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TWO-COLOUR PHOTOMETRIC STUDIES OF THE ECLIPSING BINARY SYSTEMS TW DRACONIS, Z HERCULIS AND RS VULPECULAE

BY

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Summary.

The observations on which this study is based were obtained in 1950 and 1951. Part of the observations were made at the David Dunlap Observatory, but the greater part were made at the Steward Observatory of the University of Arizona.

The three eclipsing variables, TW Draconis, Z Herculis and RS Vulpeculae, were chosen for observation in the hope that the circumstances of the eclipses might prove favourable for the estimation of the limb darkening of the components by showing annular, or deep total, eclipses. The system TW Drac shows a deep total eclipse at primary minimum, and it is possible to derive an estimate of the limb darkening in two colours from a reduction of the observations in the primary minimum by the method of least squares. This variable has been previously studied by R. H. Baker,[1] who reported an asymmetric light curve. I find the asymmetry to be much smaller than reported by Baker.

Nijland's visual observations of the system Z Herc were interpreted by Fetlaar[2] as being due to an annular eclipse. I find, however, a partial eclipse, and the solution is too indeterminate to allow a determination of the limb darkening effect. It is possible to estimate the difference in the degree of limb darkening in two colours.

The system RS Vulp has been studied by Baker[3] and by Dugan[4]. Baker found an annular eclipse at primary minimum, Dugan a partial. The present study agrees with Dugan's. Again the uncertainties in the solution of a system exhibiting a partial eclipse do not allow a determination of the limb darkening, but an estimate can be made of the differential limb darkening.

Corrections to the times of minima predicted from the ephemerides were found to be necessary in all three systems. The reason for the departure of the systems TW Drac and Z Herc from the ephemeris is not clear. A small correction to the period of RS Vulp is suggested.

A study of the colour of the reflection effect shows that in each system the colour of the reflected light is the same as the colour of the brighter component.

The measurements of the ellipticity effect are not sufficiently exact to allow conclusions to be drawn about the photometric behaviour of the distorted components.

1. *Observational Material—General.*

The measurements in this study were made with the aid of the photoelectric photometers of the David Dunlap Observatory and of the Steward Observatory, Tucson, Ariz. (April-June 1951). Experiments in photoelectric photometry using the 1P21 photomultiplier were begun at the David Dunlap Observatory late in 1948, and have continued. At present the measuring instrument is a null-indicating D.C. amplifier mounted on a 19-inch telescope. The photometer used at the Steward Observatory is mounted on a 36-inch telescope and uses a light chopper with an A.C. amplifier.

Atmospheric conditions were mostly much better at Tucson, where the bulk of the observations were made, than at Toronto. Pettit has made an extensive study of the transmission of the atmosphere at Tucson. On some few nights measurements were made through dust, haze or broken cloud. In general the transparency was good. The comparison stars were chosen to be as like the variables in colour, and as close in position, as possible. Corrections for differential extinction are believed to be negligible in comparison with observational scatter. There was no convenient constant light source for determining the changes in the transmission of the atmosphere. The measurements were made in two colours of effective wave-lengths 4370A. and 5100A. A group of four comparisons in the two colours took 20 to 30 minutes to complete, and the resulting normal point of observation is thought to have a probable error of between $0^m.005$ and $0^m.010$.

2. *Observational Material—TW Draconis.*

The system TW Drac was observed on 32 nights. Approximately 400 measures were made which were averaged into some 130 normal points. The normal points are tabulated by phase in the appendix. The comparison star used was H.D. 140512, R.A. (1900) 15 h. 38 m., Dec. (1900) 62° 11′, A5; the variable is H.D. 139319, R.A. (1900) 15 h. 32 m., Dec. (1900), 64° 14′, A6 and G2.

Primary minimum seems quite symmetrical. The harmonic analysis of the observations outside eclipse shows a small asymmetry which appears to be real, but may be the result of a bad distribution of observations, or intrinsic variability of one of the components of the eclipsing system. The eclipsing system TW Drac has a small visual companion, about 3 seconds of arc away. The measures of the system include the light of the visual companion. The light of the visual companion was measured independently; it proved difficult and only on a few nights was the seeing sufficiently steady to allow the two components to be observed independently. Nineteen measurements were made on three nights and the brightness of the companion star to the system TW Drac was found to be 0.051 in both colours, the light unit being the luminosity of the companion star H.D. 140512. This amount was subtracted from the observations before analysis.

Plots of the observations are shown in figures 1 and 2. Secondary minimum appears to be later than the half-period by $0^d.010$ to $0^d.020$. The observed epoch of primary minimum was 1951 June 1.128, J.D. 2433798.628.

3. Observational Material—Z Herculis.

The system Z Herc was observed on 26 nights. Approximately 400 measures were made which were averaged into 95 normal points shown in the appendix. A plot of the observations is given in figures 3 and 4. The comparison stars used were H.D. 164043, R.A. (1900) 17 h. 54.2 m., Dec. (1900) 14° 52′, F8; and H.D. 162705 R.A. (1900) 17 h. 47.3 m, Dec. (1900) 15° 01′, F0. The variable is H.D. 163950, R.A. (1900) 17 h. 53.6 m., Dec. (1900) 15° 09′, F2 and F5.

The estimated epoch of primary minimum was 1951 June 3.503, J.D. 2433801.003.

4. Observational Material—RS Vulpeculae.

The system RS Vulp was observed on 31 nights. Approximately 500 measures were made, which were averaged into some 130 normal points, shown in the appendix. Plots of the observations are shown in figures 5 and 6. The comparison stars used were H.D. 180889, R.A. (1900) 19 h. 13.2 m., Dec. (1900) 21° 38′, A3; and H.D. 180811, R. A. (1900) 19 h. 12.9 m., Dec. (1900) 22° 15′, B9. The variable is H.D. 180939, R.A. (1900) 19 h. 13.4 m., Dec. (1900) 22° 16′, B8 and A5.

Primary minimum is well covered, but unfortunately it proved impossible to cover the ascending branch of secondary minimum. Secondary minimum occurs later than the half-period by $0^d.030$ to $0^d.040$. The epoch of primary minimum from these observations was 1951 June 4.787, J.D. 2433802.287.

5. Analysis of Observations—TW Draconis.

Harmonic analysis of observations outside eclipse yielded

$$\begin{array}{lllll} L = & 2.012 & -\ 0.0198\cos\theta & -\ 0.0613\cos^2\theta & +\ 0.0172\sin\theta, \\ & (\pm\ 0.001\ \text{m.e.}) & (\pm\ 0.001\ \text{m.e.}) & (\pm\ 0.005\ \text{m.e.}) & (\pm\ 0.001\ \text{m.e.}) \end{array}$$

in the yellow, and

$$\begin{array}{lllll} L = & 1.887 & -\ 0.0191\cos\theta & -\ 0.0635\cos^2\theta & +\ 0.0098\cos\theta. \\ & (\pm\ 0.001\ \text{m.e.}) & (\pm\ 0.001\ \text{m.e.}) & (\pm\ 0.005\ \text{m.e.}) & (\pm\ 0.001\ \text{m.e.}) \end{array}$$

in the blue.

A preliminary graphical study led to an estimate of k about 0.76 to 0.78 and the coefficient of limb darkening in the two colours about

0.6. As the observations seemed to justify a more careful study the measures were grouped into normal points and rectified as shown in Table I.

TABLE I

OBSERVATIONS CORRECTED FOR LIGHT OF VISUAL COMPANION, AND GROUPED INTO NORMAL POINTS, TW DRACONIS

Phase degrees	L (obs.) (yellow)	Refl. Light	L	Ellipt. Effect	L (rect.)	Corr.
29.200	0.9469	4	0.9465	0.9726	0.9731	− 9
25.692	0.8949	3	0.8946	0.9707	0.9216	− 27
23.800	0.8368	3	0.8365	0.9698	0.8625	− 32
20.846	0.7572	2	0.7570	0.9687	0.7814	− 32
17.456	0.6394	2	0.6392	0.9672	0.6608	− 3
15.166	0.5548	1	0.5547	0.9664	0.5738	+ 32
13.170	0.4691	1	0.4690	0.9660	0.4856	+ 48
11.800	0.4207	1	0.4206	0.9656	0.4355	+ 55
10.367	0.3610	1	0.3609	0.9653	0.3739	+ 56
8.716	0.2976			0.9650	0.3083	+ 52
7.945	0.2712			0.9647	0.2811	+ 50
7.172	0.2429			0.9645	0.2518	+ 45
6.663	0.2262			0.9644	0.2345	+ 41
6.060	0.2106			0.9644	0.2183	+ 37
5.436	0.1960			0.9643	0.2032	+ 31
5.004	0.1930			0.9642	0.2001	+ 25
4.486	0.1800			0.9642	0.1866	+ 20
3.888	0.1733			0.9640	0.1797	+ 5
0.000	0.1687			0.9640	0.1750	0
	(Blue)					
29.227	0.9569	4	0.9565	0.9772	0.9788	− 9
25.637	0.8927	3	0.8924	0.9757	0.9146	− 30
23.696	0.8442	3	0.8439	0.9749	0.8656	− 35
21.200	0.7815	2	0.7813	0.9738	0.8823	− 38
17.403	0.6154	2	0.6152	0.9727	0.6324	0
15.030	0.5147	1	0.5146	0.9721	0.5293	+ 41
13.572	0.4477	1	0.4476	0.9716	0.4606	+ 53
12.080	0.3790	1	0.3789	0.9713	0.3900	+ 57
10.927	0.3313	1	0.3312	0.9710	0.3410	+ 59
9.352	0.2731			0.9707	0.2813	+ 57
8.109	0.2198			0.9706	0.2264	+ 51
6.851	0.1775			0.9705	0.1828	+ 43
6.000	0.1560			0.9704	0.1607	+ 37
5.646	0.1476			0.9703	0.1521	+ 34
5.186	0.1345			0.9703	0.1386	+ 29
4.658	0.1278			0.9702	0.1317	+ 19
4.037	0.1154			0.9702	0.1189	+ 10
0.000	0.1094			0.9700	0.1127	0

The observations of a portion of the primary minimum made on May 12 show an interesting deviation from those made on the same portion of the light curve on May 15 and June 12. The measures

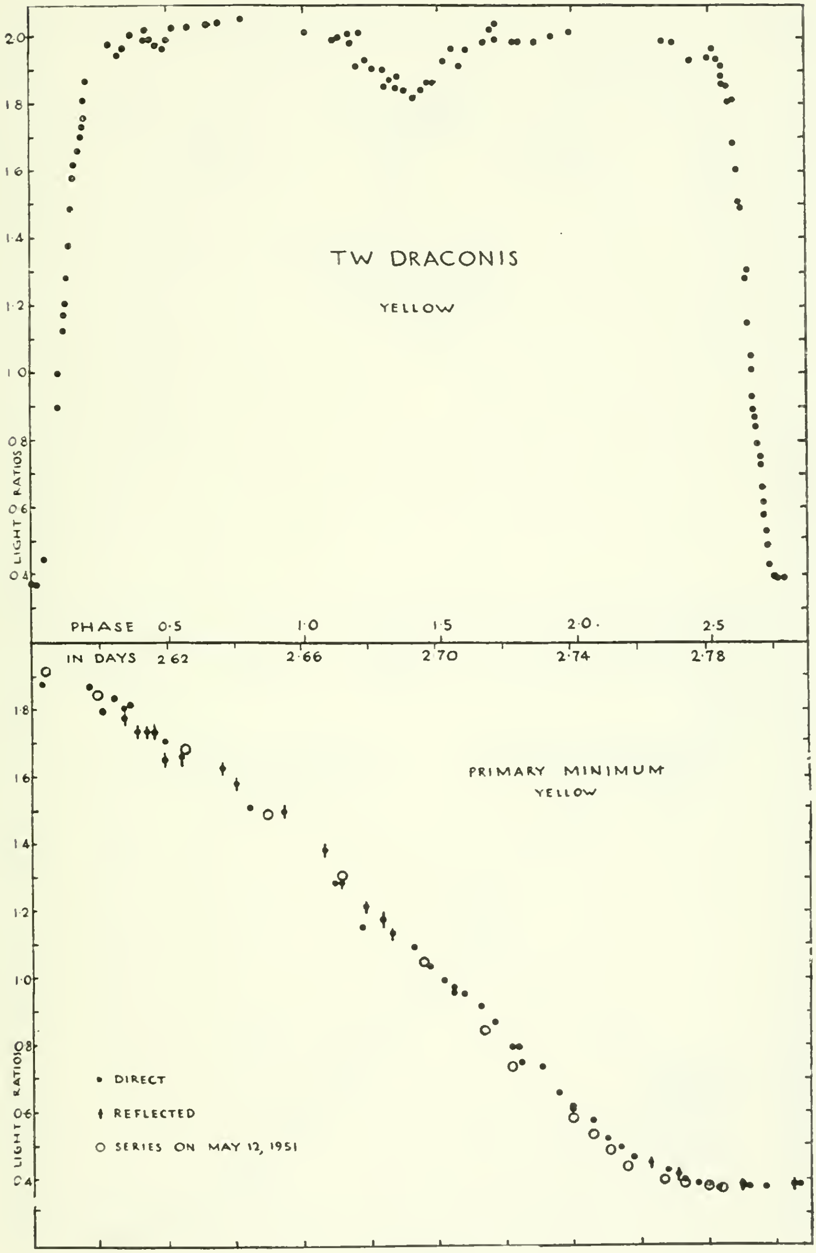
TW DRACONIS
YELLOW
LIGHT RATIOS
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
PHASE 0·5 1·0 1·5 2·0 2·5
IN DAYS 2·62 2·66 2·70 2·74 2·78
PRIMARY MINIMUM
YELLOW
• DIRECT
ϕ REFLECTED
○ SERIES ON MAY 12, 1951

FIG. 1

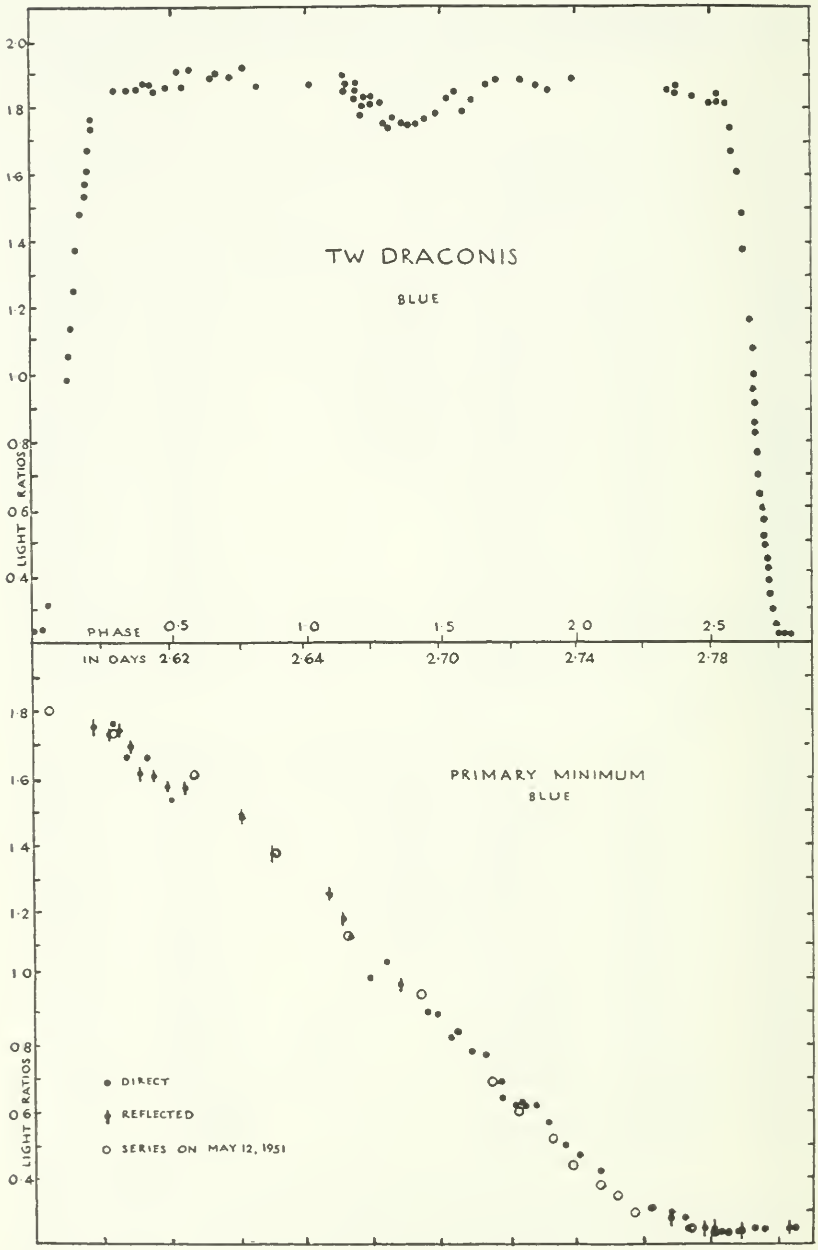
TW DRACONIS
BLUE
LIGHT RATIOS
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
PHASE
0·5
1·0
1·5
2·0
2·5
IN DAYS
2·62
2·64
2·70
2·74
2·78
PRIMARY MINIMUM
BLUE
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
● DIRECT
REFLECTED
○ SERIES ON MAY 12, 1951

Fig. 2

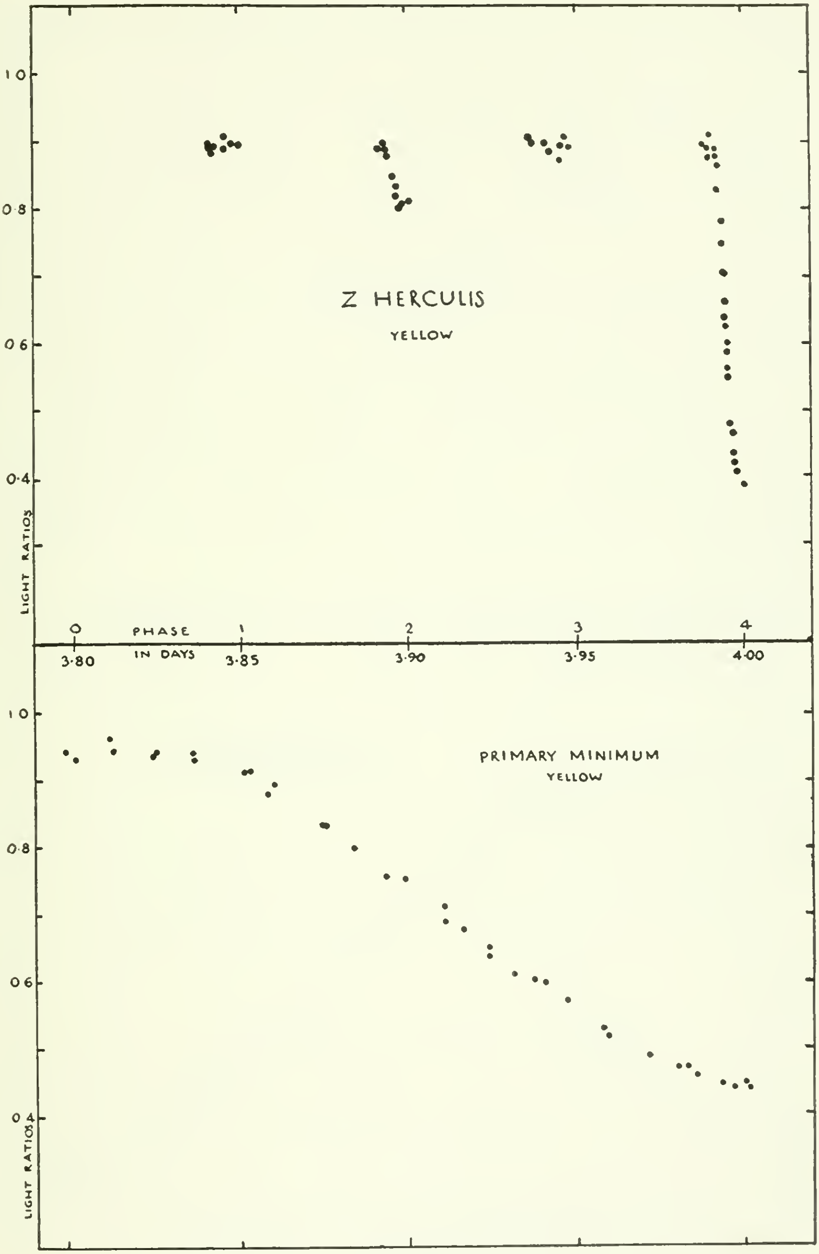
Z HERCULIS
YELLOW
LIGHT RATIOS
1·0
0·8
0·6
0·4
PHASE
IN DAYS
0
1
2
3
4
3·80
3·85
3·90
3·95
4·00
PRIMARY MINIMUM
YELLOW
1·0
0·8
0·6
0·4
LIGHT RATIOS

Fig. 3

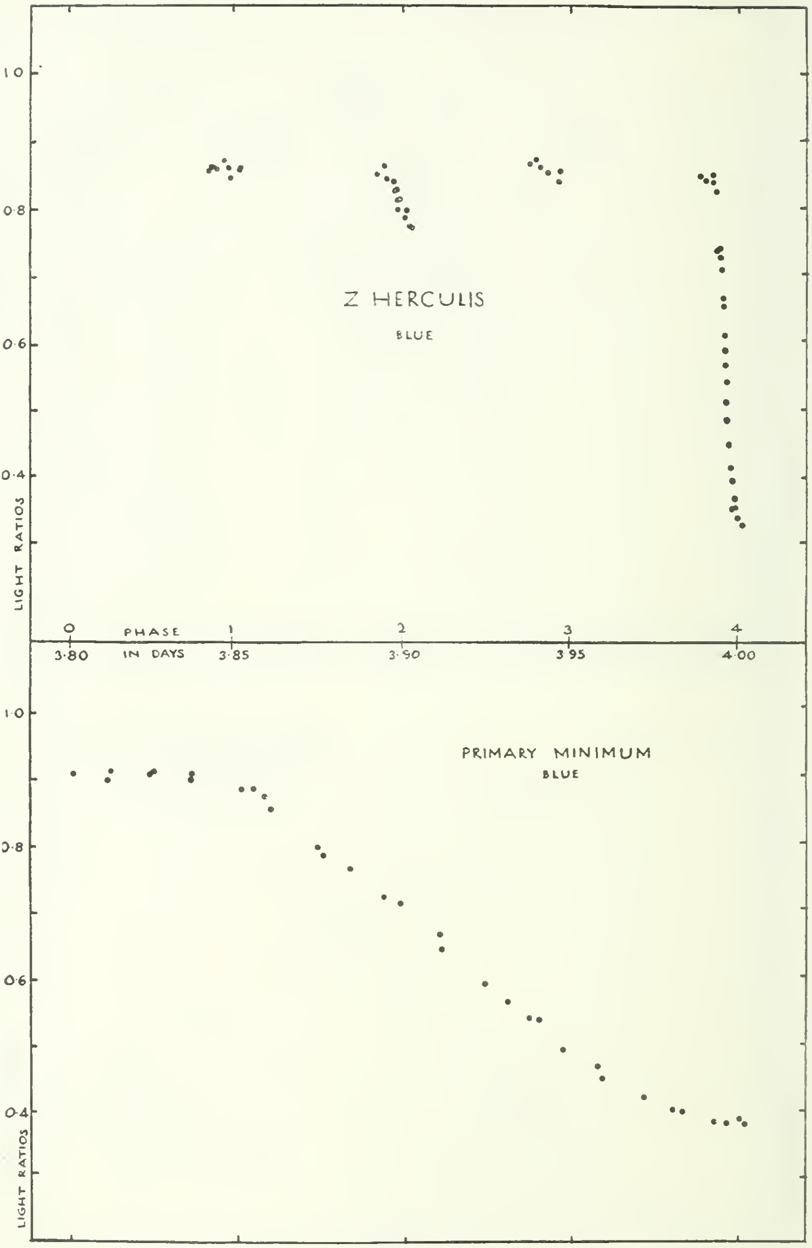
Z HERCULIS
BLUE
PRIMARY MINIMUM
BLUE
LIGHT RATIOS
LIGHT RATIOS
1·0
0·8
0·6
0·4
1·0
0·8
0·6
0·4
PHASE
IN DAYS
0
1
2
3
4
3·80
3·85
3·90
3·95
4·00

FIG. 4

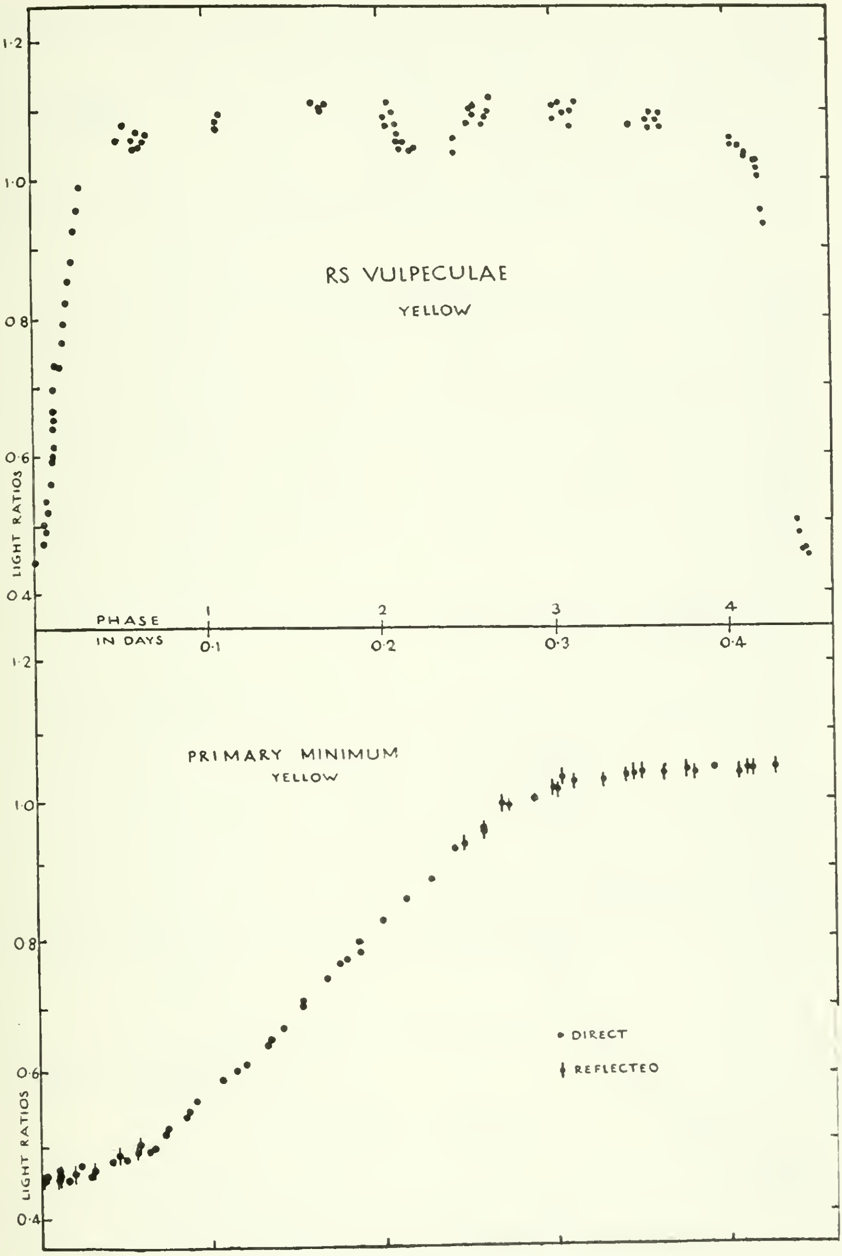
RS VULPECULAE
YELLOW
LIGHT RATIOS
1·2
1·0
0·8
0·6
0·4
PHASE
IN DAYS
1
2
3
4
0·1
0·2
0·3
0·4
PRIMARY MINIMUM
YELLOW
LIGHT RATIOS
1·2
1·0
0·8
0·6
0·4
● DIRECT
ɸ REFLECTED

Fig. 5

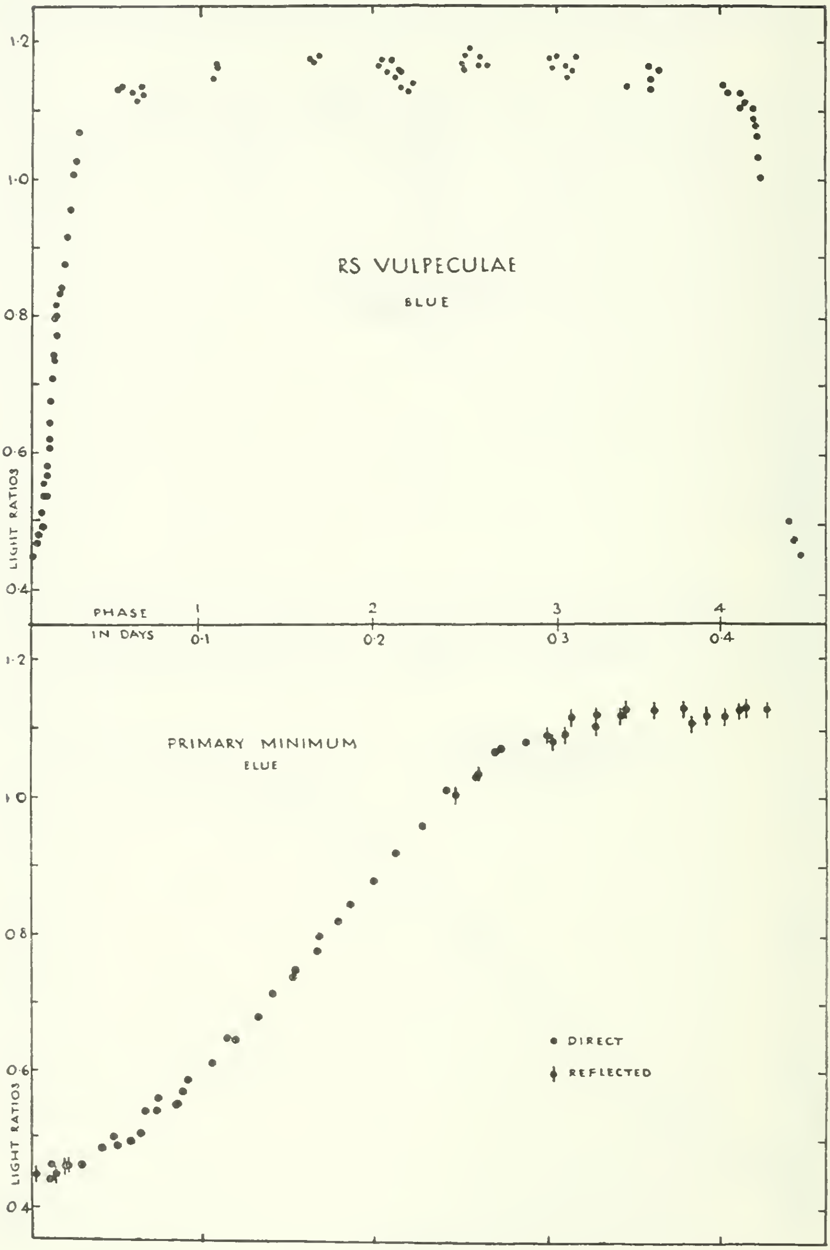
RS VULPECULAE
BLUE
LIGHT RATIOS
1·2
1·0
0·8
0·6
0·4
PHASE
IN DAYS
1
2
3
4
0·1
0·2
0·3
0·4
PRIMARY MINIMUM
BLUE
DIRECT
REFLECTED
LIGHT RATIOS
1·2
1·0
0·8
0·6
0·4

FIG. 6

made on May 12 began by falling among the measures made on other nights, but quite suddenly began to fall below the other measures by 0.02 to 0.03 light units until totality neared, when the differences diminished. The disagreement of the observations of May 12 with those of May 15 and June 12 disappeared at the onset of totality. It is believed that this variation is a real phenomenon. It is not known what cause could operate to block off 2 to 3 per cent. of the light of the brighter component of the system. Similar observations have been reported by Kron.[5]

The preliminary elements were corrected by the method of least squares, using the tables of Tsesevitsch[6] and Irwin[7]. The corrections to the preliminary elements were rather large, and the process had to be repeated. The reduction was made with the observations of primary minimum only. The secondary minimum is shallow, annular and afflicted with the reflection effect. The uncertainties of the process of "rectification" were thought to be large. The reduction was made in the two colours simultaneously. This requires some justification. The preliminary graphical analysis did not show any large difference in the elements derived from the two light curves independently. After the least-squares reduction the residuals of computed from observed points showed a similar trend in each light curve. This would not be the case if there was a significant difference in the geometric elements defined by the two colours. It is believed that the determination of the elements is considerably strengthened by making the reduction in the two colours simultaneously.

The fainter component of the system TW Drac is considerably distorted. The distortion was estimated using Pierce's estimate of the mass ratio quoted by Wood[8]. An attempt was made to correct for the effect of the distorted form of the faint component following methods suggested by Kopal.[9] Following Kopal, if we write for the error within the minima of the spherical model,

$$L = - L_1[(1 - x)h^u + xh^d],$$

then I have used

$$h^u = \frac{1}{k^2}(V_1 - V_2)I_1{}^0{}_{,0}$$

$$\frac{2}{3}h^d = \frac{1}{k^2}(V_1 - V_2)I_1{}^0{}_{,1}\left(\frac{\delta}{r_2}\right)^{\frac{1}{2}},$$

where V_1 and V_2 are the rotational distortions of the bright and faint components respectively. I find the additional terms suggested by Kopal are sufficiently small to be neglected.

The α-function corrected for "perturbations" is shown in Table II.

TABLE II

α-FUNCTIONS, TW DRACONIS

Phase degrees	Yellow α (corr.)	α (uncorr.)	Phase degrees	Blue α (corr.)	α (uncorr.)
29.200	0.0336	0.0326	29.227	0.0249	0.0239
25.692	0.0983	0.0950	25.637	0.0996	0.0962
23.800	0.1705	0.1667	23.696	0.1554	0.1515
20.846	0.2688	0.2650	21.200	0.2270	0.2228
17.456	0.4115	0.4111	17.403	0.4142	0.4143
15.166	0.5126	0.5165	15.030	0.5258	0.5305
13.170	0.6176	0.6235	13.572	0.6019	0.6079
11.800	0.6775	0.6842	12.080	0.6810	0.6875
10.367	0.7515	0.7589	10.927	0.7360	0.7427
8.716	0.8321	0.8384	9.352	0.8035	0.8100
7.945	0.8653	0.8714	8.109	0.8661	0.8718
7.172	0.9014	0.9069	6.851	0.9161	0.9210
6.663	0.9229	0.9279	6.000	0.9417	0.9459
6.060	0.9430	0.9475	5.646	0.9517	0.9556
5.436	0.9620	0.9658	5.186	0.9675	0.9708
5.004	0.9665	0.9696	4.658	0.9764	0.9786
4.486	0.9835	0.9859	4.037	0.9918	0.9930
3.888	0.9936	0.9943			

These values of the α-function are to be compared with the values in the tables for a spherical model darkened to the limb according to the usual law. In order to see what was the effect of allowing for the distortion by Kopal's methods, the least-squares reduction was made twice, once on the α-function derived from the observations allowing for the perturbations, and once on the directly observed α-function. The elements derived in the two ways hardly differ significantly. The corrections for the effect of distortion, estimated from the preliminary elements are not quite right, but since their effect is barely significant it was not thought worth-while recomputing them.

The starting point for the final least-squares correction was $r_2 = 0.3020$, $r_1 = 0.2143$, $k = .71$, $x(\text{yellow}) = 0.2$, $x(\text{blue}) = 0.3$. With these parameters the p-function was calculated for the phase of each of the normal points of observation. The α-function was computed from the tables of Tsesevitsch[6], and the residuals were used to form a number of equations of observation for computing corrections to the parameters with the aid of Irwin's tables[7] of differential coefficients. The normal points were weighted according to the number of observations included. The observations of the deeper part of the eclipse were also given greater weight, since the accuracy of the observations gets better during the deeper part of the eclipse.

The values of the parameters resulting from the solution which takes account of Kopal's correction terms, together with the estimated probable error of the determination, were

$r_2 = 0.3064 \pm 0.0002$; $r_1 = 0.2118 \pm 0.0010$; $\cos^2 i = 0.0059 \pm 0.0005$; $x(\text{yellow}) = 0.11 \pm 0.12$; $x(\text{blue}) = 0.27 \pm 0.11$.

The solution obtained by ignoring Kopal's corrections gives the same geometrical elements, but the estimates of the limb darkening are altered to $x(\text{yellow}) = 0.25$, and $x(\text{blue}) = 0.37$. According to either solution the difference in the degree of limb darkening between the two colours is 0.15, and this difference should have good precision, since it is free from many of the systematic sources of error which make the determination of the absolute value of the limb darkening so difficult.

TABLE III

RESIDUALS OF COMPUTED FROM OBSERVED VALUES OF THE α-FUNCTION RESULTING FROM THE LEAST-SQUARES CORRECTIONS, TW DRACONIS

Comp.	Obs.	Resid.	Comp.	Obs.	Resid.
0.0199	0.0336	+0.0137	0.0182	0.0249	+0.0067
0.1002	0.0983	−0.0019	0.0997	0.0996	−0.0001
0.1599	0.1705	+0.0106	0.1527	0.1554	+0.0027
0.2711	0.2688	−0.0023	0.2537	0.2270	−0.0267
0.4187	0.4115	−0.0072	0.4236	0.4142	−0.0094
0.5194	0.5176	−0.0018	0.5532	0.5258	−0.0274
0.6215	0.6176	−0.0039	0.6081	0.6019	−0.0062
0.6808	0.6775	−0.0033	0.6822	0.6810	−0.0012
0.7557	0.7515	−0.0042	0.7370	0.7360	−0.0010
0.8315	0.8321	+0.0006	0.8104	0.8035	−0.0069
0.8641	0.8653	+0.0012	0.8647	0.8661	+0.0014
0.8955	0.9014	+0.0059	0.9135	0.9158	+0.0023
0.9156	0.9229	+0.0073	0.9416	0.9417	+0.0010
0.9382	0.9430	+0.0058	0.9517	0.9517	0.0000
0.9588	0.9620	+0.0032	0.9651	0.9675	+0.0024
0.9749	0.9665	−0.0079	0.9782	0.9764	−0.0018
0.9871	0.9835	−0.0036	0.9918	0.9918	0.0000
0.9965	0.9936	−0.0029			

The residuals show a marked systematic trend. The reason for this is unknown. It may arise from a systematic error in the method of reduction, for example in the process of rectification, or a systematic error in the analysis, for example in the assumption of linear darkening to the limb, or it may be the effect of one or two individual observations with large residuals.

Accepting Pierce's estimate of the mass ratio as 3.6 we find the relative orbit comes out to be 11.6×10^6 km. The dimensions of the components are 3.53 and 5.10 times the sun in radius, the surface gravities, 0.480 and 0.060, the mean densities 0.130 and 0.012 for the bright and faint components respectively, all in terms of the sun. The residuals of the computed α-function from the observed are shown in Table III.

6. *Analysis of Observations—Z Herculis.*

The season of 1951 promised to be unusually favourable for observing this difficult system. Unfortunately the minima were found to be occurring later than predicted by the ephemeris and it appeared doubtful whether or not the phase of conjunction was reached. The analysis of the observations based on the shape of the light curve strongly suggests that conjunction occurred between predicted phases 4.00d. and 4.01d. Although it would be a good deal more comforting to have rounded the minimum, I do not think that the estimated time of conjunction is out by more than 10 minutes. I accept the observation of earlier workers that the duration of the total phase, if any, is less than 0.015d. In combination with the shape of the light curve, this rules out the possibility of a total eclipse at primary minimum.

A few spectrographic observations in 1949 and 1951 seem to show the velocity of centre of mass greater than found by Adams and Joy[10] by 10 km./sec. If these observations are correct they may show a third body motion of the system, but further spectrographic observations will be needed to establish this point.

For photometric measures only the one comparison star H.D. 164043 was used. The two comparison stars were intercompared on 14 occasions. There was no significant evidence of variability of the comparison stars.

Harmonic analysis of the observations outside eclipse yielded

$$\begin{array}{l} L = 0.893 - 0.0028 \cos\theta - 0.0089 \cos^2\theta \\ \quad\;\; (\pm 0.001) \quad (\pm 0.001) \quad (\pm 0.005) \end{array}$$

in the yellow, and

$$\begin{array}{l} L = 0.851 - 0.0049 \cos\theta + 0.0060 \cos^2\theta \\ \quad\;\; (\pm 0.001) \quad (\pm 0.001) \quad (\pm 0.005) \end{array}$$

in the blue.

Normal points of observation were read off a smooth curve and the observations "rectified" for ellipticity in the usual way, giving the loss of light, as a function of predicted phase, shown in Table IV.

TABLE IV

RECTIFICATION, Z HERCULIS

Phase days	Rect. Lum. (yellow)	Loss of Light	Rect. Lum. (blue)	Loss of Light
3.83	1.010	0.010	1.002	0.002
3.84	0.983	0.017	0.990	0.010
3.85	0.969	0.031	0.974	0.026
3.86	0.930	0.070	0.934	0.066
3.87	0.895	0.105	0.884	0.116
3.88	0.851	0.149	0.845	0.155
3.89	0.810	0.190	0.802	0.198
3.90	0.770	0.230	0.755	0.245
3.91	0.726	0.274	0.707	0.293
3.92	0.684	0.316	0.653	0.347
3.93	0.643	0.357	0.610	0.390
3.94	0.601	0.399	0.557	0.443
3.95	0.557	0.443	0.510	0.490
3.96	0.520	0.480	0.469	0.531
3.97	0.494	0.506	0.439	0.561
3.98	0.469	0.531	0.412	0.588
3.99	0.446	0.554	0.397	0.603
4.00	0.440	0.560	0.388	0.612
4.01	0.431	0.569	0.380	0.620
1.83	1.006		1.010	
1.85	1.006		1.010	
1.87	1.000		1.003	
1.89	0.982		0.994	
1.91	0.964		0.980	
1.93	0.942		0.961	
1.95	0.924		0.941	
1.97	0.910		0.928	
1.99	0.906		0.922	
2.01	0.902		0.922	

The phases are the predicted phases from the elements given in Kukarkin and Parenago, "Catalogue of Variable Stars". The predicted time of primary minimum is at phase 3.993 d. The estimated time of primary minimum occured at about phase 4.01 d.

The observations cannot be well represented as an annular eclipse, or as a partial transit. The representation as a partial occultation is good, with a rather wide range of the parameters. The ratio of the radii, $k = 0.6$ or $k = 0.7$ gives a good representation, but k cannot be as small as 0.5, and is probably not as large as 0.8. Various trials were made of the assumed phase of conjunction from predicted phase 4.00d. to predicted phase 4.030d., but the best representation is obtained by taking conjunctions as occurring at predicted phase 4.010d.

The estimates of L_1 and L_2 were obtained by a comparison of the relative depths of the two minima at corresponding points,

$k = 0.6$ $L_1 = 0.666$ $L_2 = 0.334$ (yellow)
$L_1 = 0.720$ $L_2 = 0.220$ (blue)
$k = 0.7$ $L_1 = 0.723$ $L_2 = 0.277$ (yellow)
$L_1 = 0.783$ $L_2 = 0.217$ (blue)

These estimates give the values of the α-function shown in Table V.

TABLE V

α-FUNCTIONS, Z HERCULIS

Phase days	$k = 0.6$			$k = 0.7$		
	α (yell.)	α (blue)	$\Delta\alpha$	α (yell.)	α (blue)	$\Delta\alpha$
3.84	0.025	0.014	− 11	0.024	0.013	− 11
3.85	0.046	0.036	− 10	0.043	0.033	− 10
3.86	0.105	0.091	− 14	0.097	0.084	− 13
3.87	0.158	0.157	− 1	0.145	0.144	− 1
3.88	0.223	0.215	− 8	0.206	0.197	− 9
3.89	0.285	0.275	− 10	0.263	0.253	− 10
3.90	0.345	0.340	− 5	0.318	0.313	− 5
3.91	0.411	0.407	− 4	0.379	0.374	− 5
3.92	0.474	0.481	+ 7	0.437	0.443	+ 6
3.93	0.536	0.541	+ 5	0.493	0.497	+ 4
3.94	0.598	0.614	+ 16	0.551	0.566	+ 15
3.95	0.665	0.680	+ 15	0.613	0.625	+ 12
3.96	0.720	0.736	+ 16	0.664	0.676	+ 12
3.97	0.760	0.778	+ 18	0.699	0.716	+ 17
3.98	0.808	0.815	+ 7	0.745	0.751	+ 6
3.99	0.831	0.836	+ 5	0.765	0.770	+ 5
4.00	0.849	0.849	+ 9	0.773	0.781	+ 7
4.01	0.854	0.860	+ 6	0.785	0.791	+ 6

The systematic run of the differences in the α-functions is attributed to differential limb darkening. The run of $\Delta\alpha$, with the run of differences to be expected theoretically, is shown in figure 7. When the difference in the degree of limb darkening is estimated by a method suggested by Irwin[7] there results $\delta x = 0.22$ for $k = 0.6$ or 0.7. This estimate depends systematically on the assumptions made in the analysis, and is perhaps uncertain by ± 0.1.

The elements derived by assuming the limb darkening 0.4 in the yellow are

$$i = 83^\circ - 84^\circ,$$
$$r_1 = 0.11 - 0.13 \odot,$$
$$r_2 = 0.19 - 0.18 \odot.$$

Taking the dimensions of the relative orbit as 10.5×10^6 km., there result

radii of components	$1.8 \odot$ and $2.8 \odot$,
surface gravities	$0.46 \odot$ and $0.18 \odot$,
mean densities	$0.26 \odot$ and $0.06 \odot$,

for the brighter and fainter components respectively.

7. Analysis of Observations—RS Vulpeculae.

The phases predicted from the elements given in *Rocznik Astronomiczny*, No. 22, were corrected by 0.007d. An examination of the residuals of computed from observed phases for the last thirty years suggests strongly that the currently accepted period is a little too long. I suggest that better elements would be epoch J.D. 2420606.623, period 4.477660d.

The two comparison stars were intercompared on 36 occasions. There was no significant evidence of variability. During the deep eclipse the fainter comparison star was used, at other times the brighter comparison star.

Harmonic analysis of the observations outside eclipse yielded

$$\begin{array}{lllll} L = & 1.1576 & - 0.031 \cos\theta & - 0.016 \cos^2\theta & + 0.0006 \sin\theta, \\ & (\pm 0.001 \text{ m.e.}) & (\pm 0.001 \text{ m.e.}) & (\pm 0.003 \text{ m.e.}) & (\pm 0.001 \text{ m.e.}) \end{array}$$

in the blue, and

$$\begin{array}{lllll} L = & 1.0876 & - 0.0346 \cos\theta & - 0.0226 \cos^2\theta & + 0.0033 \sin\theta, \\ & \pm 0.001 \text{ m.e.} & \pm 0.001 \text{ m.e.} & \pm 0.003 \text{ m.e.} & \pm 0.001 \text{ m.e.} \end{array}$$

in the yellow. The units are those of the brighter comparison star.

For analysis the observations were plotted on a large scale and normal points were read off a smooth curve at intervals of 2 degrees in phase. The primary minimum was "rectified" by subtracting the contribution of the reflected light as estimated from the harmonic analysis outside eclipse. The "rectification" of ellipticity was carried out by division in the usual way. This gave the values for the loss of light, as a function of phase, shown in Table VI.

This analysis of the system RS Vulp is considerably weakened by the scant observations of the secondary minimum. The best estimate of the depth of secondary minimum, rectified for ellipticity, is 0.044 in the blue and 0.058 in the yellow, with an estimated uncertainty of 0.005.

The amount of light reflected at full phase is 0.054 in the blue and 0.063 in the yellow, so that if our estimate of the depth of the secondary minimum is correct the secondary minimum cannot be a

TABLE VI
RECTIFICATION, RS VULPECULAE

Phase	L (Yellow)	Refl. Light	L	Ellipt. Effect	L (Rect.)	1-L
0°	0.448	0.0000	0.448	1.034	0.433	0.567
2	0.451		0.451		0.436	0.564
4	0.475		0.475		0.459	0.541
6	0.517		0.517		0.500	0.500
8	0.578		0.578		0.559	0.441
10	0.630	0.0005	0.629	1.034	0.609	0.391
12	0.695	0.0008	0.694	1.035	0.671	0.329
14	0.760	0.0010	0.759		0.734	0.266
16	0.825	0.0013	0.824	1.035	0.796	0.204
18	0.883	0.0017	0.881	1.036	0.850	0.150
20	0.943	0.0021	0.941	1.036	0.908	0.092
22	0.992	0.0025	0.990	1.037	0.954	0.046
24	1.015	0.0030	1.012	1.037	0.976	0.024
	(Blue)					
0	0.448	0.0000	0.448	1.109	0.404	0.596
2	0.457		0.457		0.412	0.588
4	0.485		0.485		0.437	0.563
6	0.532		0.532		0.479	0.521
8	0.586		0.586		0.530	0.470
10	0.655	0.0005	0.654	1.109	0.590	0.410
12	0.732	0.0007	0.731	1.110	0.658	0.342
14	0.805	0.0009	0.804		0.724	0.276
16	0.877	0.0012	0.876	1.110	0.789	0.211
18	0.952	0.0015	0.951	1.111	0.856	0.144
20	1.015	0.0019	1.013		0.912	0.088
22	1.060	0.0023	1.066	1.111	0.959	0.041
24	1.090	0.0027	1.087	1.112	0.978	0.022

total eclipse. This is in agreement with the conclusion reached by Dugan[4].

On the hypothesis of a partial eclipse the observed secondary minimum is partly due to the eclipse of the reflected light and partly due to the eclipse of the intrinsic light of the faint component. I have made separate estimates of the contribution of each to the secondary minimum, assuming the reflected light is completely darkened to the limb, the intrinsic light partially. Comparison of the depths of the two minima then give estimates of the luminosity of each component for assumed values of k ranging from 0.6 to 0.9. These estimates of the luminosity were used to calculate the α-function during primary eclipse, and the hypothesis was tested by methods due to Kopal.[9]

I find the observations of RS Vulp are not well satisfied by the hypothesis of a transit at primary minimum. They were well satisfied by the hypothesis of an occultation at primary minimum with

k 0.7 or 0.8. The true value of k may be somewhat smaller than 0.7, but cannot be as small as 0.6, and is probably not as large as 0.9.

The estimates of α_0, L_1 and L_2 for the various assumed values of k were,

k	α_0	L_1	L_2	
0.6	0.677	0.837	0.163	yellow
	0.684	0.872	0.128	blue
0.7	0.648	0.876	0.124	yellow
	0.655	0.911	0.089	blue
0.8	0.622	0.912	0.108	yellow
	0.636	0.937	0.063	blue
0.9	0.601	0.932	0.068	yellow
	0.620	0.953	0.047	blue

For the values of k = 0.7 and k = 0.8 these lead to the values of the α-function shown in Table VII. The difference of the α-functions for the two colours is attributed to the difference in limb darkening. Evaluating $\Delta\alpha$ by a method suggested by Irwin[7] the best estimate is

TABLE VII

α-FUNCTIONS, RS VULPECULAE

Phase degrees	k = 0.7			k = 0.8		
	α (yell.)	α (blue)	$\Delta\alpha$	α (yell.)	α (blue)	$\Delta\alpha$
0	0.648	0.655	+8	0.621	0.636	+15
2	0.644	0.646	+2	0.618	0.628	+10
4	0.618	0.619	+1	0.595	0.601	+6
6	0.565	0.571	+6	0.548	0.556	+8
8	0.503	0.516	+13	0.483	0.502	+19
10	0.447	0.451	+4	0.429	0.437	+8
12	0.375	0.376	+1	0.361	0.365	+4
14	0.304	0.303	−1	0.292	0.295	+3
16	0.233	0.232	−1	0.224	0.225	+1
18	0.171	0.158	−13	0.165	0.154	−11
20	0.105	0.096	−9	0.101	0.094	−7
22	0.052	0.045	−7	0.050	0.043	−7
24	0.027	0.024	−3	0.026	0.023	−3

$$x = 0.18 \text{ for } k \text{ assumed} = 0.7,$$
$$x = 0.27 \text{ for } k \text{ assumed} = 0.8.$$

The values depend systematically on the assumption about the depths of the minima, and may be uncertain by about ± 0.1.

The geometrical elements are derived assuming x = 0.4 in the yellow, then

$$i = 78°,$$
$$r_1 = 0.19 - 0.21,$$
$$r_2 = 0.28 - 0.26.$$

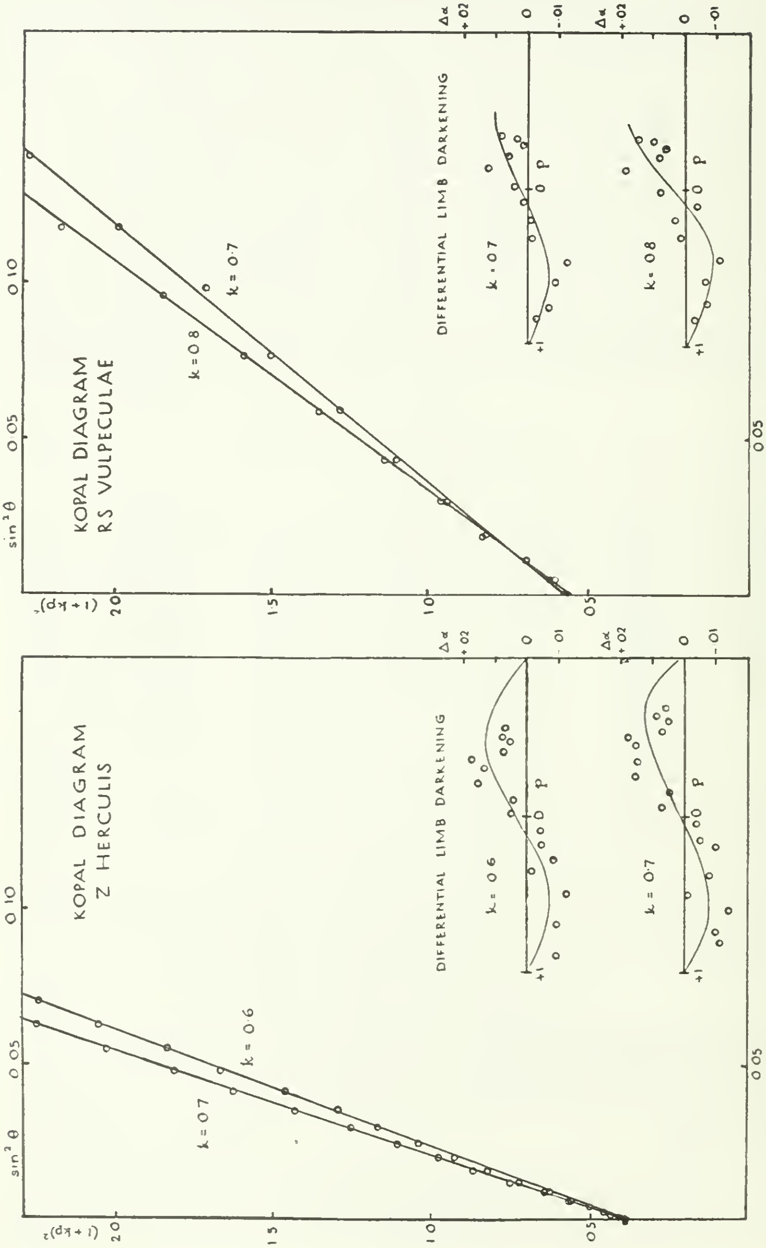
KOPAL DIAGRAM
RS VULPECULAE
KOPAL DIAGRAM
Z HERCULIS
sin²θ
0.05
0.10
(1 + kp)²
2.0
1.5
1.0
0.5
k = 0.8
k = 0.7
k = 0.6
k = 0.7
DIFFERENTIAL LIMB DARKENING
k = 0.7
k = 0.8
k = 0.6
k = 0.7
Δα
+.02
0
-.01
+1
0
p

FIG. 7

Taking the dimensions of the relative orbit as about 14.5×10^6 km., these result in

radii of components $4.2 \odot$ and $5.6 \odot$,
surface gravities $0.25 \odot$ and $0.045 \odot$,
mean densities $0.06 \odot$ and $0.008 \odot$,

for the brighter and fainter components respectively.

8. *Discussion.*

It is a valuable feature of the systems TW Drac, Z Herc and RS Vulp that we have spectrographic observation of the spectral types and of the masses of their components. These are of varying weights. The spectral types of the components of TW Drac and Z Herc are well determined, the spectral type of the faint component of RS Vulp is more uncertain. The masses of the components of Z Herc[10] and RS Vulp[11] are well determined, those of TW Drac are probably quite uncertain, and for it I use an estimate by Pierce, quoted by Wood.[8] The dimensions and densities of the components derived with the aid of the spectrographic data are quoted above. Except for the system Z Herc, the components seem to lie pretty well on the main sequence.

A simple photometric datum which can be compared with the spectrographic data is the colour index of the components of these systems. I am obliged to Prof. E. F. Carpenter for making available to me his measures of the colour indices of a number of stars of known spectral type, as observed with the Tucson photometer. The luminosities of the components are given in terms of the light of the comparison star as unit. The observed colour index of the comparison star (reduced to the zenith) can be used to calculate the colour index of the components. This colour index can be used to give an estimate of the spectral type based on Carpenter's data, and the result can be compared with the spectral type determined by the aid of the spectrograph. The results are as follows:

	C.I.	Est. Sp.	Obs. Sp.
TW Drac	− 0.20	A5	A6
	+ 0.47	K5	K2
RS Vulp	− 0.40	G	A2
	+ 0.0	F	F2
Z Herc	+ 0.04	F	F2
	+ 0.4	G-K	F5

The agreement seems reasonable except for the system Z Herc.

Another way of comparing the photometric and spectrographic data is to estimate the difference in spectral type by comparing the observed depths of minima, which are approximately in the ratio of the surface brightness of the components. The estimate is crude since it is based on the black body assumption and an assumed temperature scale which may not be valid. Russell[12] has given a table by means of which such an estimate may be made. I have used the rectified depths of minima, but have corrected them as carefully as possible for the reflected light, so that the ratios represent the undisturbed disks as nearly as possible.

The data used were:

	Depths of Min.	Log Ratio	Sp. from Russell's Table
TW Drac	0.825, 0.078	1.02	A6 and M
	0.887, 0.048	1.27	A6 and later than M
RS Vulp	0.566, 0.04	1.2	B5 and A2
	0.596, 0.03	1.3	B5 and A5
Z Herc	0.569, 0.10	0.75	F and K
	0.620, 0.08	0.84	F and later than M

The agreement seems fair for the systems TW Drac and RS Vulp. The latter system has been considered to be anomalous in this respect but my measures do not indicate very much of an anomaly. The discrepancy between the estimates of spectral type and the observed spectral type of the system Z Herc is very great, however, and it is difficult to see how the cool star can have a spectrum of type F.

Another comparison can be made with the spectroscopic data by comparing the determination of the difference in magnitude of the components of the systems RS Vulp and Z Herc which have been found spectrophotometrically by Petrie,[13] with those found in this study. Petrie finds for the system RS Vulp, Δm 3.4 ± 0.9 while I find Δm to be 2.7 to 2.9 (in the blue). The agreement seems reasonable. For the system Z Herc, however, there is a considerable discrepancy between his estimate of Δm 0.4 ± .06 and mine of Δm 1.1 to 1.4.

Another simple datum to be derived from the photometric measurements is the colour index of the reflected light. As the reflection effect is small this colour index is not very precise, but it is of interest to calculate it and compare it with the observed colour index of the components of the system. I find the colour index of the reflected light in these systems to be

	C.I. of Refl. Light
TW Drac	− 0.2,
RS Vulp	− 0.1,
Z Herc	− 0.6.

Not much weight can be attached to the measured colour for the system Z Herc as the coefficients are so small. The colour of the reflected light of the system TW Drac is the same as the colour of the brighter component. This is contrary to Milne-Eddington theory of the reflection effect. A similar observation has been made by Walter[14] on the system ζ Aurigae. The reflection effect in the system RS Vulp shows a similar tendency but the effect is smaller. The colour of the reflected light of the system is different from the colour of the fainter component, whether we consider the observed colour of the inner hemisphere (facing the brighter component) or the outer hemisphere. It does not seem, therefore, that the discrepancy can be explained by allowing for the higher temperature of the inner hemisphere. These data suggest that whatever the processes producing the reflection effect are, they are more complicated than the simple picture of absorption followed by re-emission at a lower temperature.

The calculations based on the Milne formulae give the right order of magnitude for the reflection effect except in the case of TW Drac. The computed reflection effect and the observed reflection effect for the three systems are

		Computed	Observed	
TW Drac	yellow	0.029	0.010	
	blue	0.032	0.010	
RS Vulp	yellow	0.027	0.031	$k = 0.7$
	blue	0.028	0.027	
	yellow	0.023	0.031	$k = 0.8$
	blue	0.024	0.027	
Z Herc	yellow	0.008	0.005	$k = 0.6$
	blue	0.009	0.006	
	yellow	0.007	0.005	$k = 0.7$
	blue	0.008	0.006	

The comparison of the observed ellipticity effect with the theoretical ellipticity effect is subject to considerable uncertainties arising from the uncertain contribution of the reflection effect to the second harmonic, and uncertainty about the spectral type of the faint component, and its darkening to the limb. Accepting the mass ratios for the systems which have been given from spectrographic evidence, I find the oblateness of the components due to tidal distortion to be

	Bright Component	Faint Component
TW Drac	0.004	0.150
RS Vulp	0.003 – 0.004	0.106 – 0.085
Z Herc	0.001 – 0.003	0.012 – 0.010

If the disks of the stars were uniformly bright the observed ellipticity effect would be the mean of these oblatenesses (weighted according to the luminosities of the components) multiplied by $\sin^2 i$. The observed effect may be greater by as much as a factor of 1.6 if the disks are completely darkened to the limb.

The weighted means of the oblatenesses for these systems are:

	Yellow	Blue	
TW Drac	0.027	0.020	
RS Vulp	0.016	0.012	$k = 0.7$
	0.013	0.009	$k = 0.8$
Z Herc	0.005	0.004	$k = 0.6$ or 0.7

Before a comparison can be made the observed effect must be increased by an amount estimated from the reflection effect. This amount must be considered to be rather uncertain in any case, and particularly for the system TW Drac, for which the theory does not seem to give a very good estimation of the reflection effect.

The observed ellipticity effect together with the estimated corrections from the reflection effect which are to be added to it are:

		Observed Effect	Reflection Effect	Sum
TW Drac	yellow	0.031	0.013	0.044
	blue	0.032	0.013	0.045
RS Vulp	yellow	0.014	0.011	0.025
	blue	0.020	0.011	0.031
Z Herc	yellow	0.010	0.004	0.014
	blue	0.007	0.004	0.011

The data are pretty uncertain but there seems to be an indication that there is an additional factor besides the ordinary limb darkening contributing to the observed ellipticity effect. It is not possible to estimate the magnitude of this factor. Even this conclusion would have to be altered if we have overestimated the contribution of the reflected light to the second harmonic variation.

It remains to compare the observed limb darkening effects with the theoretical effects. This effect was best observed in the system TW Drac in which the differential limb darkening is quite accurately determined, and certain limits placed on its absolute value, namely, that it is probably equal to 0.3 or less for wave-length 5100 A., and 0.5 or less for wave-length 4370 A. These determinations are to be compared with certain data calculated by Chandrasekhar and Münch[15] on the basis of a model in which the continuous absorption is due to the negative hydrogen ion, and to neutral hydrogen. The data given show that for a given spectral type the limb darkening coefficient is very nearly a linear function of the wave-length, at least in the visible region. The values they suggest for an A5 star are 0.77 for wave-length 4570A. and 0.65 for wave-length 5100A. But they remark that the linear approximation for the darkening to the limb is not very good for the visual region. Accordingly, the theoretical coefficient ought to be reduced somewhat. By considering a hypothetical eclipse of a star like the Chandrasekhar-Münch model I estimate the correction to the theoretical coefficient can hardly be greater than 0.1.

Actually the observed values of the limb darkening for A stars are smaller than the theoretical values by amounts of 0.2 and more. It happens that estimates of the limb darkening have been made for three A stars as shown here:

		Obs. Coeff.	Theor. Coeff.
TW Drac	4370A.	0.22 ± 0.12	0.77
	5100A.	0.11 ± 0.11	0.65
YZ Cass[16]	4500A.	0.40 ± 0.04	0.73
	6700A.	0.33 ± 0.03	0.44
AR Cass[17]	4500A.	0.0 ± 0.04	0.73

These measurements strongly suggest that in the visual region, at least, the observed coefficients of limb darkening are considerably smaller than the approximate theory would lead us to expect. It also appears that there are real differences in the degree of limb darkening of stars of the same spectral class.

As the limb darkening is a nearly linear function of the wave-length the expression of the difference of the degree of limb darkening in different wave-lengths as a gradient is strongly suggested. It is the nature of the analysis of eclipsing variables that the difference in the degree of limb darkening can be measured with much more reliability than the absolute amount of limb darkening, since many of the systematic uncertainties in the determination of the absolute amount of limb darkening cancel out when we form the gradient. Thus we have for the gradient (difference in limb darkening coefficient divided by difference in wave-length in thousands of Angstroms).

	Observed	Theoretical
TW Drac	0.15	0.15
YZ Cass	0.07	0.13
Z Herc	0.2 ± 0.1	0.12
RS Vulp	0.2 ± 0.1	0.12?

The gradients for TW Drac, RS Vulp and Z Herc are derived from this study. The gradient for YZ Cass is derived from Kron's measures. Strictly speaking the gradient derived from Kron's measures will be slightly affected by the fact that he reduced his measures in the two colours independently, and there is a small difference in the radii he finds for the stars in reducing the light curves in the two colours. The difference is probably not real, and a better estimate of the gradient would be obtained by forcing the geometrical elements to be the same in each colour. The effect would probably be to increase the gradient of the limb darkening estimated from the observations. The theoretical estimate of the gradient for the B5 star of the system RS Vulp is only rough as it is derived from an interpolation between the value Miss Underhill[18] gives for an O9.5 star, and the values given by Chandrasekhar and Münch for an A star.

It seems from these data that the wave-length dependence of the limb darkening coefficient is well represented by current theories.

This brief study is perhaps sufficient to show that accuracy of observation is now sufficiently good to allow us to obtain data on the limb darkening effect in a variety of systems. At the least, observations in two colours will allow a determination of the differential limb darkening effect, if systems showing a well-defined total phase are chosen. There are a number of systems which offer some hope of determining the absolute value of the limb darkening coefficient, among them TX Cass, VV Orio, TT Auri, CP Orio and TW Andr.

Some of these are systems in which two spectra are observed. Observations of a number of such systems offer hope of clearing up some of the uncertainties about the reflection and ellipticity effects. A better understanding of these effects would go far towards removing the existing systematic uncertainties in the analysis of the light curves of eclipsing binaries.

The writer desires to express his gratitude to Prof. E. F. Carpenter of the Steward Observatory for the generous facilities he provided, and to colleagues at the David Dunlap Observatory for steady support and encouragement. He is much indebted to the late Prof. F. S. Hogg, who initiated the construction of a photometer at the David Dunlap Observatory.

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Richmond Hill, Ontario
May 28, 1952

APPENDIX

NORMAL POINTS OF OBSERVATION FOR TW DRACONIS
Heliocentric phase from epoch 2433032.351 Julian Date
Period 2,8067655 days. The phases of the Ephemeris have been corrected by − 0.030 D.

1951	Phase	L (yell.)	L (blue)	1951	Phase	L (yell.)	(L blue)
Jun. 14/15	0.102	0.908		Jun. 20/21	0.505	1.994	1.864
	0.119	1.133	0.987	Jun. 23/24	0.659	2.030	1.891
	0.123	1.175	1.059		0.683	2.038	1.901
	0.128	1.213		May 12/13	0.832	1.934	1.857
	0.135	1.283	1.143	May 15/16	1.031	2.009	1.866
	0.140	1.384	1.255	May 4/5	1.162	2.000	1.850
	0.157	1.495	1.375		1.182	1.997	1.860
	0.166	1.580	1.490		1.200	1.979	1.868
	0.170	1.628			1.219	1.914	1.776
	0.182	1.660			1.207	2.008	1.851
	0.183		1.573		1.235	1.928	1.808
	0.187	1.708			1.261	1.908	1.828
	0.188		1.573		1.323	1.867	1.738
	0.192	1.734	1.611		1.349	1.880	1.750
	0.197	1.813		May 18/19	1.375	1.840	1.745
	0.198		1.746		1.131	1.993	1.899
	0.202	1.834			1.163	2.000	1.875
	0.203		1.763		1.190	2.004	1.831
May 31	0.187	1.650	1.530		1.232	2.007	1.836
Jun. 1	0.191	1.734			1.257	1.918	1.811
	0.192		1.610	May 21/22	1.299	1.912	1.820
	0.195	1.770	1.672		1.322	1.850	1.754
	0.199	1.763	1.669	Jun. 7/8	1.341	1.847	1.772
	0.204	1.795			1.433	1.817	1.748
	0.205		1.735		1.467	1.840	1.766
	0.209	1.870			1.479	1.861	1.775
	0.210		1.755		1.503	1.860	1.780
Jun. 3/4	0.297	1.979	1.852		1.548	1.932	1.829
	0.327	1.945	1.841	May 10/11	1.577	1.966	1.846
	0.345	1.965	1.850		1.613	1.910	1.785
	0.363	1.992	1.856	Jun. 9/10	1.631	1.962	1.824
	0.383	1.993	1.870		1.695	1.984	1.868
	0.411	2.015	1.868		1.720	2.013	1.873
May 20/21	0.350	2.005	1.844	May 27/28	1.726	2.020	1.880
	0.395	1.997	1.874	Apr. 29/30	1.739	1.994	1.851
May 6/7	0.397	1.989	1.844		1.824	1.982	1.882
	0.417	1.974	1.852	May 16/17	1.861	1.978	1.866
Apr. 22/23	0.492	1.967	1.860	May 2/3	1.986	2.000	1.849
	0.534	2.025	1.909	Apr. 21/22	2.019	2.009	1.879
	0.584	2.026	1.916		2.371		1.854
Jun. 6/7	0.527	2.010	1.859		2.372	1.980	
	0.548	2.018	1.863	May 8/9	2.404	1.980	1.842
May 26/27	0.737	2.049	1.892	Apr. 25/26	2.402	1.971	1.856
	0.785	2.046	1.919		2.469	1.932	1.832

Normal Points of Observation for TW Drac. (*cont.*)

1951	Phase	L (yell.)	(L blue)	1951	Phase	L (yell.)	L (blue)
May 11/12	2.535	1.938	1.816	Jun. 11/12	2.709		0.773
	2.552	1.962	1.836		2.713	0.918	0.773
	2.568	1.930	1.815		2.717	0.868	
	2.584	1.915	1.806		2.718		0.698
	2.603	1.847	1.739		2.722	0.792	0.638
	2.626	1.682			2.726	0.747	
	2.627		1.601		2.727		0.628
	2.650	1.487			2.731	0.736	
	2.651		1.379		2.732		0.576
	2.672	1.302			2.736	0.657	
	2.673		1.163		2.737		0.507
	2.695	1.049			2.740	0.617	
	2.696		0.903		2.741		0.475
	2.714	0.843			2.746	0.572	
	2.715		0.695		2.747		0.428
	2.722	0.733			2.750	0.520	
	2.723		0.608		2.754	0.493	
	2.733		0.524		2.758	0.462	
	2.739		0.447		2.762		0.316
	2.740	0.583			2.768	0.426	0.286
	2.746	0.531			2.773	0.396	0.257
	2.747		0.389		2.777	0.389	
	2.751	0.486			2.778		0.257
	2.752		0.352		2.780	0.381	
	2.756	0.435			2.781		0.253
	2.757		0.303		2.783	0.372	
	2.767	0.398			2.784		0.242
	2.768		0.302		2.785	0.377	
	2.773	0.385			2.786		0.240
	2.774		0.253		2.790	0.374	
	2.780	0.378			2.791		0.247
	2.781		0.244		2.792	0.374	
	2.785	0.370			2.793		0.254
	2.786		0.248		2.796		0.253
Apr. 27/28	2.584	1.879	1.802		2.797	0.376	
	2.607		1.666		0.000	0.382	
	2.608	1.804			0.002		0.256
	2.641		1.481	May 14/15	2.705	0.972	0.845
	2.644	1.508			2.724	0.794	0.642
	2.670	1.281			2.740	0.608	
	2.671		1.181		2.741		0.480
Jun. 11/12	2.678	1.153			2.771	0.412	
	2.679		1.009		2.772		0.289
	2.693	1.091			2.789		0.259
	2.694		0.957		2.790	0.3855	
	2.698	1.032			0.002	0.383	
	2.699		0.915		0.004		0.253
	2.702	0.992			0.023	0.390	0.252
	2.703		0.828		0.044	0.450	0.317
	2.708	0.952					

NORMAL POINTS OF OBSERVATION FOR Z HERCULIS

Heliocentric phase from epoch 2413086.365 Julian Date
Period 3.992795 days

1951	Phase	L (yell.)	L (blue)	1951	Phase	L (yell.)	L (blue)
May 2/3	0.8034	0.8875	0.8423	May 16/17	2.7602	0.9060	0.8663
	0.8220	0.8858	0.0456		2.790	0.8949	0.8552
	0.8389	0.8885	0.8530		2.924	0.8681	0.8343
	0.9089	0.8858	0.8563	May 24/25	2.8607	0.9020	0.8445
May 6/7	0.8081	0.8906	0.8508		2.8739	0.8876	0.8510
	0.8475	0.8893	0.8504	Jun. 13/14	2.9349	0.8994	0.8510
May 10/11	0.8085	0.8828	0.8525		2.9475	0.9031	0.8485
	0.8267	0.8796	0.8484	May 17/18	3.7729	0.8933	0.8442
	0.9400	0.8929	0.8528		3.7852	0.8820	0.8414
May 22/23	0.8760	0.8895	0.8560		3.7978	0.8779	0.8360
May 26/27	0.9024	0.9010	0.8656		3.8111	0.8852	0.8391
Jun. 15/16	0.9438	0.8917	0.8372		3.8240	0.8836	0.8391
	0.9675	0.8916	0.8477		3.8352	0.8838	0.8363
	0.9808	0.8898	0.8514		3.8517	0.8590	0.8213
	0.9868	0.8897	0.8536		3.8592	0.8383	0.7928
May 3/4	1.8502	0.8924	0.8582		3.8745	0.7781	0.7249
May 7/8	1.8587	0.8841	0.8584		3.9099	0.6311	0.5845
May 15/16	1.8562	0.8803	0.8486		3.9232	0.5837	0.5326
May 27/28	1.8139	0.8851	0.8445		3.9363	0.5494	0.4824
	1.8262	0.8827	0.8478		3.9430	0.5164	0.4464
	1.8364	0.8916	0.8437	May 9/10	3.8005	0.8729	0.8435
	1.8512	0.8850	0.8418		3.8280	0.8531	0.8078
	1.8636	0.8857	0.8373		3.8569	0.8218	0.7838
	1.8779	0.8728	0.8371		3.8732	0.7788	0.7371
	1.8932	0.8675	0.8395		3.8925	0.7014	0.6632
	1.9064	0.8430	0.8231		3.9151	0.6208	0.5651
	1.9242	0.8366	0.8333		3.930	0.5548	0.5071
	1.9442	0.8392	0.8201	May 25/26	3.8102	0.9053	0.8347
	1.9600	0.8170	0.8080		3.8227	0.8782	0.8454
Jun. 16/17	1.9481	0.8123	0.7908		3.8353	0.8736	0.8361
	1.9567	0.8060	0.7831		3.8503	0.8554	0.8206
	1.9660	0.8008	0.7763		3.8825	0.7422	0.7063
	1.9760	0.8042	0.7796		3.8977	0.6994	0.6517
	1.9851	0.8017	0.7838		3.9096	0.6566	0.6075
	1.9888	0.7982	0.7733		3.9233	0.5955	0.5378
	2.0018	0.8069	0.7693		3.9395	0.5425	0.4808
Jun. 20/21	1.9468	0.8145	0.8197		3.957	0.4750	0.4100
	1.9508	0.8101	0.7931	Jun. 14/15	3.9825	0.4112	0.3470
	1.9563	0.8113	0.8073		3.9926	0.3913	0.3273
	1.9744	0.8015	0.7924		3.9961	0.3885	0.3259
	1.9772	0.7989	0.7882	Jun. 22/23	3.9583	0.4606	0.3929
	1.979	0.7949	0.7906		3.9713	0.4303	0.3633
May 8/9	2.8293	0.8930	0.8480		3.9912	0.4014	0.3481
	2.8453	0.8805	0.8456		3.9999	0.3924	0.3305
	2.930	0.8980	0.8481		4.0090	0.3838	0.3233
May 16/17	2.7422	0.9021	0.8618				

NORMAL POINTS OF OBSERVATION FOR RS VULPECULAE
Heliocentric phase from epoch 2428760.434 Julian Date
period 4.477666 days

1950	Phase	L (yell.)	L (blue)	1951	Phase	L (yell.)	L (blue)
Jun. 8/9	2.082	1.069		Jun. 26/27	0.1972	0.822	0.872
	2.087	1.065			0.2114	0.852	0.913
	2.098	1.061			0.2250	0.882	0.953
	2.102	1.061			0.2390	0.926	1.005
	2.109	1.050			0.2560	0.955	1.024
	2.126	1.034			0.2698	0.988	1.069
	2.132	1.030		May 8/9	0.4868	1.053	1.128
	2.142	1.038		May 26/27	0.5399	1.077	1.128
Jun. 15	2.210	1.045		Jun. 13/14	0.5725	1.053	1.125
Jun. 17/18	2.419	1.035			0.5801	1.042	1.124
Jul. 5/6	2.247	1.039			0.5986	1.067	1.130
Sep. 6/7	2.434	1.057			0.6134	1.045	1.101
					0.6275	1.051	1.132
1951				Jun. 4/5	0.6493	1.053	1.120
May 12/13	0.0024	0.4465	0.4464	May 22/23	1.0644	1.070	1.148
	0.0154	0.4494	0.4447	Jun. 1	1.0609	1.083	1.163
	0.0283	0.453	0.456		1.0775	1.092	1.160
	0.0572	0.5014	0.4928	Jun. 9/10	1.0458	1.072	1.144
Jun. 8/9	0.0140	0.4562	0.4468		1.0689	1.089	1.163
	0.0333	0.4618	0.4690		1.0896	1.083	1.161
	0.0486	0.4718	0.4861	Jun. 13/14	1.6641	1.097	1.172
	0.0650	0.4922	0.5358		1.6821	1.105	1.177
	0.0830	0.5391	0.5487	Jun. 23/24	1.6012	1.109	1.173
	0.1177	0.6136	0.6402		1.6272	1.101	1.170
	0.1344	0.651	0.715		1.6516	1.100	1.172
	0.1510	0.697	0.7325		1.6701	1.104	1.179
	0.1644	0.7301	0.7709	Jun.1/2	2.0237	1.080	1.162
	0.1738	0.760	0.799		2.0486	1.100	1.165
	0.1814	0.776	0.831		2.0691	1.095	1.158
May 21/22	0.0103	0.4641	0.4575		2.0972	1.069	1.147
	0.0234	0.4678	0.4550		2.1243	1.054	1.157
	0.0408	0.4730	0.4819		2.1458	1.061	1.154
	0.0574	0.4910	0.4911	Jun. 10/11	2.0229	1.086	1.171
	0.0726	0.5142	0.5358		2.0371	1.076	1.172
	0.0858	0.5459	0.5654		2.0482	1.075	1.151
Jun. 26/27	0.0492	0.4762	0.4994		2.0614	1.075	1.159
	0.0622	0.4874	0.5107		2.0741	1.075	1.159
	0.0743	0.5206	0.5542		2.0883	1.076	1.171
	0.0889	0.5607	0.5805		2.0979	1.070	1.148
	0.1039	0.5911	0.6064		2.1144	1.050	1.150
	0.1142	0.6064	0.6423		2.1322	1.042	1.141
	0.1296	0.6408	0.6744		2.1475	1.035	1.130
	0.1386	0.6672	0.7091		2.1619	1.043	1.131
	0.1519	0.7045	0.7396		2.176	1.037	1.127
	0.1650	0.7337	0.7934		2.199	1.043	1.139
	0.1764	0.7650	0.8160	May 10/11	2.5253	1.078	1.157
	0.1838	0.7918	0.8381	Jun. 6/7	2.5434	1.100	1.163

Normal points of Observation for RS Vulp. (*cont.*)

1951	Phase	L (yell.)	L (blue)	1951	Phase	L (yell.)	L (blue)
Jun. 6/7	2.5685	1.101	1.160	Jun. 16/17	3.5808	1.072	1.138
	2.5914	1.089	1.162	May 16/17	4.0282	1.057	1.134
	2.6081	1.091	1.162	Jun. 12/13	4.058	1.043	1.126
	2.6345	1.097	1.161		4.0747	1.044	1.124
Jun. 15./16	2.5330	1.087	1.178		4.0885	1.033	1.115
	2.5542	1.079	1.186		4.1049	1.035	1.103
	2.5062	1.092	1.165		4.1406	1.031	1.124
	2.5927	1.076	1.174		4.1579	1.024	1.099
	2.6172	1.087	1.163		4.1701	1.021	1.088
	2.6375	1.116	1.190		4.1862	1.011	1.077
May 2/3	3.0017	1.106	1.172		4.1997	0.998	1.076
May 15/16	3.0190	1.102	1.166	Jun. 21/22	4.0403	1.045	1.123
May 24/25	3.0434	1.108	1.177		4.0618	1.042	1.128
Jun. 2/3	3.0118	1.088	1.160		4.0874	1.043	1.116
	3.0293	1.094	1.173		4.0999	1.041	1.127
	3.0444	1.092	1.167		4.1182	1.035	1.123
	3.0578	1.087	1.159		4.1381	1.030	1.115
	3.0685	1.095	1.160		4.1515	1.036	1.117
	3.0875	1.090	1.162		4.1646	1.029	1.110
	3.1032	1.073	1.153		4.1774	1.022	1.100
	3.1175	1.099	1.158		4.1898	1.010	1.084
Jun. 11/12	3.1504	1.110	1.176		4.2109	0.990	1.060
Jun. 20/21	3.1040	1.096	1.146		4.2200	0.950	1.029
	3.1295	1.094	1.155		4.2336	0.931	0.999
May 2/3	3.4438	1.075	1.133	May 21/22	4.4205	0.5002	0.5002
May 20/21	3.586	1.073	1.142		4.4330	0.4834	0.4704
Jun. 7/8	3.5722	1.081	1.161		4.4467	0.4654	0.4683
	3.585	1.093	1.160		4.4675	0.4586	0.4500
	3.5976	1.086	1.160	Jun. 8/9	4.4586	0.4586	0.4556
	3.6229	1.093	1.145		4.4686	0.4482	0.4385
	3.6372	1.072	1.155		4.4753	0.4477	0.4421