude l, for all not-too-steeply outwardly increasing rotation curves, the maximum observed relative radial velocity was that of the orbit closest to the Galactic centre along the line of sight. We will again assume circular orbits.

3 (a)

(2 pts) Write an expresion for relative velocity along the line of sight at Galactic longitude l for all observed objects at any radius R in terms of the rotation velocity at that radius $V(R)$, the angle l, and our radius R_0.

3 (b)

(5 pts) Prove that for a flat rotation curve $V(R) = V_0$, the maximum observed velocity occurs for objects with an orbit at $R = R_{\min} = R_0 \sin l$.

3 (c)

(5 pts) For power-law rotation curves as in the question on Oort contstants, at what minimum p is this no longer true?

3 (d)

(5 pts) Assuming spherical distribution of masses, and starting with $V(R)^2 = GM/R$, find the density distribution $\rho(R)$ necessary for the rotation curve to have the minimum p calculated above.

3 (e)

(3 pts) Why does the Tangent Point Method not work for radii $R > R_0$?

3 (f)

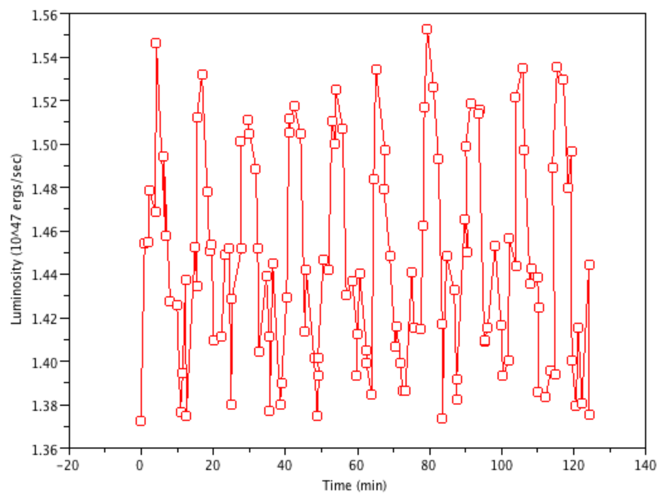
(3 pts) Using our known $R_0 = 8\,\mathrm{kpc}$ and $V_0 = 220\,\mathrm{km\,s^{-1}}$, estimate the mass of the Milky Way interior to the Sun, assuming it is distributed spherically.

3 (g)

(2 pts) Assuming the mass above is mostly dark matter, and distributed spherically, find the average interior dark matter density. The local stellar density is approximately $0.1\,\mathrm{pc}^{-3}$; if the Sun is a typical-mass star, is there enough local density to make the local rotation curve deviate significantly from Keplarian - that is, is the local density likely comparable to the average interior density?

Question 4 - Supermassive Black Holes [25 pt]

Plotted below is xray data taken from an object believed to be a central black hole in a galaxy found in the Non-Existant Galaxy Survey, NEGS 1234. The data is also found on the course webpage under problem sets.



4 (a)

(5 pts) Plot the data, using the tool of your choice (a tutorial on data plotting with a few different tools will be available shortly on the course website).

4 (b)

(5 pts) By eye, estimate the periodicity of the signal, in minutes, and derive an upper limit of the size of the emitting object.

4 (c)

(5 pts) Estimate in your plotting tool (preferably) or by eye the mean luminosity of the emitting object. If this is accretion-powered luminosity, to what accretion rate (in solar masses per year) does this correspond?

4 (d)

(5 pts) If, from dynamical arguments, this black hole is believed to have the same mass as the central object in our own galaxy (approximately $3.5 \times 10^6 M_\odot$), what would its Schwarzschild radius be?

4 (e)

(5 pts) Our own supermassive black hole dominates the dynamics of the galaxy within the inner $100\ \mathrm{pc}$ or so; let's investigate if it affects dynamics at the Local Standard of Rest significantly. You have above estimated the mass of the galaxy interior to the solar circle. Imagine that our central black hole somehow was instantaneously whisked away, leaving the rest of the Milky Way behind. Once the galaxy had adjusted, what would the new circular rotation velocity be at the current solar circle? Is this a significant change?