

PUBLICATIONS OF
THE DAVID DUNLAP OBSERVATORY
UNIVERSITY OF TORONTO

VOLUME I

NUMBER 10

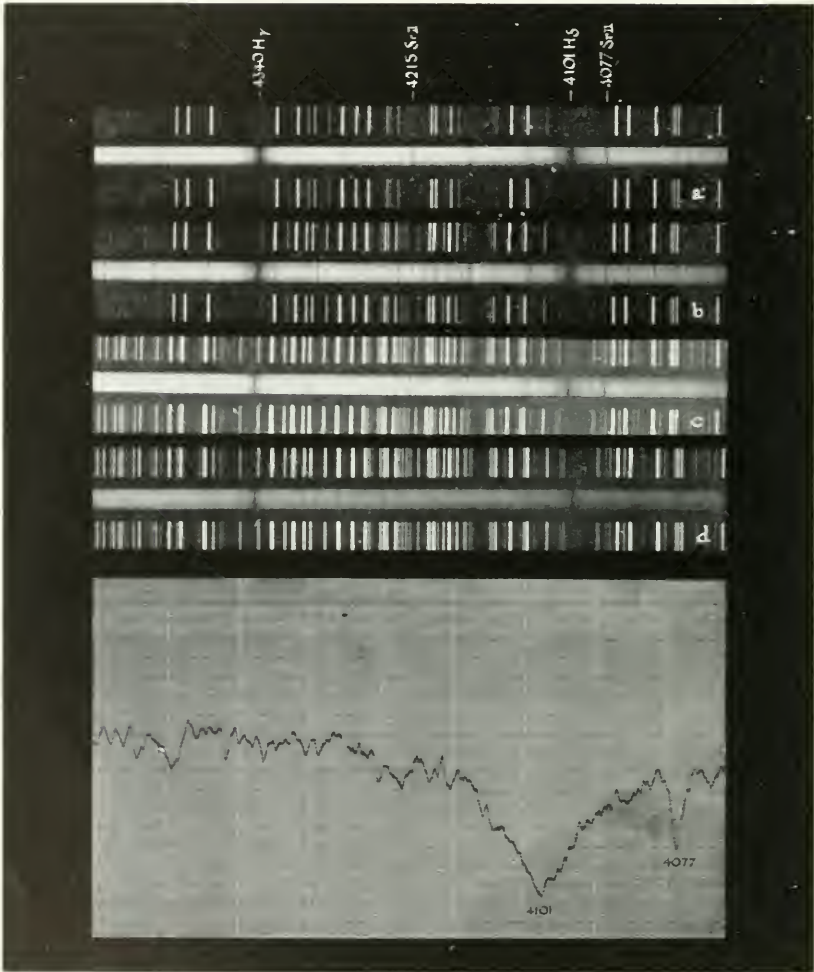
THE SPECTRA OF PECULIAR STRONTIUM STARS

BY

A. F. BUNKER

1941
THE UNIVERSITY OF TORONTO PRESS
TORONTO, CANADA

PLATE XXVII



Typical Spectra of Peculiar Strontium Stars.

The enlargements shown are, *a*) ι Cass A5p; *b*) β CorB F0p; *c*) γ EquI F0p; and *d*) the normal star σ Boot F0.

The scale of the microphotometer tracing of the spectral region of ι Cass near H δ and Sr II 4077 is 3.3 times that of *a*). It is a 0.4 reduction of the original tracing.

THE SPECTRA OF PECULIAR STRONTIUM STARS*

By A. F. BUNKER

(With Plate XXVII)

ABSTRACT

The equivalent widths of the spectral lines of seven peculiar strontium stars (A2p-F0p) and three comparison stars have been measured. Curves of growth have been constructed and values of $\log X$, the optical depth, and turbulence found. The abundances of Sr II atoms in the lower states have been found. The ratio of the $\log X$'s of the peculiar stars and comparison stars has been plotted against excitation potential to determine the differential excitation temperature. In general, the peculiar stars were found to be cooler than the comparison stars, while the degrees of ionization were nearly the same.

THE theory of equivalent widths developed by Menzel¹ and the method employed by Goldberg² of determining the absolute abundances of elements offer a means of studying the spectra of the stars with abnormally strong ionized strontium lines. By constructing curves of growth, and fitting these to the theoretically determined curves, the optical depth, $\log X$, of any line can be found. A study of these values should reveal whether atmospheric conditions are abnormal, or if there is simply an abnormal abundance of strontium atoms. In this paper the condition of temperature is compared by the method used by Russell.³

OBSERVATIONAL MATERIAL

In the present program, the seven peculiar strontium stars, Boss 3506 A2p, Boss 2443 A3p, ι Cassiopeiae A5p, γ Equulei F0p, β Coronae Borealis F0p, θ^2 Tauri F0, and τ Cygni F0, and the comparison stars β Trianguli A5, γ Bootis A5, and σ Bootis F0, were studied.

The stars chosen for the comparison are as nearly identical to the peculiar stars in spectral type, absolute magnitude and line width as could be found within convenient reach of the 74-inch telescope with contrast slow plates. The data concerning these stars are given in Table I.

*A paper submitted in partial fulfilment of the requirements of the degree of Master of Arts at the University of Toronto.

TABLE I

Name	H.D.	Type	M_t	M_s	T	Line width
Boss 3506	118022	A2p	1.2	1.4	9200	11
Boss 2443	78209	A3p		1.8	7000	10
ι Cass	15089	A5p	1.3	1.0	8100	15
γ Equi	201601	F0p	1.5	1.1	7000	10
β CorB	137909	F0p	1.3	0.9	7200	9
θ^2 Taur	28319	F0	0.7	1.3	8400	20
τ Cygn	202444	F0	2.3	2.4	6900	17
γ Boot	127762	A5	2.0	2.0	7800	20
β Tria	13161	A5	-1.3	1.4	8600	19
σ Boot	128167	F0	3.5	3.2	8200	9

The columns give Henry Draper number, the spectral types, the trigonometric and spectroscopic magnitudes from Schlesinger,⁴ the colour temperature corrected by Kuiper⁵ to represent the effective temperature, and a measure of the line width determined by averaging the values of $\Delta\lambda/(1-r_c)$ for several unblended lines. Here r_c is the residual intensity at the centre of the line.

Plates were taken at the David Dunlap Observatory with the one-prism spectrograph attached to the 74-inch telescope. Only the 25-inch camera was used, giving a dispersion of 33A per millimetre at $H\gamma$. Most spectra were taken on Eastman process plates, while a few were taken on Eastman 33. The 11-spot tube-sensitometer of the observatory was used with a blue filter to impress the sensitization spots on the emulsions. The plates were tray-developed for eight minutes in the routine manner of the observatory.

METHODS OF REDUCTION

Tracings of the spectra were made by the Beals⁶ type micro-photometer constructed at the David Dunlap Observatory. With this machine the galvanometer deflection is recorded on a fogged background cut by regularly spaced unfogged lines parallel to the length of the paper. The fogging light was extinguished at every half-millimetre of the plate, leaving an unfogged reference line. Tracings were made using a magnification of 50, at the second highest speed, requiring about 15 minutes to record from $\lambda 3950$ to $\lambda 4600$. The circuits were left closed for about an hour previous to a run to insure constancy of the zero point and sensitivity, thus increasing the accuracy and ease of reduction. The characteristic curve of each plate was determined in the usual way. Much tedious

reduction was eliminated by replotting the log I of the characteristic curve on a strip cut from the tracing. By placing this strip on the tracing, being sure that the lines of each were coincident, the log I of the continuous background and the centre of a line could be read directly. To measure the widths of lines, a scale was made and reduced photographically so that the distance between the half-millimetre reference lines was divided into fifty equal parts. The number of Angstrom units per division for each spectral region was computed.

The equivalent widths of the narrower absorption lines were computed by assuming, after other workers, that the lines can be considered as triangles. For lines strong enough for damping broadening to be effective, several points on the profile had to be measured, and the area found by a summation.

Two problems of equivalent width measures are the drawing of the continuous background, and the correction for blending. The continuous background can be drawn with a fair degree of confidence by following the rule of drawing it tangent to the tops of the lines in many-line spectra and through the plate grain in the cases of earlier type spectra.

The problem of blending was not solved in this work. Bad blends were either measured as a unit or discarded. In the cases of lesser blends the lines were reconstructed by noting the shape of other unblended lines. With the small dispersion and resolving power available, blending is a serious handicap, as most lines have some degree of blending. Lines blended with the hydrogen lines are difficult to evaluate, as Thackeray⁷ has shown that a weakening results if the profile of the blending line is used as the continuous background. When it seemed desirable to measure such lines to complete the multiplet, a value of the continuous background above the blending profile was used. The accuracy of such measurements is admittedly low.

An indication of the consistency of the measurements was obtained by applying Peter's formula for probable errors to fifteen consecutive lines of Boss 2443. Five measurements of each line were available. The probable error of the fifteen determinations of $\Delta\lambda$, the width of the spectral line at the continuous background, and r_c were averaged to give an average probable error.

TABLE II

λ	E.P.	Boss 3506		Boss 2443		ι Cass		γ Equul		β CorB	
		r_c	$W \log X$	r_c	$W \log X$	r_c	$W \log X$	r_c	$W \log X$	r_c	$W \log X$
4077	0.00	0.63	712	0.51	1080	0.56	1345	0.42	1925	0.50	1200
4161	2.93	0.76	288	0.77	282	0.81	187	0.66	429	0.68	423
4215	0.00	0.67	454	0.67	444	0.65	863	0.47	1110	0.63	465
4305	3.03	0.82	146	0.81	262	0.83	189	0.74	314	0.71	390
4005	1.55	0.77	178	0.54	547	0.82	163	0.73	316	0.57	498
4045	1.48	0.75	258	0.56	590	0.80	236	0.59	661	0.55	621
4052	3.35	0.81	154	0.83	130	0.88	286	0.73	248
4063	1.55	0.79	211	0.63	540	0.80	254	0.68	527	0.61	557
4067	2.82	0.82	168	0.64	450	0.845	236	0.74	330	0.66	453
4071	1.60	0.82	175	0.70	446	0.85	208	0.76	276	0.71	400
4132	1.60	0.70	366	0.65	504	0.77	290	0.65	476	0.50	618
4143	1.55	0.77	320	0.65	575	0.86	239	0.76	396	0.69	299
4147	1.48	0.67	72	0.87	117	0.87	885	0.73	258
4150	3.32
4154	2.82	0.70	338	0.85	181	0.8	0.70	351
4158	3.42	0.72	316	0.6	0.76	235	0.71	262
4176	3.38	0.77	177	0.70	354	1.0	0.71	248
4181	2.82	0.76	252	0.74	252	0.88	128	0.80	230	0.70	342
4202	1.48	0.82	191	0.73	352	0.90	55	0.80	204	0.74	285
4210	2.47	0.80	217	0.84	137	0.87	169	0.83	150	0.80	181
4222	2.41	0.85	136	0.83	157	0.92	76	0.87	97	0.85	126
4235	2.42	0.83	194	0.77	293	0.6	146	0.83	206	0.79	194
4260	2.39	0.79	249	0.71	348	0.9	149	0.69	426
4454	2.82	0.84	175	0.82	273	0.4	191	0.68	592	0.70	519

TABLE II—Continued

λ	E.P.	θ° Taur		τ Cygn		γ Boot		β Tria		σ Boot	
		r_c	$W' \log X$	r_c	$W' \log X$	r_c	$W' \log X$	r_c	$W' \log X$	r_c	$W' \log X$
4077	0.00	0.72	676	0.75	726	0.74	604	0.72	553	0.72	376
4161	2.93	0.89	127	0.85	168	0.88	122	0.87	109	0.86	94
4215	0.00	0.81	303	0.76	480	0.86	263	0.85	222	0.75	238
4305	3.03	0.80	168
4005	1.55	0.80	401	0.73	573	0.87	239	0.79	322	0.67	330
4045	1.48	0.78	415	0.66	675	0.81	236	0.77	333	0.59	449
4052	3.35	0.88	90
4063	1.55	0.79	359	0.73	561	0.85	257	0.80	261	0.70	305
4067	2.82	0.84	242	0.78	392	0.88	152	0.85	191	0.81	206
4071	1.60	0.81	320	0.78	394	0.84	266	0.83	234	0.77	234
4132	1.60	0.79	457	0.74	620	0.81	306	0.83	235	0.77	226
4143	1.55	0.82	335	0.74	501	0.84	253	0.83	303	0.75	246
4147	1.48	0.93	41
4150	3.32	0.85	208	0.90	104
4154	2.82	0.84	193	0.80	250	0.88	111	0.78	233
4158	3.42	0.81	125
4176	3.38	0.81	401	0.76	515	0.78	178
4181	2.82	0.79	195
4202	1.48	0.82	219	0.78	351	0.87	163	0.87	171	0.81	171
4210	2.47	0.89	112	0.83	140
4222	2.44	0.91	108	0.88	71
4235	2.42	0.88	148	0.81	287	0.91	122	0.81	171
4260	2.39	0.84	288	0.78	424	0.87	246	0.89	212	0.73	278
4454	2.82	0.84	326	0.80	519	0.85	317	0.91	284	0.79	307

$$\text{P.E.} = 0.845 \frac{\sum v}{n\sqrt{n}} = 0.076 \Sigma v$$

Av. P.E. for $\Delta\lambda = 0.11\text{\AA}$

Av. P.E. for $r_c = 0.69$ of 1 per cent.

These values do not, of course, give any idea of errors due to blending which is the greatest source of error, or any systematic errors.

It was originally intended that four plates of each star be taken and measured. Only one plate each of γ Equulei and τ Cygni was obtained and two of σ Bootis. In other cases, in which fewer than four measurements were made, plates were discarded because exposures were either too weak or too strong, or characteristic curves too poorly determined. In Table II the equivalent widths of the Sr II lines and the neutral iron lines used later in the temperature comparison are tabulated. The three columns give: r_c , the average value of the residual intensity at the centre of the line; W , the equivalent width expressed in milli-Angstroms determined by $W = \Delta\lambda(1 - r_c)/2$, and for the Fe I lines, $\log X$, the optical depth determined from the curves of growth.

For use in constructing the curves of growth, the values of $\log W/\lambda$ were computed for all lines. Between 60 and 150 lines per star were measured in the region $\lambda 3900 - \lambda 4600$. For identifying lines, wave-length measurements were made on six plates of different stars for about 120 lines. These were averaged to give the wave-lengths of the important lines. For other lines, the wave-lengths were found by direct interpolation on the tracing. Identification was determined by wave-length, presence of other members of multiplets, and multiplet intensities. Much valuable information was obtained from Miss Moore's multiplet tables.⁸

CONSTRUCTION OF CURVES OF GROWTH

The theoretical relation between the equivalent width of a spectral line and the number of atoms above the photosphere that are producing the line has been developed by Menzel.¹ Assuming a definite radiating surface surrounded by an atmosphere transparent to all wave-lengths, except those near an absorption line, the expression $r(\nu) = 1/(1 + Na_\nu)$ is adopted as an approximation of the ratio of the spectral intensity at a frequency ν inside the line to the intensity outside the line. The atomic absorption coefficient a_ν is given by

$$a_\nu = \frac{\pi \epsilon^2}{mc} f \left[\frac{1}{\sqrt{\pi}} \frac{c}{v \nu_0} e^{-(\nu - \nu_0)^2 c^2 / \nu_0^2 v^2} + \frac{\Gamma}{4\pi^2} \frac{1}{(\nu - \nu_0)^2} \right]$$

where f is the oscillator strength, v the root mean square kinetic velocity and Γ the damping constant of the atomic transition. The first term arises from the Doppler effect, while the second arises from the radiational or collisional damping.

The expression for the equivalent width, $\Delta\nu = \int_0^\infty \frac{Na_\nu}{1 + Na_\nu} d\nu$, has

been solved for three cases, when Na_ν is small, intermediate, and large. The resulting relations are

$$\log W/\lambda = 1/2 \log \pi + \log v_0/c + \log X_0$$

$$\log W/\lambda = \log 2 + \log v_0/c - 1/2 \log 0.434 + 1/2 \log \log X_0$$

$$\log W/\lambda = 1/4 \log \pi - \log 2 + \log v_0/c + 1/2 \log \Gamma/\nu + 1/2 \log X_0$$

The X_0 introduced is the optical depth of the line. In quantum mechanical terms it is

$$X_0 = \frac{N_a}{b(T)} e^{-x/kT} \frac{1}{3\pi R} \frac{\pi \epsilon^2}{mc} \frac{c}{v_0} \phi S \frac{s}{\Sigma_s}$$

$e^{-x/kT}/b(T)$ gives the Boltzmann distribution of electrons in the various states of the atom. $\phi Ss/\Sigma_s$ expresses the spectroscopic strength of any line, ϕ being the square of the radial integral divided by $4l^2 - 1$ representing the strength of the transition, S the relative multiplet strength and s/Σ_s the strength of a line within a multiplet. The v_0 is the kinetic velocity of the atoms and equal to $1.289 \times 10^4 \sqrt{T/\mu}$. N_a is the number of atoms of one element in a given stage of ionization per square centimeter above the photosphere.

To determine the theoretical curve of growth for A2-F0 stars, it is necessary only to substitute the proper values in the three equations, plot $\log W/\lambda$ against $\log X_0$ and draw a smooth curve through the three sets of points. For these stars the assumed values are, $T=8000^{\circ}$, and $\mu=56$, since iron lines were used most frequently in determining the empirical curve of growth. The value $\Gamma/\nu=1.52 \times 10^{-6}$ which Menzel⁹ found to give the best fit in the case of the sun, was adopted. Using these values the equations reduce to:

$$\log W/\lambda = \log X_0 - 5.04$$

$$\log W/\lambda = 1/2 \log \log X_0 - 4.83$$

$$\log W/\lambda = 1/2 \log X_0 - 5.73.$$

For the construction of the actual curves of growth of the stars,

Russell's¹⁰ table of multiplet intensities was used as the main source of relative line strengths. The method of construction was that used by Allen¹¹ in making the curve of growth of the sun. It consists of plotting the $\log W/\lambda$ of a line as observed in the star against the logarithm of the strength of the line within the multiplet. After the lines of several multiplets have been plotted, each multiplet was moved horizontally as a unit. Guided by the slope of the multiplet and its height, the various multiplets were combined to form as smooth a curve as possible. The scatter of the points is considerable because of the blending effects, errors in measurement, and any irregularities in the multiplet intensities because of the failure of the *LS* coupling. This scatter and the relatively few multiplets made it necessary to seek other sources of material. A satisfactory source of spectroscopic data is contained in Allen's¹² tables of the equivalent widths in the solar spectrum. By using the curve of growth of the sun computed by Menzel,¹⁵ the value of $\log X_0$ can be read for each value of $\log W/\lambda$ from Allen's work. The X_0 contains the spectroscopic data of the line, the abundance of the element in the sun, and the Boltzmann factor. When the lines of one element are used, only the Boltzmann factor need be changed when applying the data to stars of different temperatures. The change can be effected by putting the desired temperature in the factor

$$e^{-\chi(1/k[1/T-1, T_0])}$$

For the temperature change $4500^\circ - 6500^\circ$ the correction to $\log X_0$ is simply 0.343χ . With this material additional lines were utilized which were unclassified or in weak multiplets. These lines were plotted and moved horizontally as a unit and combined with other multiplet lines.

In view of the small number of plotted points, usually defining only a portion of the curve of growth, it seemed inadvisable to draw a curve through the mean position of the points and accept that curve as the curve of growth of the star. A better method, the one finally adopted, is to use the plotted points to define a particular theoretical curve of growth and accept this as the true curve of growth of the star.

In a study of B-type stars, Goldberg¹⁴ found that many stars had curves of growth whose intermediate sections were higher than the theoretical one for a given temperature. Presumably this was the result of a turbulent motion of the atoms in the stellar atmospheres. This effect was introduced into the curve of growth equations by the turbulence factor $V = \log v' - \log v_0$, giving

$$\log W/\lambda = \log X_0 - 5.04 + V$$

$$\log W/\lambda = 1/2 \log \log X_0 - 4.81 + V$$

$$\log W/\lambda = 1/2 \log X_0 - 5.73 + V/2.$$

The method of selecting the proper curve of growth is, then, to plot several curves with different values of V and move the plotted observed points horizontally until the best fit is obtained with some computed turbulence curve. The curve was traced through the points and used as the curve of growth of the star. The curves and observed points are reproduced in Figure 1.

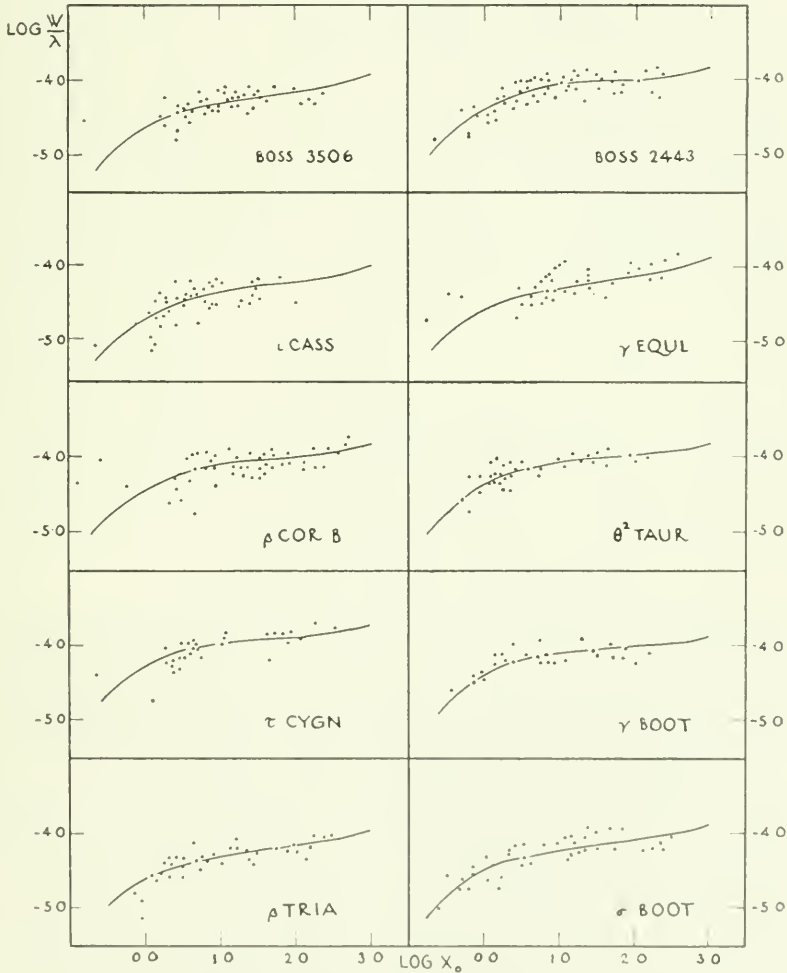


Figure 1—Curves of growth of peculiar and normal stars.

In choosing the proper curve, the turbulence in the stellar atmosphere is determined. In all cases, the values are positive and lay between 0.5 and 0.9. These have been plotted against a colour temperature corrected by Kuiper⁵ to represent the effective temperature of the star. A definite correlation between temperature and turbulence was found, and is shown in Figure 2. The turbulence is greater for lower temperatures. It is interesting to note that the opposite effect was found for the O and B stars.

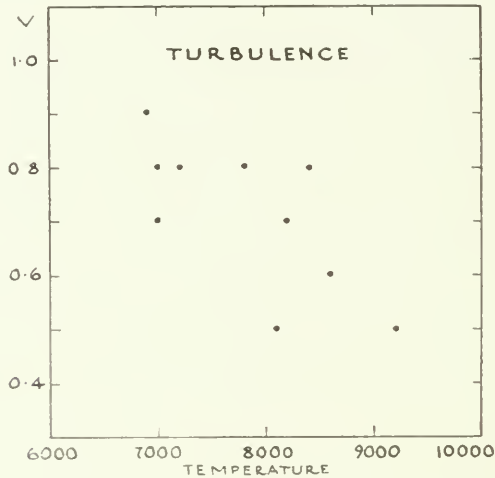


Figure 2—Correlation between turbulence and temperature.

THE CURVE OF GROWTH FOR SR II

Goldberg¹⁵ has shown how the absolute abundance of an element may be found when the curve of growth of the element and the absolute strengths of the lines are known. The same method is applied here to determine the abundance of ionized Sr atoms, but small changes have been made to meet the varied conditions.

Thus to determine the abundance of Sr II atoms, two things must be found: the value of $\phi S_s / \Sigma s$ for the transitions involved, and the form of the curve of growth, for which the value of Γ , the damping constant is required.

It has been possible to compute the absolute strengths and damping constants of Sr II lines through the generosity of Dr.

Leo Goldberg, who kindly made available values of ρ , the radial quantum integral for the transitions involved. Thus since

$$\phi = \rho^2 / (4l^2 - 1),$$

$$S = (2S + 1)(2L + 1)l(l - 1),$$

and $s/\Sigma s$ can be found from Russell's table of multiplet strengths, the necessary values can be found easily.

	λ	ρ	$\phi S \frac{s}{\Sigma s}$
5s-5p	4077	5.62	1.62
	4215	5.62	1.32
5p-6s	4305	1.59	0.53
	4161	1.59	0.23

The damping constant can be found knowing the strength of the line. Γ is equal to the sum of the reciprocal mean lives of the two levels involved. For the transition ${}^2S_{1,2} - {}^2P^0_{1,1/2}$, the reciprocal mean life of the term ${}^2S_{1,2}$ is zero, for it is the ground state. For the ${}^2P^0_{1,1/2}$ term only the transition to the ground state need be included in the summation. Thus for the line $\lambda 4077$

$$\Gamma = 3.15 \times 10^8.$$

For use with the curve of growth

$$\log \Gamma/\nu = -6.37.$$

With the value of $\log \Gamma/\nu$, the theoretical curve of growth of Sr II can be computed. When $\mu = 88$ and $T = 8,000^\circ$, the equations reduce to:

$$\log W/\lambda = \log X_0 - 5.14 + V$$

$$\log W/\lambda = 1/2 \log \log X_0 - 4.90 + V$$

$$\log W/\lambda = 1/2 \log X_0 - 6.05 + V/2.$$

The factor V was added since turbulence is present in the atmosphere. These theoretical curves were plotted, using several different values of V .

It follows from the previously used equation that for Sr II

$$\log X_0 = -11.452 + \log N_a/b(T) - \frac{5040}{T_{ex}} \chi - 1/2 \log T + \log \phi S \frac{s}{\Sigma s}$$

or if $\log X'_0 \equiv \log \phi S \frac{s}{\Sigma s} - \frac{5040}{T_{ex}} \chi$ and there is turbulence present,

$$\log X_0 = -11.452 + \log N_a/b(T) - 1/2 \log T - V + \log X'_0$$

since X_0 is inversely proportional to V .

In the paper cited the value $\Delta = \log X_0 - \log X_0'$ was introduced, which here becomes

$$\Delta = -11.452 + \log N_a/b(T) - 1/2 \log T - V.$$

This gives the means of determining the absolute abundance of atoms in certain states in the atmosphere.

$$\log N = \log \omega + \log N_a/b(T) - \frac{5040}{T_{ex}} \chi$$

or, by substitution, and letting $T = 8,000^\circ$

$$\log N = \log (2S+1)(2L+1) + 13.40 + \Delta + V - \frac{5040}{T_{ex}} \chi.$$

To evaluate this equation Δ and V must be found by a comparison of the theoretical and observed curves of growth. The observed curve is constructed by plotting the observed $\log W/\lambda$ of the Sr II lines against $\log X_0'$.

$$\text{The values of } \log X_0' = \log \phi S \frac{s}{\Sigma s} - \frac{5040}{T_{ex}} \chi$$

for the different lines can now be found since $\phi S s / \Sigma s$ has been computed. The value $T_{ex} = 7,000^\circ$ has been assumed as the excitation

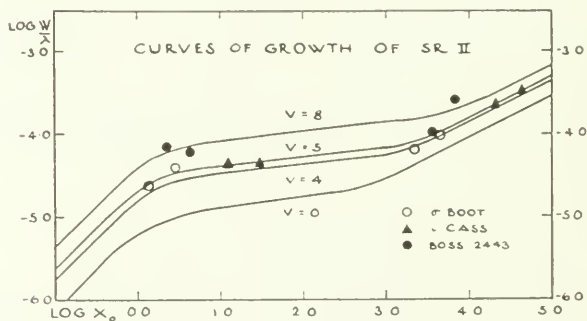


Fig. 3.

temperature which is lower than the effective or kinetic temperature. The values used are then

λ	$\log X_0'$
4077	1.62
4215	1.32
4305	-1.60
4161	-1.90

The observed curves are moved horizontally until the best fit is

obtained with some theoretical turbulence curve. The values Δ and V become known immediately as the best fit is found. With these evaluated, the log N for any state can be found. The determined values of the logarithm of the number of Sr II atoms per square centimetre above the photosphere are tabulated.

TABLE III

Star	Sp	Δ	$V_{Sr II}$	$\log N(^2S)$	$\log N(^2P^0)$	$V_{gen.}$
Boss 3506	A2p	2.4	0.5	16.6	14.9	0.5
Boss 2443	A3p	2.2	0.8	16.7	15.0	0.8
ι Cass	A5p	3.0	0.5	17.2	15.5	0.5
γ Equi	F0p	3.0	0.7	17.4	15.7	0.7
β CorB	F0p	2.3	0.8	16.8	15.1	0.8
θ^2 Taur	F0	2.1	0.6	16.4	14.7	0.8
τ Cygn	F0	2.3	0.6	16.6	14.9	0.9
β Tria	A6	2.1	0.5	16.3	14.6	0.6
γ Boot	A5	2.1	0.5	16.3	14.6	0.8
σ Boot	F0	2.0	0.4	16.1	14.4	0.7

The tabulated abundances of the Sr II atoms in the $^2P^0$ states are not independent of the temperature, and have been evaluated on the assumption of $T_{ex} = 7,000^\circ$. The temperatures of the individual stars differ from this value, making the $^2P^0$ column only an approximation.

Better fits between the observed equivalent widths and theoretical curves of growth would have been obtained had individual temperatures been used, but these values were not available. In Figure 3 the values of $\log W/\lambda$ of the Sr II lines of Boss 2443, ι Cass and σ Boot are shown plotted on theoretical curves.

COMPARISON OF EXCITATION TEMPERATURE AND IONIZATION

Having determined the numbers of Sr II atoms in the normal and peculiar stars, and the optical depths of lines of other elements, an attempt has been made to find out whether abnormal conditions exist in the atmospheres of the peculiar stars or whether one is forced to accept a difference in chemical composition. Two conditions, excitation temperature and electron pressure, can be compared by a comparison of the intensities of spectral lines in the two kinds of stars.

The excitation temperature may be compared by the method

used by Russell. The fundamental relation used is the Boltzmann factor:

$$N = \omega \frac{N_a}{b(T)} e^{-\chi/kt}$$

or logarithmically,

$$\log N = \log \omega + \log \frac{N_a}{b(T)} - \frac{5040}{T} \chi.$$

If similar equations are written for two stars and the differences taken, the following equation is obtained:

$$\log \frac{N}{N'} = c + 5040\chi(1/T' - 1/T).$$

The values of $\log N/N'$ can be found from the ratios of the $\log X_0$'s of the two stars determined from the curves of growth, since the $\log X_0$ contains the abundance, N . When the ratios of one element in the same state of ionization are plotted against the excitation potential of the lower level, the slope of the resulting line is a measure of the difference in temperature of the two stars. In this way, the peculiar stars were compared with the normal stars. Figure 4 shows four typical comparisons.

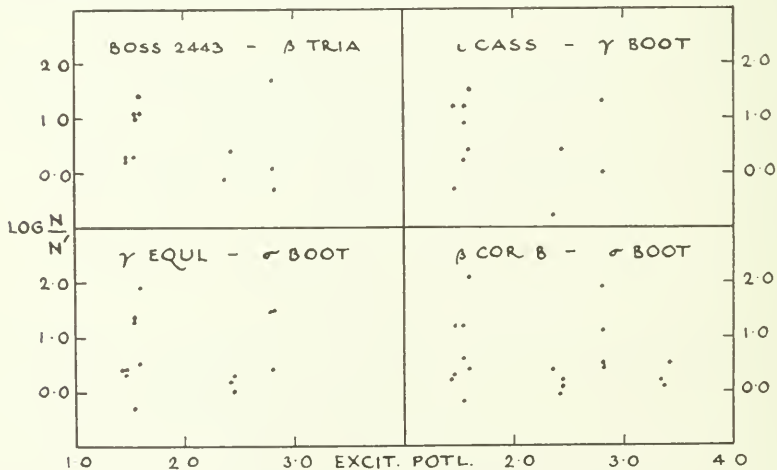


Figure 4—Comparison of excitation temperatures.

Unfortunately the number of suitable lines for comparison is very small. Only the neutral iron lines were sufficiently abundant to make a comparison profitable. Since only 20 iron lines were

used, after blends were excluded, the determined slopes are subject to some uncertainty. The slopes were used in view of this uncertainty, to tell which of two stars is the hotter and not to determine the exact difference in temperature. In this way, by numerous inter-comparisons, the stars used have been arranged in order of decreasing temperature, as follows: γ Boot, ι Cass, Boss 3506, σ Boot, β Tria, β CorB, τ Cygn, θ^2 Taur, γ Equi, and Boss 2443 being the coolest. The most noticeable characteristic of the order is that the peculiar stars are cooler than the comparison stars of the same spectral class. Thus ι Cass, A5p, is cooler than γ Boot, A5, and γ Equi, F0p, and β CorB, F0p, are cooler than σ Boot, F0. One exception is β Tria, which is cooler than σ Boot. This phenomenon of a temperature decrease in passing from normal stars to peculiar stars of the same spectral class might be a clue to the explanation of the abnormal abundance of Sr II atoms. Since the second stage of ionization is small, in A2-F0 stars, the rise in intensity of $\lambda 4077$ with advancing spectral type is due mainly to the change in electron concentration from the higher states to the ground level. The lower temperatures of the peculiar stars would then produce an increase in the strength of the Sr II lines of lower excitation potential.

For normal stars a decrease in temperature would infer a simple change to a later spectral type. For a star to have a lower temperature than the average for a spectral class, there must be some difference in the electron pressure so that the degree of ionization remains nearly the same. To test this, the electron pressures have been computed for as many stars as measurements of the K line of calcium II are available. Only on plates of β Tria, ι Cass, and θ^2 Taur were the K lines exposed strongly enough to be measured. Values for γ Boot, β CorB, and σ Boot were used from Hynek's¹⁶ paper on F-type spectra. The equivalent widths of Ca I 4227 were measured for all stars.

Adapting the Saha formula for use with the curve of growth, the equation becomes for Ca II, the primes referring to the ionized states:

$$\log P_e = \log X_0 - \log X_0' - 5.92 \frac{5040I}{T} + 5/2 \log T$$

since

$$\log \frac{x}{1-x} = \log \frac{N'}{N} = \log \frac{X_0'}{X_0} \frac{b'T}{b(T)} \frac{\phi S s / \Sigma s}{\phi' S' s' / \Sigma s'} = \log X_0' - \log X_0 - 0.28$$

which is believed to be a close approximation. The log X 's were found from the curves of growth and substituted in the equation. The values derived by this equation are:

	T	Pe
β Tria.	7,000	2.5×10^{-3}
	8,000	1.2×10^{-2}
	9,000	5.0×10^{-2}
ι Cass.	7,000	6.3×10^{-4}
	8,000	3.2×10^{-3}
	9,000	1.2×10^{-2}
γ Boot.	8,000	4.4×10^{-3}
	9,000	4.0×10^{-2}
	10,000	9.5×10^{-2}
θ^2 Taur.	7,000	2.0×10^{-4}
	8,000	1.0×10^{-3}
	9,000	4.0×10^{-3}
σ Boot.	6,000	2.4×10^{-4}
	7,000	1.9×10^{-3}
β CorB.	6,000	2.1×10^{-4}
	7,000	1.7×10^{-3}

From these values and assuming temperatures in accordance with the results of the temperature comparisons, the most likely conditions in the atmospheres of ι Cass and γ Boot are: ι Cass, $T = 8,000^\circ$, $Pe = 3.2 \times 10^{-3}$, γ Boot, $T = 9,000^\circ$, $Pe = 4 \times 10^{-2}$. If now, the temperature of γ Boot were reduced to $8,000^\circ$ while holding the degree of ionization constant, an electron pressure of 4.4×10^{-3} would result, which is approximately the pressure in ι Cass. This indicates the same degree of ionization in each star. In a similar manner, σ Boot and β CorB can be shown to have the same degree of ionization, yet a difference in temperature. Thus peculiar Sr II stars of a given spectral class have a lower temperature than normal stars but approximately the same degree of ionization.

β Tria, A6, which was noted previously to be an exception in that it was cooler than σ Boot, F0, has an electron pressure and degree of ionization characteristic of F0 stars.

It is regretted that a quantitative value of the thermal differences could not be extracted, for it would show definitely whether the difference is the sole cause of the peculiarity or merely a contributing cause. In a plot of temperature order against abundance of Sr II, most of the stars fall in a roughly defined curve, while ι Cass and γ Equal are much displaced toward greater abundances. This suggests that some peculiar stars might be produced by a lower

temperature, or an absolute magnitude effect, while others require some further explanation.

In determining the abundances of Sr II atoms, it was noted that for normal stars and a few peculiar ones of somewhat lower abundance, the turbulence value found by the Sr II atoms was less than the turbulence found by the general curve of growth. Since little is known of the cause of turbulence, it is difficult to see the real significance of this difference. It might be suggested that it is the result of a stratification of Sr II atoms at different layers in the atmosphere. This scheme, however, leads to many serious objections.

It is a pleasure to acknowledge my indebtedness to Dr. Leo Goldberg of Harvard College Observatory for making available quantum mechanical evaluations invaluable to the present work.

David Dunlap Observatory,
Richmond Hill, Ontario,
February, 1941.

REFERENCES

1. *Ap. J.*, v. 84, p. 462, 1936.
2. *Ap. J.*, v. 89, p. 623, 1939.
3. *Ap. J.*, v. 68, p. 9, 1928.
4. Schlesinger, *General Catalogue of Stellar Parallaxes*, 1935.
5. *Ap. J.*, v. 88, p. 469, 1938.
6. *M. N.*, v. 96, p. 730, 1936.
7. *Ap. J.*, v. 84, p. 433, 1936.
8. Moore, *A Multiplet Table of Astronomical Interest*.
9. *Ap. J.*, v. 84, p. 466, 1936.
10. *Ap. J.*, v. 83, p. 129, 1936.
11. *Mem. Commonwealth Solar Obs.*, v. 1, mem. 5, 1934.
12. *Mem. Commonwealth Solar Obs.*, v. 2, mem. 6, 1939.
13. *Op. cit.*
14. *Ap. J.*, v. 89, p. 623, 1939.
15. *Op. cit.*
16. *Ap. J.*, v. 82, p. 338, 1935.