

AST 222 Winter 2011

Assignment #1

Due 12pm, Wed Feb 2

This assignment will be marked out of 100, and makes up the first of five assignments this term. Your term mark will be made up 50% from your assignments, taken from the top four of your five assignment grades.

This assignment is due before class on Feb 2; that is, at 12:10 **sharp**. Assignments handed in late — including at the end of class — will lose 50% credit. Answers should be posted online soon afterwards; once answers are posted, of course, no further assignments will be accepted. We'll have them returned promptly.

Assignments may be handed in in person at class or submitted through the dropbox on blackboard.

The instructor will have office hours after class on Mon and Wed; the TAs have office hours 3-4pm Tues and 4-5 on Thursday. Questions can also be asked by email to ast222@astro.utoronto.ca, or posted on the blackboard forum (this is especially good for questions you think others will have.) An online office hour will be announced for 6pm Feb 1, the day before the assignment.

Show your work, use a sensible number of significant digits, and good luck!

Question 1 - Short Answer [15pt]

1 (a)

(6 pts) Milky Way Scavenger Hunt: Where (of the stellar halo, dark matter halo, disk, or bulge) would one be most likely to find the following?

- Young stars? — **Disk**
- Old stars? — **Stellar Halo**
- A huge black hole? — **Bulge**
- Magnetic fields? — **Disk**
- Gas and Dust? — **Disk**
- Globular Clusters? — **Stellar Halo**

1 (b)

(2 pts) The Milky Way's supermassive black hole is thought to have a mass of approximately $3.5 \times 10^6 M_{\odot}$. Give two independent reasons why we believe that that mass is enclosed in a small area (and thus, why we think it may be a black hole).

We see nearby stars orbiting Sgr A* with orbits that pass within 200 AU; and the quasi-regular 17 minute flaring of the Xray source implies an object of size less than approximately 17 light-minutes (2 AU).

1 (c)

(2 pts) The black hole is understood to be contained in a radius of less than 200 AU. Compare this to a large dense stellar system, the Omega Centauri globular cluster, with a similar mass but a radius of approximately 5×10^6 AU. Assuming the object is spherical, how does its density (in M_{\odot}/AU^3) compare to Omega Centauri? The local disk stellar density (about $0.1 M_{\odot} \text{pc}^{-3}$)?

A lower limit for the density (which could be significantly higher if the object is less than 200 AU in size) would be

$$\rho = \frac{3.5 \times 10^6 M_{\odot}}{\frac{4}{3}\pi(200 \text{ AU})^3} = 0.10 M_{\odot} \text{AU}^{-3}.$$

In comparison, Omega Centauri's density is of the order

$$\rho = \frac{5 \times 10^6 M_{\odot}}{\frac{4}{3}\pi(5 \times 10^6 \text{ AU})^3} = 6.68 \times 10^{-15} M_{\odot} \text{AU}^{-3}.$$

A more accurate mass is $5 \times 10^6 M_{\odot}$, so a better mass density is $9.55 \times 10^{-15} M_{\odot} \text{AU}^{-3}$. (This average density for Omega Centauri is much lower than the central density, which can be as high as 350,000 times the local stellar density - but that still only on order $4 \times 10^{-12} M_{\odot} \text{AU}^{-3}$).

In these units, the local stellar density is

$$\rho = 0.1 M_{\odot} \text{pc}^{-3} = 0.1 M_{\odot} \text{pc}^{-3} \left(\frac{1 \text{ pc}}{206264.806 \text{ AU}} \right)^3 = 1.14 \times 10^{-17} M_{\odot} \text{AU}^{-3}.$$

That puts the lower limit on the density of Sgr A* at least a trillion times the other densities considered here, and ten million times the density of the sun.

1 (d)

(2 pts) Why, briefly, did star count-based approaches to examining the local Universe place the Sun too close to its centre?

Because of extinction due to dust in the disk of the galaxy (where the vast majority of the visible stars are), there is a limit to how far can be seen in any direction — so that the Sun naturally seemed towards the centre of any distribution of stars seen in the optical.

1 (e)

(2 pts) The radial scale height of the disk is observed to be approximately 2.25 kpc. If at our location ($R_0 = 8$ kpc out) the local stellar density is, say, 0.1pc^{-3} , what would we expect it to be at $R = 10.75$ kpc?

Over one scale height, the density will fall by a factor of e ; or, more formally,

$$\begin{aligned} n(R_0) &= n_0 e^{-R_0/h_R} \\ n(R_0 + h_R) &= n_0 e^{-(R_0+h_R)/h_R} \\ &= n_0 e^{-R_0/h_R - 1} \\ &= n_0 e^{-R_0/h_R} e^{-1} = n(R_0) e^{-1} \end{aligned}$$

so that $n(R_0 + h_R) \approx 0.0368$.

1 (f)

(1 pt) When looking in the general direction of the galactic centre, where would you see a higher density of stars in the sky - at high galactic latitude ($b \approx 90$) or at low galactic latitude ($b \approx 0$)?

The galactic disk is at $b = 0$; the galactic pole is at $b = 90$. So the density of stars is highest at $b = 0$, and falls off dramatically towards $b = 90$.

Question 2 - Oort Constants [35 pts]

Seeking fame and fortune, the AST222 students quit school and head out to mine the asteroid belt for gold (as seen in the Discovery Network reality TV show, "Gold Rush Asteroid". Their instructor sets up a home base on Ceres, a large body approximately 2.7663 AU from the sun, while the students begin exploring neighbouring smaller bodies, each remaining about 1.5×10^5 km (0.001 AU) from the Ceres base. We assume circular orbits.

2 (a)

(2 pts) What is Ceres' orbital speed, in km s^{-1} (assuming, correctly, that this is dominated by the Sun's mass?)

The circular rotation speed is simply given by

$$\begin{aligned} V(R) &= \sqrt{\frac{GM_{\odot}}{R}} \\ &= \sqrt{\frac{887.18 (\text{km s}^{-1})^2 \text{ AU } M_{\odot}^{-1} \cdot 1M_{\odot}}{2.7663 \text{ AU}}} \\ &= 17.9 \text{ km s}^{-1}. \end{aligned}$$

Note that it's often convenient to have constants like G handy in various units. Various freely-available tools (I'm fond of Google calculator, <http://www.google.com/help/features.html#calculator>, Wolfram Alpha (<http://www.wolframalpha.com/>) is another) are good at doing such conversions.

2 (b)

(3 pts) At Ceres' position, what is the slope of the rotation curve (dV/dr) in $\text{km s}^{-1} \text{ AU}^{-1}$?

We can get the slope of the rotation curve from the circular rotation speed:

$$\begin{aligned} V^2(R) &= \frac{GM_{\odot}}{R} \\ 2V(R) \frac{dV(R)}{dR} &= -\frac{GM_{\odot}}{R^2} \\ \frac{dV(R)}{dR} &= -\frac{GM_{\odot}}{2V(R)R^2} \\ &= -\frac{1}{2} \frac{GM_{\odot}}{R} \frac{1}{RV(R)} \\ &= -\frac{1}{2} V^2(R) \frac{1}{RV(R)} \\ &= -\frac{1}{2} \frac{V(R)}{R} \\ &= -\frac{1}{2} \frac{17.9 \text{ km s}^{-1}}{2.7663 \text{ AU}} \\ &= -3.24 \text{ km s}^{-1} \text{ AU}^{-1} \end{aligned}$$

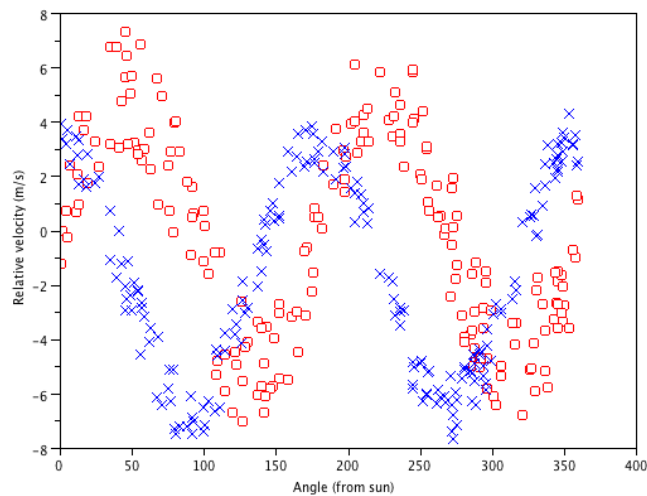
2 (c)

(3 pts) In the same units, what are the expected values for the Oort Constants A and B around Ceres?

From our work on the Oort constants,

$$\begin{aligned}
 A &= -\frac{1}{2} \left(\left. \frac{dV}{dR} \right|_{R_0} - \frac{V_0}{R_0} \right) \\
 B &= -\frac{1}{2} \left(\left. \frac{dV}{dR} \right|_{R_0} + \frac{V_0}{R_0} \right) \\
 A &= -\frac{1}{2} \left(-3.24 \text{ km s}^{-1} \text{ AU}^{-1} - \frac{17.9 \text{ km s}^{-1}}{2.7663 \text{ AU}} \right) \\
 &= 4.86 \text{ km s}^{-1} \text{ AU}^{-1} \\
 B &= -\frac{1}{2} \left(-3.24 \text{ km s}^{-1} \text{ AU}^{-1} + \frac{17.9 \text{ km s}^{-1}}{2.7663 \text{ AU}} \right) \\
 &= -1.62 \text{ km s}^{-1} \text{ AU}^{-1}
 \end{aligned}$$

Using the surveying equipment brought for the mining expedition, the instructor observes the relative radial and tangential velocities of the asteroids being explored by the students, and records the data below. Sadly the instructor trips over a rocket pack and is flung towards Jupiter before labelling the graph as to which line is which. (Moral of the story: Always label your graphs right at the beginning! At least he had time to put units on his axes).



2 (d)

(2 pts) Which line is which, and why?

From the definition of the Oort constants,

$$\begin{aligned}
 V_r &= Ad \sin(2l) \\
 V_t &= Ad \cos(2l) + Bd.
 \end{aligned}$$

This tells us that the red 'o's must represent the radial velocities, and the blue 'x's must be the tangential velocities.

Thinking about this a little more deeply than just looking at the equations - the radial velocities of the nearby objects must average out to zero, or else the bodies would be moving on average towards or away from Ceres, and the asteroid belt would be getting denser (or less dense) at that point — contradicting our assumptions of circular motions. On the other hand, the tangential velocities can have a non-zero average; it just means that there's a net velocity shear, which is normal in a rotating system.

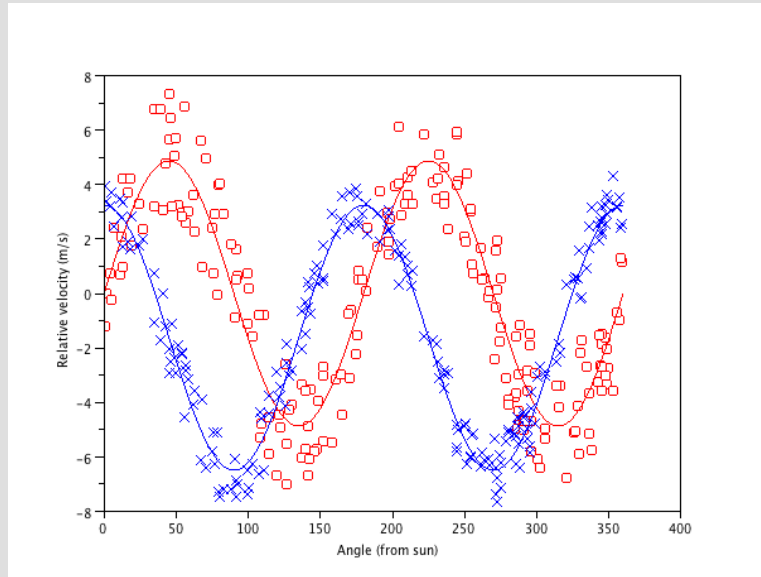
2 (e)

(4 pts) By eye, estimate A and B and quote uncertainties (eg, what are the highest and lowest numbers you could plausibly assign). Are they consistent with the expected values?

Answers will slightly vary. Looking at the red 'o's, the peaks on either side are between 3 m s^{-1} and 7 m s^{-1} , so $Ad = 5 \pm 2\text{ m s}^{-1}$; since $d = 0.001\text{ AU}$, so $A = 5 \pm 2\text{ km s}^{-1}\text{ AU}^{-1}$.

The tangential velocity peaks similarly go from between -6 m s^{-1} and -8 m s^{-1} on the low side, and $+2\text{ m s}^{-1}$ and $+4\text{ m s}^{-1}$ on the high side; that's consistent with an amplitude of Ad , but with a shift of $-2 \pm 1\text{ m s}^{-1}$. That would make $Bd = -2 \pm 1\text{ m s}^{-1}$, or $B = -2 \pm 1\text{ km s}^{-1}\text{ AU}^{-1}$.

The theoretical expected values all fall comfortably within these ranges.



The asteroid belt, of course, has mass (approximately $3.5 \times 10^{21}\text{ kg}$) and so might affect the local rotation curve. Consider the asteroid belt to be a uniform cylindrical annulus of density ρ and height $h = 0.1\text{ AU}$ between the $R_i = 2.06\text{ AU}$ and $R_o = 3.27\text{ AU}$, so that the mass interior to an orbit at any point within the annulus is $M(< R) = 1 M_\odot + \pi\rho[R^2 - R_i^2]h$.

2 (f)

(3 pts) Write an expression for $dM(< R)/dR$ in terms of the local density ρ .

$$\begin{aligned} \frac{dM(< R)}{dR} &= \frac{d}{dR} [1 M_\odot + \pi\rho[R^2 - R_i^2]h] \\ &= 2\pi h\rho R \end{aligned}$$

2 (g)

(5 pts) Using the equation of circular orbital velocities,

$$V(R)^2 = \frac{GM(< R)}{R}$$

show that, to get a flat rotation curve, the local density ρ must be comparable to the average density in the enclosed volume $\approx M/R^3$.

The slope of the rotation curve, as above but with a varying $M(< R)$, can be calculated from

$$\begin{aligned} V^2(R) &= \frac{GM(< R)}{R} \\ 2V(R)\frac{dV(R)}{dR} &= G\left(-M(< R)R^{-2} + R^{-1}\frac{dM(< R)}{dR}\right) \\ \frac{dV(R)}{dR} &= -\frac{GR}{2V(R)}\left(\frac{M(< R)}{R^3} - \frac{1}{R^2}\frac{dM(< R)}{dR}\right) \\ &= -\frac{GR}{2V(R)}\left(\frac{M(< R)}{R^3} - 2\pi\frac{h}{R}\rho\right) \end{aligned}$$

For the slope to be zero, the two terms in brackets must be equal, meaning that

$$\begin{aligned} \frac{M(< R)}{R^3} &= 2\pi\frac{h}{R}\rho \\ \rho &= \frac{1}{2\pi}\frac{R}{h}\frac{M(< R)}{R^3} \\ \rho &= 4.4\frac{M(< R)}{R^3} \end{aligned}$$

2 (h)

(4 pts) Calculate an average density for the asteroid belt. Is it enough to significantly alter the rotation curve within the belt (is ρ comparable to M_{\odot}/R^3 ?)

Taking a mass of the asteroid belt of 3.5×10^{21} kg, the average density within the asteroid belt would be

$$\begin{aligned} \bar{\rho} &= \frac{3.5 \times 10^{21} \text{ kg}}{\pi(0.1 \text{ AU})[(3.27 \text{ AU})^2 - (2.06 \text{ AU})^2]} \\ &= \frac{1.76 \times 10^{-9} M_{\odot}}{\pi(0.1 \text{ AU})[(3.27 \text{ AU})^2 - (2.06 \text{ AU})^2]} \\ &= 8.69 \times 10^{-10} M_{\odot} \text{ AU}^{-3} \end{aligned}$$

whereas

$$\begin{aligned} \frac{M_{\odot}}{R^3} &= \frac{1 M_{\odot}}{(2.7663 \text{ AU})^3} \\ &= 4.7 \times 10^{-2} M_{\odot} \text{ AU}^{-3} \end{aligned}$$

so the mass of the asteroid belt will not be enough to greatly modify the keplarian rotation curve.

It's worth exploring what the Oort constants would be if the asteroid belt changed the rotation curve significantly from Keplerian.

2 (i)

(6 pts) Consider rotation curves at Ceres' current position (R_c) and rotational velocity (V_c), in the form of a power law

$$V(R) = V_c \left(\frac{R}{R_c} \right)^p,$$

and find dV/dR , and then A and B , as a function of p . What would Ceres' Oort constants A and B be if the rotation curve was locally flat ($p = 0$)? Solid body ($p = 1$)?

If the rotation curve is of the form above, then

$$\begin{aligned} \frac{dV}{dR} &= V_c \frac{d}{dR} \left(\frac{R}{R_c} \right)^p \\ &= \frac{p}{R_c} V_c \left(\frac{R}{R_c} \right)^{p-1} \\ &= \frac{p}{R} V_c \left(\frac{R}{R_c} \right)^p \\ &= \frac{p}{R} V(R) \end{aligned}$$

In that case, the Oort constants take a fairly simple form:

$$\begin{aligned} A &= -\frac{1}{2} \left(\left. \frac{dV}{dR} \right|_{R_c} - \frac{V_c}{R_c} \right) \\ &= -\frac{1}{2} \left(\frac{p}{R_c} V(R_c) - \frac{V(R_c)}{R_c} \right) \\ &= -\frac{p-1}{2} \frac{V(R_c)}{R_c} \\ B &= -\frac{1}{2} \left(\left. \frac{dV}{dR} \right|_{R_c} + \frac{V_c}{R_c} \right) \\ &= -\frac{p+1}{2} \frac{V(R_c)}{R_c} \end{aligned}$$

We already know $V_c/R_c = 6.47 \text{ km s}^{-1} \text{ AU}^{-1}$; so for a flat ($p = 0$) rotation curve,

$$\begin{aligned} A &= +3.24 \text{ km s}^{-1} \text{ AU}^{-1} \\ B &= -3.24 \text{ km s}^{-1} \text{ AU}^{-1} \end{aligned}$$

while for a solid body ($p = 1$) rotation curve,

$$\begin{aligned} A &= 0 \text{ km s}^{-1} \text{ AU}^{-1} \\ B &= -6.47 \text{ km s}^{-1} \text{ AU}^{-1} \end{aligned}$$

as compared to our expected Keplerian values of $A = 4.86 \text{ km s}^{-1} \text{ AU}^{-1}$ and $B = -1.62 \text{ km s}^{-1} \text{ AU}^{-1}$.

2 (j)

(3 pts) Are the Oort constants sensitive enough measures to distinguish between these cases – if with more measurements, the students' measured values of the Oort constant at Ceres were uncertain by 10%, would they still be able to rule out flat and solid body rotation curves? How about with the data as plotted above?

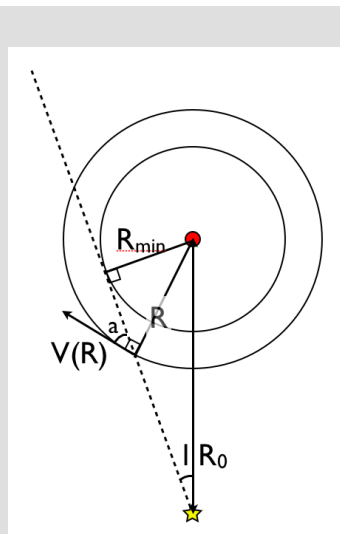
Note that even with the large uncertainties from the data in the plot above, we could easily rule out solid body rotation, whereas flat rotation might still be possible. However, if we had 10% uncertainties, so that $A = 4.9 \pm 0.5 \text{ km s}^{-1} \text{ AU}^{-1}$ and $B = -1.6 \pm 0.2 \text{ km s}^{-1} \text{ AU}^{-1}$, both other rotation curves would be firmly ruled out.

Question 3 - Tangent Point Method [25 pt]

In class, we discussed the Tangent Point Method, and stated (with a qualitative geometric argument) that when observing at a line of sight at Galactic longitude l , for all not-too-steeply outwardly increasing rotation curves, the maximum observed relative radial velocity was that of the orbit closest to the Galactic centre along the line of sight. We will again assume circular orbits.

3 (a)

(2 pts) Write an expression for relative velocity along the line of sight at Galactic longitude l for all observed objects at any radius R in terms of the rotation velocity at that radius $V(R)$, the angle l , and our radius R_0 .



Consider the geometry as sketched to the left; the solar position is at R_0 , we are looking along galactic longitude l , seeing objects at radii from just within R_0 to $R_{\min} = R_0 \sin(l)$. At any intermediate radius R , we see objects moving radially away from us with speed $V_r(R) = V(R) \cos(a)$.

To calculate $\cos a$, it is enough to note that the angle opposite R_0 is $\pi/2 + a$, and so by law of sines,

$$\begin{aligned} \frac{R_0}{\sin(\frac{\pi}{2} + a)} &= \frac{R}{\sin l} \\ \frac{R_0}{\sin \frac{\pi}{2} \cos a + \cos \frac{\pi}{2} \sin a} &= \frac{R}{\sin l} \\ \frac{R_0}{\cos a} &= \frac{R}{\sin l} \\ \cos a &= \frac{R_0 \sin l}{R} \end{aligned}$$

and thus we have

$$V_r(R) = V(R) \cos a = V(R) \frac{R_0 \sin l}{R}$$

3 (b)

(5 pts) Prove that for a flat rotation curve $V(R) = V_0$, the maximum observed velocity occurs for objects with an orbit at $R = R_{\min} = R_0 \sin l$.

For a flat rotation curve we have $V_r(R) = V_0 R_0 \sin l / R$, so for fixed l , this is just proportional to $1/R$; the largest value for this is for the smallest value R which we can see looking at angle l , which is $R_{\min} = R_0 \sin l$.

More formally, if we take the derivative to look for minima or maxima of $V_r(R)$,

$$\frac{dV_r}{dR} = -\frac{V_0 R_0 \sin l}{R^2},$$

we see that there are no zeros and thus no minima or maxima within $[R_{\min}, R_0)$, so that the extremal values must occur at the ends of the interval; testing each, we see the maximum clearly occurs at R_{\min} .

3 (c)

(5 pts) For power-law rotation curves as in the question on Oort constants, at what minimum p is this no longer true?

For a power law rotation curve,

$$V(R) = V_0 \left(\frac{R}{R_0} \right)^p$$

so that we have

$$\begin{aligned} V_r(R) &= V(R) \frac{R_0 \sin l}{R} \\ &= V_0 \left(\frac{R}{R_0} \right)^p \frac{R_0 \sin l}{R} \\ &= V_0 \sin l \left(\frac{R}{R_0} \right)^{p-1} \end{aligned}$$

and the derivative is

$$\frac{dV_r}{dR} = V_0 \sin l (p-1) \left(\frac{R}{R_0} \right)^{p-2}$$

Note that the slope of the observed radial velocity does not change sign over the interval; the velocity is monotonically increasing ($p > 1$), decreasing ($p < 1$) with R , or constant at $p = 0$. If the observed radial velocity is monotonically decreasing, then the maximum is at the smallest R — that is, $R_{\min} = R_0 \sin l$. On the other hand, for monotonically increasing $V_r(R)$, the maximum is at the maximum R seen — which is R_0 , and isn't very useful (because we already know the rotation velocity at R_0).

The p for which the Tangent Point Method breaks down, then, is $p = 1$ (solid body rotation) where $V_r(R)$ goes from monotonically decreasing to increasing. At this point, the fact that $V(R)$ increases linearly with R cancels out the $1/R$ in the expression for $V_r(R)$, and the observed radial velocity is constant, and no special R is picked out by the method.

3 (d)

(5 pts) Assuming spherical distribution of masses, and starting with $V(R)^2 = GM/R$, find the density distribution $\rho(R)$ necessary for the rotation curve to have the minimum p calculated above.

We have $V(R)^2 = GM(< R)/R$ and we want to find the $\rho(R)$ that gives an $M(< R)$ that gives $V(R) = V_0(R/R_0)^1$.

$$\begin{aligned} V(R)^2 &= \frac{GM(< R)}{R} \\ V_0^2 \frac{R^2}{R_0^2} &= \frac{GM(< R)}{R} \\ M(< R) &= \frac{V_0^2}{GR_0^2} R^3 \\ \int_{r=0}^R 4\pi r^2 \rho(r) dr &= \frac{V_0^2}{GR_0^2} R^3 \\ \frac{d}{dR} \int_{r=0}^R 4\pi r^2 \rho(r) dr &= \frac{d}{dR} \frac{V_0^2}{GR_0^2} R^3 \\ 4\pi R^2 \rho(R) &= \frac{3V_0^2}{GR_0^2} R^2 \end{aligned}$$

$$\rho(R) = \frac{3V_0^2}{4\pi GR_0^2}$$

So we need a constant density (so that $M(< R) \propto R^3$) to get solid body rotation. This is difficult to achieve in practice, since gravitation will normally produce a peaked density distribution as things fall in towards the centre.

3 (e)

(3 pts) Why does the Tangent Point Method not work for radii $R > R_0$?

The Tangent Point Method hinges crucially on there being a minimum radius observed for any given l , at which there can be a maximum of velocity. When looking outwards there is no longer any special radius — we see all $R > R_0$ for which there is Galactic matter — except for our own, which isn't very helpful.

If there was a sharp edge to our Galaxy, which there isn't, there could be a minimum (for $p < 1$) or maximum (for $p > 1$) there, but even then it would be only one data point — $V(R_{\max})$, which would be the same for all lines of sight, which again isn't very helpful.

3 (f)

(3 pts) Using our known $R_0 = 8 \text{ kpc}$ and $V_0 = 220 \text{ km s}^{-1}$, estimate the mass of the Milky Way interior to the Sun, assuming it is distributed spherically.

$$\begin{aligned} V(R_0)^2 &= \frac{GM(< R_0)}{R_0} \\ M(< R_0) &= \frac{R_0 V(R_0)^2}{G} \\ &= 9.00 \times 10^{10} M_\odot \end{aligned}$$

3 (g)

(2 pts) Assuming the mass above is mostly dark matter, and distributed spherically, find the average interior dark matter density. The local stellar density is approximately 0.1 pc^{-3} ; if the Sun is a typical-mass star, is there enough local density to make the local rotation curve deviate significantly from Keplerian - that is, is the local density likely comparable to the average interior density?

The average dark matter density within the solar circle would be:

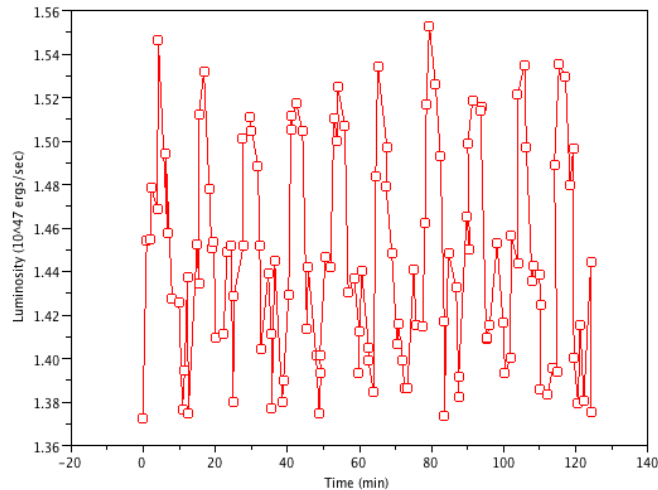
$$\begin{aligned} \bar{\rho} &= \frac{M(< R_0)}{\frac{4}{3}\pi R_0^3} \\ &= \frac{9.00 \times 10^{10} M_\odot}{\frac{4}{3}\pi (8 \text{ kpc})^3} \\ &= 2.69 M_\odot \text{ pc}^{-3} \end{aligned}$$

Note that this average density is an upper limit on the local density, as we expect the density distribution to be peaked centrally.

The local stellar density would be on order $0.1 M_\odot \text{ pc}^{-3}$ — which, while small (about 5% of the number calculated above) is at least in the same ballpark as the interior density. And as we'll discuss later, the stellar density is just the tip of the iceberg.

Question 4 - Supermassive Black Holes [25 pt]

Plotted below is xray data taken from an object believed to be a central black hole in a galaxy found in the Non-Existant Galaxy Survey, NEGS 1234. The data is also found on the course webpage under problem sets.



4 (a)

(5 pts) Plot the data, using the tool of your choice (a tutorial on data plotting with a few different tools will be available shortly on the course website).

Your plot, of course, should look like the one above.

4 (b)

(5 pts) By eye, estimate the periodicity of the signal, in minutes, and derive an upper limit of the size of the emitting object.

Answers may slightly vary, but there are 10 peaks (and thus 9 periods) between approximately 5 minutes and 115 minutes, so the period is something like 12.2 min. From our speed-of-light argument, this places a constraint on the upper size of the pulsing object of

$$R = c\Delta t = 2.2 \times 10^8 \text{ km} = 1.47 \text{ AU}$$

4 (c)

(5 pts) Estimate in your plotting tool (preferably) or by eye the mean luminosity of the emitting object. If this is accretion-powered luminosity, to what accretion rate (in solar masses per year) does this correspond?

The mean luminosity of the data given is $1.4455 \times 10^{47} \text{ erg s}^{-1}$, though answers may vary somewhat. That corresponds to a mass accretion rate of

$$\dot{E} = \frac{1}{4} \dot{M} c^2$$

$$\begin{aligned}
 \dot{M} &= \frac{4\dot{E}}{c^2} \\
 &= \frac{4 \times 1.4455 \times 10^{47} \text{ erg s}^{-1}}{c^2} \\
 &= 10.2 M_{\odot} \text{ yr}^{-1}
 \end{aligned}$$

4 (d)

(5 pts) If, from dynamical arguments, this black hole is believed to have the same mass as the central object in our own galaxy (approximately $3.5 \times 10^6 M_{\odot}$), what would its Schwarzschild radius be?

The Schwarzschild radius is given by

$$\begin{aligned}
 R_s &= \frac{2GM}{c^2} \\
 &= \frac{2G(3.5 \times 10^6 M_{\odot})}{c^2} \\
 &= 1.03 \times 10^7 \text{ km} = 0.069 \text{ AU}
 \end{aligned}$$

4 (e)

(5 pts) Our own supermassive black hole dominates the dynamics of the galaxy within the inner 100 pc or so; let's investigate if it affects dynamics at the Local Standard of Rest significantly. You have above estimated the mass of the galaxy interior to the solar circle. Imagine that our central black hole somehow was instantaneously whisked away, leaving the rest of the Milky Way behind. Once the galaxy had adjusted, what would the new circular rotation velocity be at the current solar circle? Is this a significant change?

We calculated above the mass interior to the solar circle based on our rotation data:

$$\begin{aligned}
 V(R_0)^2 &= \frac{GM(< R_0)}{R_0} \\
 M(< R_0) &= \frac{R_0 V(R_0)^2}{G} \\
 &= 9.00 \times 10^{10} M_{\odot}
 \end{aligned}$$

If we subtract off the $3.5 \times 10^6 M_{\odot}$ mass of the black hole, we barely even see that in the updated mass:

$$\begin{aligned}
 V'(R_0)^2 &= \frac{GM'(< R_0)}{R_0} \\
 &= G8.99965 \times 10^{10} M_{\odot} R_0 \\
 V'(R_0) &= 219.969 \text{ km s}^{-1}
 \end{aligned}$$

Since we changed the mass by 0.004%, the velocity ended up being changed by 0.62% — this is a pretty small amount and is well within the current observational errors in our velocity.